ANALYSIS OF STORAGE/RELEASE SYSTEMS IN URBAN STORMWATER QUALITY MANAGEMENT

BY

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A DISSERTATION PRESENTED TO THE GRADUATE COUNCIL OF
THE UNIVERSITY OF FLORIDA
IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE
DEGREE OF DOCTOR OF PHILOSOPHY

UNIVERSITY OF FLORIDA

1982

To My Mother, Father,
and Brothers, David, Michael,
Richard, Kenneth, and Patrick,

and

My Wife, Aulaire,
and Children, Jeffrey and Stephanie

ACKNOWLEDGEMENTS

I have benefited from the advice, knowledge, and friendship of many individuals during the course of this study and my academic career at the University of Florida. I am especially indebted to my doctoral committee chairman and mentor, Dr. James P. Heaney, who expended considerable effort to direct this dissertation and my general course of study. Dr. Heaney's unique outlook on environmental problems has certainly influenced my own. Equally appreciated is the assistance, advice, and encouragement provided by Dr. Wayne C. Huber. Dr. Huber's mastery of many areas in mathematics and engineering has been particularly valuable.

Special thanks are due to Dr. Herbert A. Bevis, Dr. Bryan E. Melton, Dr. Gary D. Lynne, Dr. Barry A. Benedict, and Dr. W. Brian Arbuckle for their assistance and review of this dissertation. I am also grateful to Mr. Robert Dickinson and the many other colleagues and friends who aided and encouraged me over the last several years.

The persistence and professional typing of Mrs. Dibbie Dunnam, Mrs. Peggy Paschall, and Mrs. Wendy Stafford contibuted greatly to the completion of this study. I am deeply grateful to them for their efforts under difficult circumstances.

Some of the work presented here was supported by the Municipal Environmental Research Laboratory, United States Environmental Protection Agency through Project CR-805664.

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Abstract of Dissertation Presented to the Graduate Council
of the University of Florida in Partial Fulfillment of the
Requirements for the Degree of Doctor of Philosophy

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December, 1982

Chairman: James P. Heaney
Major Department: Environmental Engineering Sciences

Urban stormwater runoff acts as a transport medium for a variety of wastes. This phenomenon can cause water quality problems and, as a result, corrective measures sometimes are required. Storage/release systems, consisting of a storage basin and a release mechanism, are often used for this purpose. Unfortunately, there are few data to assess their performance in this role and the techniques currently used to design and analyze these systems are generally inadequate. A comprehensive and usable approach, reflecting the dynamic aspects of this problem, is presented in order to fill this need.

This dissertation develops and presents the necessary theories and evaluation techniques to assess the long-term performance of storage/release systems, the use of the production function to summarize system performance, and several techniques to determine the most cost-effective designs. The focus is on long-term analyses rather than "design storm" or single-event approaches. Several available computer simulation and statistical models capable of evaluating long-term performance are discussed. However, these models are either too inflexible or simple to be generally applicable. To meet the need for a more sophisticated and flexible model, a computer simulator known as the Storm Water Management Model Storage/Treatment Block is introduced. The production function, a concept taken from economic theory, is used to summarize the relationship between long-term performance and the characteristics of the system. The properties of production functions, as well as several possible mathematical representations, are also discussed. Simple analytical and graphical optimization techniques, which require the information contained in the production function, are presented and developed for use in the determination of cost-effective designs.

A hypothetical case study is used to demonstrate many of the techniques and procedures, illustrate the generally expected results, and suggest an overall methodology for analyzing storage/release systems. Emphasis is also placed on the results produced by the Storage/Treatment Block and its sensitivity to changes in the assumptions governing system behavior.

CHAPTER 1

INTRODUCTION

Traditionally, urban stormwater has been viewed as a flooding and drainage problem. Urban areas, by their nature, produce more runoff than non-urban areas for a given storm event. To compound the problem, urban areas more expeditiously remove stormwater because the roughness of the land surface has been reduced and natural storage areas have been eliminated. The damages incurred by uncontrolled stormwater runoff can be impressive, dramatic, and disastrous. The need for solutions to urban drainage and flooding problems is and has been apparent.

Not so apparent is the role urban stormwater plays as a transport medium for pollutants. Undoubtedly, urban stormwater runoff is contaminated; but is it a water quality problem? It was not until the early 1960's that this question was addressed at all. The first concern on a national scale came with the Water Quality Act of 1965 (PL 89-234) which provided some assistance to state and local authorities in developing controls for stormwater discharges (including combined sewer overflows). The Federal Water Pollution Control Act Amendments of 1972 (PL 92-500), however, marked the beginning of intense national concern over the quality of the nation's waters and, as part of that concern, the following goals were established:

1) The discharge of pollutants into navigable waters is to be eliminated by 1985.

2) Navigable waters should be of sufficient quality by July 1, 1983, to protect aquatic life, wildlife, and recreation.

3) Areawide water quality management planning processes should be developed and utilized in order to meet and maintain the previously stated goals.

Other provisions included funds for research and aid for the implementation of management plans. The commitment mandated by PL 92-500 is immense and was probably a product of the general thinking of the early 1970's embodied in the phrase "if we can send men to the moon, we ought to be able to. . . ." The goals of PL 92-500 may be more difficult and costly to achieve. This act specifically recognizes nonpoint pollution sources as potentially damaging and Environmental Protection Agency (EPA) guidelines called for "an analysis of the magnitude of existing and anticipated urban stormwater problems" (p. 3-62, U.S. Environmental Protection Agency, 1976).

The tone and goals of PL 92-500 had a dramatic effect on the way in which urban stormwater was studied and analyzed. This act and subsequent EPA guidelines on its interpretation tended to define urban stormwater as a "problem" before enough field data and analysis had been compiled to categorize it as such. This dampened the importance of subsequent monitoring work and delayed answering the essential question: Is urban stormwater a water quality problem? The Clean Water Act of 1977 (PL 95-217) responded to the unanswered questions and overwhelming control cost estimates by suspending Federal funding for the treatment and control of stormwater discharges from separate sewer systems. The EPA also responded to the dearth of solid information and data by creating the Nationwide Urban Runoff Program (NURP) in 1978. This program is collecting data from twenty-eight planning efforts around the United States and providing a coordinating body through which administrative and technical support is provided to assure consistency and maximize the effectiveness of the effort.

When is urban stormwater a water quality problem? There are no obvious guidelines. It can probably be said that urban stormwater is a problem when it impairs or denies beneficial uses of a receiving water. However, all this statement accomplishes is to shift the burden of definition from "problem" to "beneficial use." Beneficial to whom or what? By whose perception? There are no absolutes. Regulatory agencies have assumed some of the specification of beneficial uses through water quality criteria, effluent standards, and the classification of receiving waters. Almost invariably, though, the substantiation for these regulations is weak or nonexistent. Often, legislation is written in such a way to make it very difficult to interpret.

The best way to determine the existence of an urban stormwater quality problem is through a combination of intense local monitoring and analysis and informed public forums to determine local perceptions. The process must be localized because the potential for a problem is highly site specific. Local factors influencing this determination are

1) climatology and hydrology;
2) characteristics of the receiving water;
3) the level of beneficial use desired; and
4) attitudes, resources, and values.

The process is partially subjective; but in a democratic society, the need to address this question must ultimately come from the populace and its ranking of urban stormwater problems in national and local priorities.

After an urban stormwater quality problem is identified, concern is directed towards providing an effective means of control. The use of storage/release systems has been the prominent means of control (Lager et al., 1977). Regulatory pressures are further solidifying the use of storage/release systems for this purpose. Storage/release systems provide storage to capture a portion of the highly variable stormwater flows and pollutant loads, detention capability to allow pollutant removal to proceed, and a release mechanism through which the storage capacity can be recharged and the contents released in a more controlled fashion. Storage/release systems are not limited to structures such as detention basins. In fact, this type of system is encountered by stormwater in the urban environment at several points and in several forms. Many have some potential to abate a stormwater quality problem. Among the storage/release systems prevalent in the urban environment are

1) rooftops,
2) parking lots (porous and nonporous),
3) catch basins (including the percolating type),
4) natural depressions and impoundments,
5) soil storage and percolation,
6) sewer systems, and/or
7) manmade detention basins.

The focus of this study is on the basic theory and analytical techniques useful to the analysis and design of storage/release systems employed in urban stormwater quality management. Nevertheless, it should be emphasized that the techniques and theory reviewed and formulated herein are applicable to a wide range of water quality management problems in which storage/release systems can play a role. This thesis emphasizes urban stormwater, primarily because the developmental work was so imbued; however, the generality of the work should not be lost.

The analysis of urban stormwater storage/release systems relies on long-term data records (i.e., many years). Long-term data can be in time series form or summarized by statistical parameters. Quantity control facilities have long been designed by a statistically based method known as the "design event." A design event is defined as a runoff event or storm with a specific duration, depth, and return period by which a facility design is based. The use of a design event is a widely accepted method of designing storage/release facilities and it is often mandated by statutory requirements. Unfortunately, the underlying reasoning behind the selection of a design event has been forgotten by many in the field of urban stormwater management. Figure 1-1 depicts the costs of constructing and operating a hypothetical storage/release system to handle specific design events and the damages incurred by any subsequent runoff events. The damages could be attributed to stormwater quality and/or quantity. Assuming the damages could be quantified in monetary terms, the rational analyst would select the event inflicting the minimum total cost (damages plus the control costs) on society (James and Lee, 1971). An alternative approach presents the reduction in damages as a "benefit" and Figure 1-1 yields to the benefit-cost curves shown in Figure 1-2. In this case, the point where the net benefits (total benefits less costs) are maximized is selected as the design event. In either case, the answer is the same. Of course, there is considerable analysis behind the development of the cost and damage or benefit information over such a wide variety of conditions and, in fact, it is usually very difficult to place a monetary value on damages

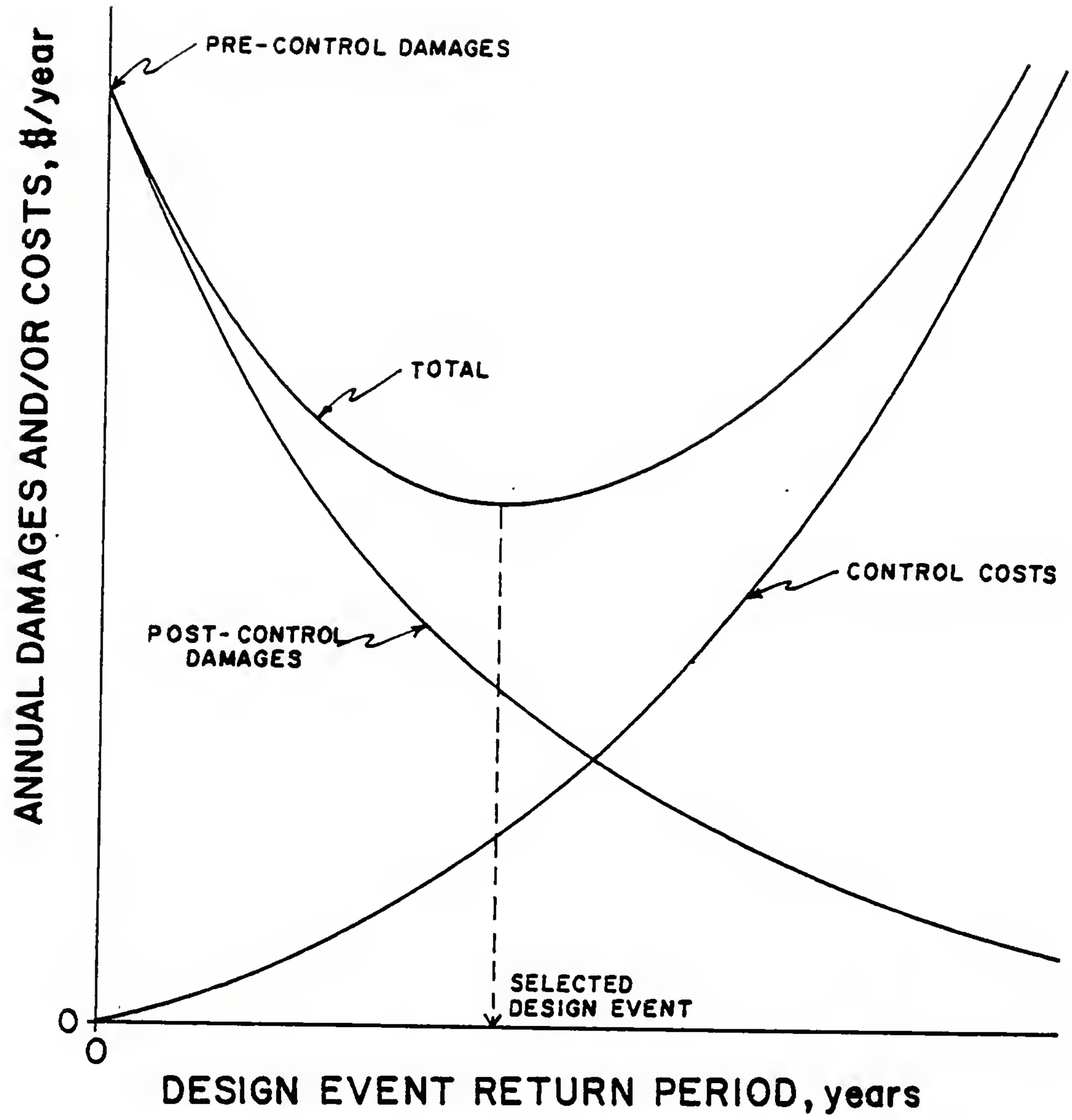


Figure 1-1. Design Event Minimizing the Sum of Control Costs and Damages.

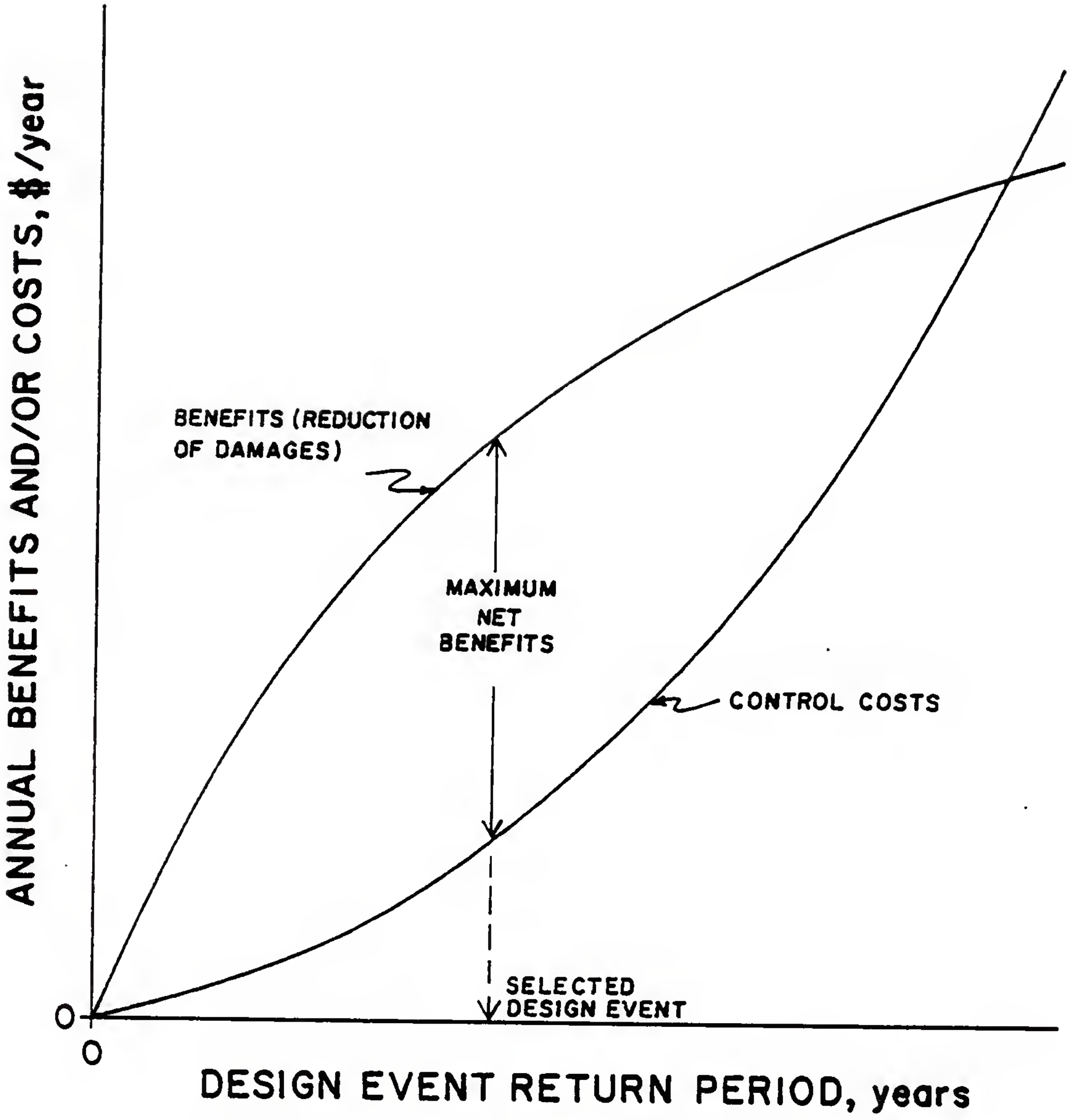


Figure 1-2. Design Event Maximizing Net Benefits.

or benefits (see Chapter 6). Nevertheless, the simple logic behind the selection of a design event appears to have been subdued by regulatory and technical dogma. A review of several manuals intended for practicing engineers and analysts produces very few references to this fundamental principle (Lager and Smith, 1974; Lager et al., 1977; Municipal Environmental Research Laboratory, 1976; American Public Works Association, 1981).

The design event method is widely used and attractive because of the relative ease with which it is applied once the design event is established. The fact that most regulations and statutes require storage/release designs based on particular design storms solidifies its popularity. The method is fundamentally sound and, given the technical atmosphere from which it evolved (i.e., pre-computer age), it has performed admirably in quantity control applications. Nevertheless, there are serious drawbacks in quantity and quality control applications:

1) Pre-event design conditions (e.g., antecedent dry period, previous storm characteristics, storage/release system status) are difficult to assess because of the nearly random nature of of storm events.
2) Additional uncertainties exist when developing a design storm for water quality purposes. This is because of the lack of historical data, the inability to measure benefits (or damages) associated with different levels of water quality, the unreliability of water quality measurements, and the unclear relationship between stormwater flows and pollutant loads (Nix et al., 1981).
3) "Design" conditions in the receiving water are also necessary.

The advent of computers has opened the door to the analysis of the behavior of storage/release systems over a long period and, thus, reduces the need for the design event. The proliferation of efficient computers with large memories allows a return to the original intent of the design event concept which is the analysis of a wide range of storage/release system designs in order to locate the most effective alternative.

Several mathematical models of the urban hydrologic cycle are available. These include nomographic or desk-top procedures based on computer results, statistical methods, or computer simulators. Unfortunately, mathematical models are not perfect representations of the physical world and this is why stormwater quantity and quality monitoring is important. Monitoring is vital to our understanding of the physical, chemical, and biological processes involved in the urban hydrologic cycle, and, more specifically, the behavior of storage/release systems. However, monitoring alone cannot provide the long-term information needed to analyze and design these systems under the highly variable conditions presented by stormwater flows and pollutant loads. Besides being very expensive, monitoring does not directly provide, a priori, the information needed to characterize the behavior of a wide range of designs. Properly constructed models can provide this predictive information. Analyses of future monitoring data will improve the models and, concurrently, the models might be able to provide guidance for the monitoring programs.

Unfortunately, many analysts are distrustful of or intimidated by models, especially computer simulators. The distrust probably comes from the extensive use of models to analyze stormwater problems in recent years at the expense of adequate monitoring. Models alone are not up to the task. Modeling and monitoring should be considered complementary, not mutually exclusive. Consider the statement by Sonnen (1980):

> Mathematical models are relevant to a consideration of data or information needs, because ostensibly the intent of these models is to provide a means by which quality prediction could obviate the need for monitoring. (p. 33)

With this commonly held view of mathematical models, it is little wonder that their use so often ends in disappointment. Mathematical models are imperfect because our knowledge of the underlying principles is imperfect. While every effort should be made to increase this understanding, properly calibrated and verified models are certainly useful and capable of providing valuable insights.

The overall objective of this thesis is to construct a comprehensive framework for analyzing storage/release systems used in urban stormwater quality management in light of the need for long-term analyses and the present condition of the data base. The specific objectives are to

1) review basic theory and available analytical techniques;
2) develop and present a flexible storage/release computer simulator;
3) explore the use of production theory and the production function in characterizing the performance of storage/release systems;
4) review simple analytical optimization techniques and develop a graphical optimization technique whereby the production information can be used to design cost-effective systems; and

5) synthesize the previous steps through the use of a case study.

To this end, Chapter 2 discusses the basic theory and available analytical techniques. Chapter 3 explores a recently developed statistical technique for analyzing storage/release systems (and urban stormwater in general). This technique serves as an analytical prelude to the more sophisticated computer simulator presented in Chapter 4. This simulator was developed for this thesis and as part of a larger package of urban stormwater simulation modules. Chapter 5 investigates the use of production functions to represent the performance of storage/release systems as urban stormwater quality control devices. Chapter 6 reviews simple analytical optimization techniques and presents a flexible, effective graphical method for guiding the cost-effective design and operation of storage/release systems. Chapter 7 provides a case study through which some observations can be made and the merits or demerits of many of the techniques explored. This application will also point out the integrated nature of the techniques and suggest a general methodology. Chapter 8 summarizes the main points and presents concluding statements.

CHAPTER 2

STORAGE/RELEASE SYSTEMS IN URBAN STORMWATER QUALITY MANAGEMENT

Introduction

Storage/release systems play an important role in urban stormwater quality management and, in fact, are the most commonly used control technology (Lager et al., 1977; Finnemore, 1982). The systems are used in a variety of situations in the urban environment. They are also predominant in a number of agricultural applications (Loehr, 1974; Meta Systems, Inc., 1979; Nix and Melton, 1979). Among potential urban stormwater applications are the following:

1) control of combined sewer overflows;
2) sediment control from disturbed sites (e.g., construction); and/or
3) control of pollutants and debris from industrial, commercial, residential, and other urban land areas.

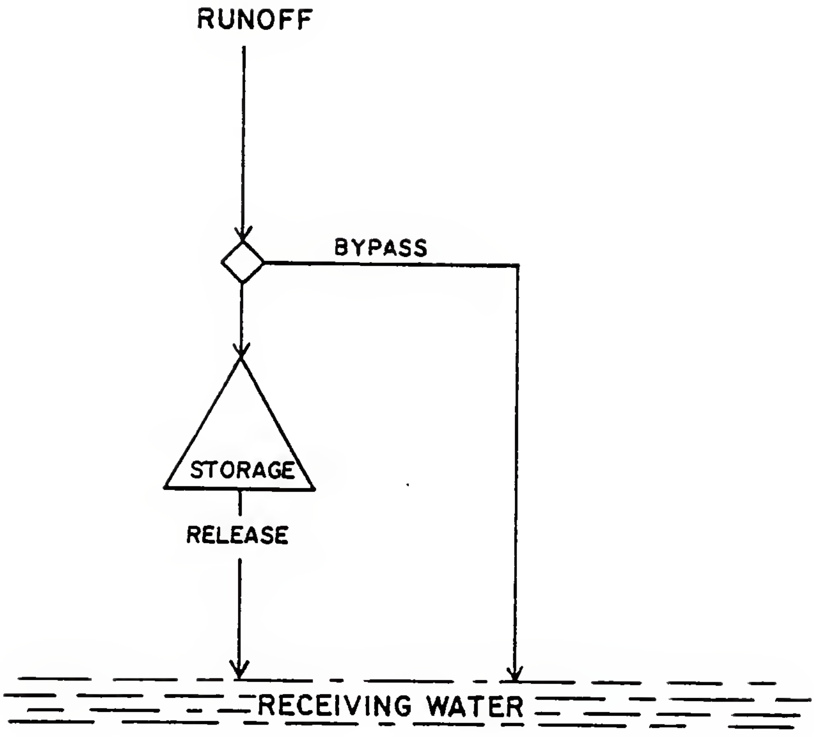
These systems are constructed in several different manners. Some are covered or uncovered concrete basins; others are earthen (lined or unlined) basins. Several are designed to serve aesthetic purposes as well as to provide pollution control. In some cases, existing ponds or lakes are used to provide the system. The actual form is dependent on site conditions, the consideration of all system purposes, economic factors, and public desires.

Urban stormwater storage/release systems have primarily been designed through the use of the design storm. As revealed in Chapter 1, this time-honored method is probably not equipped to handle the design of stormwater quality management systems. This chapter explores the basic configurations and theories needed to examine the problem in the context of long-term evaluation. The theoretical exposition provides the necessary foundation for the review and development of a series of evaluation techniques.

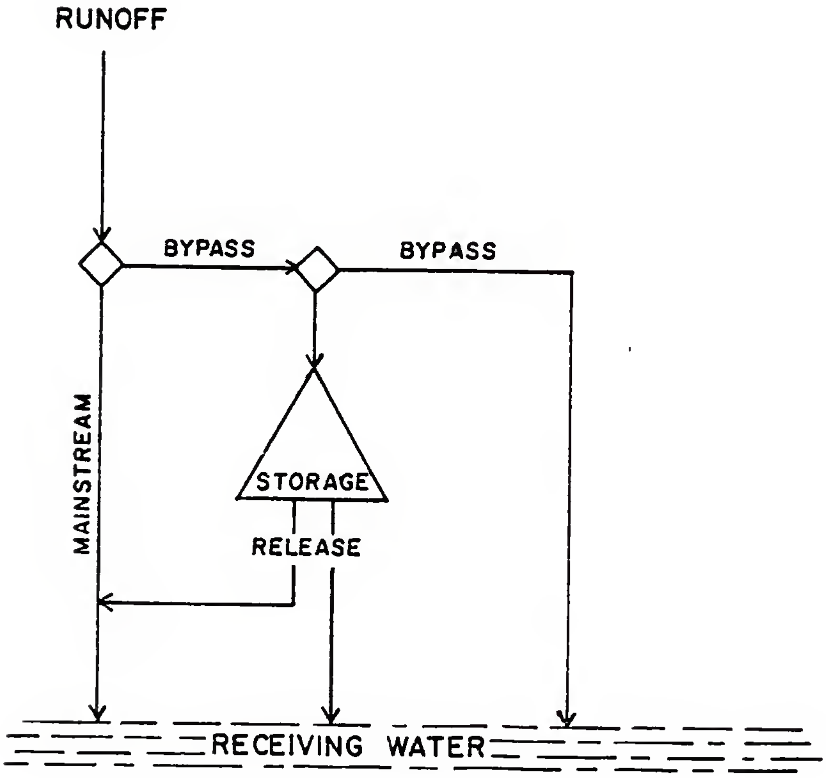
Basic Configurations

The basic storage/release system in the urban stormwater flow scheme receives a highly variable input with a large random component. This fact makes the analysis of these systems much more complex than those operating under steady state or quasi-steady state conditions (e.g., sewage treatment facilities). Not only are these systems receiving fluctuating stormwater flows but also they accept highly variable pollutant loads which further complicate the situation.

There are essentially two storage/release system configurations: 1) in-line and 2) off-line. The in-line configuration is shown in Figure 2-1(a). In this arrangement the flow is first directed to the storage unit, which either accepts the flow or bypasses all or part of it. Flows entering the storage unit are drained or released in some restrained fashion (unrestrained release implies no storage). The decision to accept or bypass flows is a function of the role of the system. It is sometimes advantageous to capture the early portions of flow events and bypass all other flows after the unit is filled. This will be called the bypass mode. In some cases, it may be useful to allow



(a) In-Line System



(b) Off-Line System

Figure 2-1. Basic Storage/Release System Configurations.

all flows to enter the unit and design a release mechanism to handle a wide range of flows and provide the necessary relief.

The off-line configuration, shown in Figure 2-1(b), only accepts flows from the mainstream after a predetermined capacity has been exceeded. The term "stream" is loosely defined to mean any movement of stormwater. The excess is routed to the storage unit which may accept or bypass part or all of the flow in the manner discussed above. The flows entering storage are released in some restrained manner but the flows returned to the mainstream must not exceed its capacity.

Theoretical Representations

Regardless of the configuration, the manner in which flows and/or pollutants are handled by the storage/release system can be conceptualized with the same basic theoretical framework. This framework will be used to establish a background for the performance evaluation techniques discussed later in this chapter and in Chapters 3 and 4.

Storage Equation

Storage/release systems attenuate input flows and pollutant loads by providing a repository from which controlled releases are made. The flow continuity relationship for a storage/release system is given by (Linsley et al., 1975)

$$\frac{dV(t)}{dt} = I(t) - O(t) \tag{2-1}$$

where $V(t)$ = volume of water in storage, L^3,

$I(t)$ = inflow rate to storage, L^3T^{-1},

$O(t)$ = outflow rate from storage, L^3T^{-1}, and

t = time, T.

Theoretically, a storage unit could become so large that it would capture all inflows and release them at a constant rate. Such a unit would totally "equalize" the flows. Obviously, physical and economic considerations often prohibit such systems. However, some level of equalization is often desirable in a number of applications. Some examples include

1) the improvement of stormwater treatment unit efficiency by minimizing peak flows;
2) the construction of smaller treatment facilities by reducing the need to design for peak flows; and
3) the attenuation of shock loads from combined sewer overflows or urban runoff on receiving waters.

Naturally, as the storage unit becomes smaller, the equalization effect is reduced.

The storage equation is a simple, elegant representation of the physical system. However, the equation is difficult to solve for all but the most trivial functions of I(t) and O(t). For example, assume that the outflow is a function of the fluid volume in the storage basin, i.e.,

$$O(t) = f[V(t)] \tag{2-2}$$

Equation 2-1 may then be written as

$$\frac{dV}{dt} = I(t) - f[V(t)] \tag{2-3}$$

Assume that a power function governs the relationship between outflow and volume, i.e.,

$$O(t) = a[V(t)-V_o]^b \tag{2-4}$$

where a, b = coefficients, and

V_o = volume of stored water at which outflow begins, L^3.

Substituting equation 2-4 into equation 2-3 yields

$$\frac{dV}{dt} = I(t) - a[V(t)-V_o]^b \tag{2-5}$$

Equation 2-5 is very difficult to solve except under very restrictive situations (Dooge, 1973). A few of the simpler cases are discussed below.

When b = 1, the relationship between 0(t) and V(t) is linear and equation 2-5 becomes

$$\frac{dV}{dt} + aV(t) = I(t) + aV_o \tag{2-6}$$

Equation 2-6 is a simple linear first-order differential equation for which the solution is (Ross, 1964)

$$V(t) = \exp(-\int_0^t a\ dt)\{\int_0^t [I(t) + aV_o]\ \exp(\int_0^t a\ dt)dt + V(0)\} \tag{2-7}$$

or

$$V(t) = \exp(-at)\{\int_0^t [I(t) + aV_o]\exp(at)dt + V(0)\} \tag{2-8}$$

where V(0) = initial volume of stored water, L^3.

The obvious possible drawback in equation 2-8 is the inflow function, I(t). Only rather trivial functions allow the complete solution of equation 2-8.

When b = 0, the outflow is constant, i.e., 0(t) = a, and equation 2-5 becomes

$$\frac{dV}{dt} = I(t) - a \tag{2-9}$$

for which the solution is

$$V(t) = \int_0^t (I(t) - a)dt \tag{2-10}$$

This solution is useful for pumped outflow. The only restriction, again, is the funciton I(t) and its ability to be integrated.

Reactor Theory and Reaction Kinetics

Reactor theory is a useful tool in the fields of chemistry and chemical engineering. This theoretical framework also provides a good structure for analyzing the behavior of pollutants (i.e., reactants) in storage units (Metcalf and Eddy, Inc., 1972; Rich, 1973; Medina, 1976; Medina et al., 1981b). The application is not necessarily limited to chemical reactions as several physical and biological processes can also be characterized by the mathematical representations found in reaction kinetics. Reactors are often segregated into three ideal reactor types: the batch reactor and two flow reactors, completely mixed and plug flow. Each type of reactor essentially routes fluid elements (along with any associated reactants, pollutants, etc.) in a different manner. Whereas the storage equation describes the continuity of mass for a storage unit, it does not provide this routing information. The flow regime is particularly important in determining the ability of the storage unit to carry out the desired reaction (i.e., pollutant removal).

Batch reactors. Batch reactors are normally charged with reactants, completely mixed, and allowed to react over time without inflow to or outflow from the reactor. The system is shown in Figure 2-2. The storage equation for a batch reactor is

$$\frac{dV}{dt} = 0 \qquad (2\text{-}11)$$

The ideal batch reactor provides a convenient point to investigate some of the basics of reaction kinetics. However, the concepts are applicable to the other reactor types.

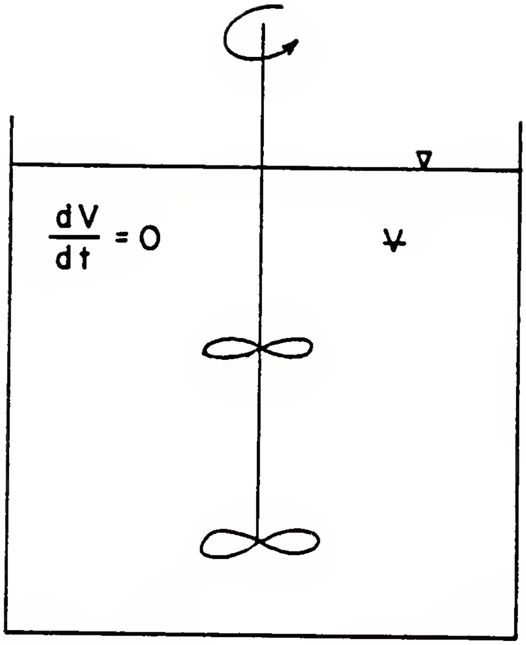


Figure 2-2. Batch Reactor.

Isothermal, irreversible reactions in a completely mixed, constant-volume batch reactor are governed by the following equation (Levenspiel, 1972):

$$\forall \frac{dC}{dt} = -K\forall C^{n}(t) \qquad (2\text{-}12)$$

or

$$-\frac{dC}{dt} = KC^{n}(t) \qquad (2\text{-}13)$$

where $C(t)$ = reactant concentration in the reactor, ML^{-3},

$\forall$ = constant fluid volume in the reactor, L^{3},

K = rate constant, T^{-1}, and

n = reaction order.

The term $-dC/dt$ represents the reaction rate. Under the assumption of first-order kinetics (i.e., $n=1$), equation 2-13 becomes

$$-\frac{dC}{dt} = KC(t) \qquad (2\text{-}14)$$

for which the solution is

$$\frac{C(t)}{C(0)} = \exp(-Kt) \qquad (2\text{-}15)$$

where $C(0)$ = initial reactant concentration in the reactor, ML^{-3}. Equation 2-15 produces the family of curves shown in Figure 2-3. First-order reactions are typified by those that are unaffected by the initial concentration, $C(0)$. The first-order assumption is commonly made because of the simplicity of the governing equation and the fact that it appears to perform adequately for many purposes. However, it does not always fit the situation and, thus, it is sometimes necessary to determine the correct reaction order.

Transforming equation 2-13 by the natural logarithm yields

$$\ln\left(-\frac{dC}{dt}\right) = n \ln[C(t)] + \ln(K) \qquad (2\text{-}16)$$

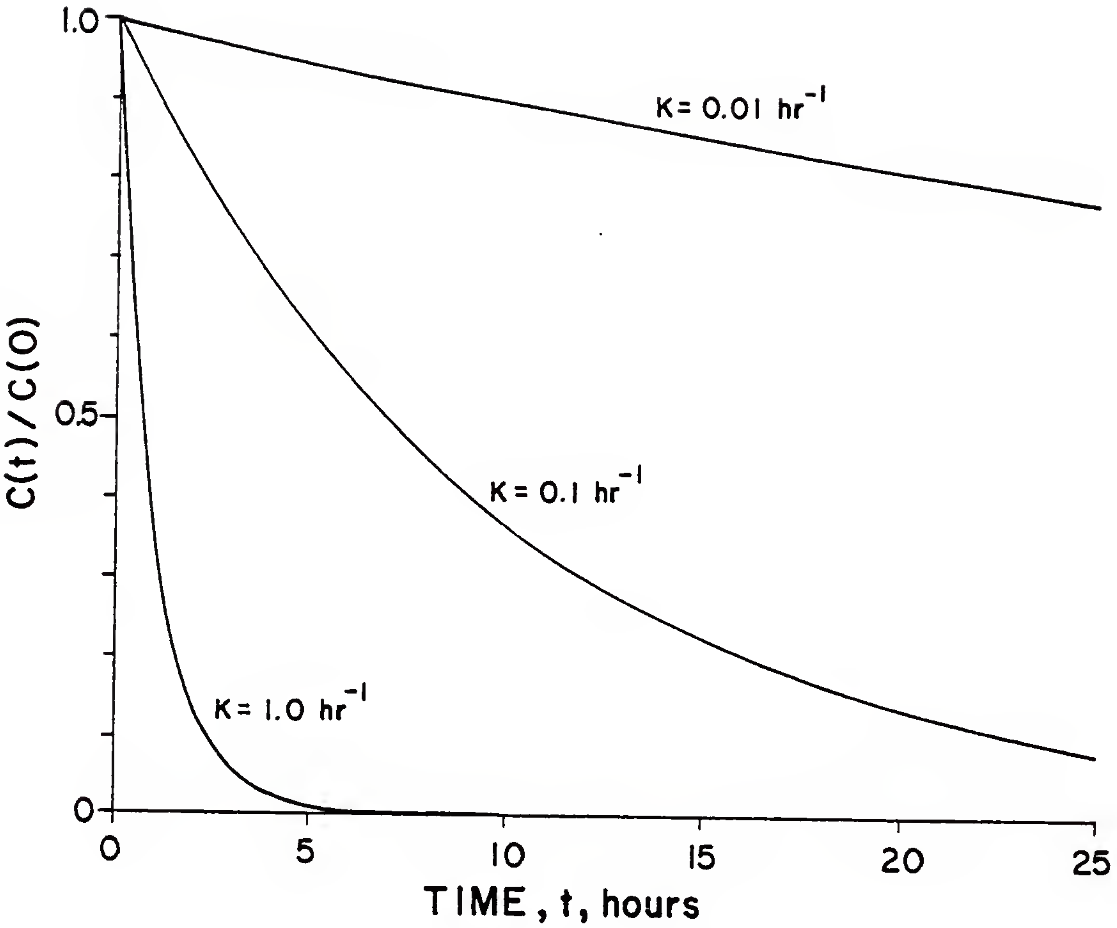


Figure 2-3. First-Order Reactions in a Batch Reactor.

This transformation allows the estimation of the reaction order, n, and the rate constant, K, through a simple linear regression of ln(-dC/dt) against ln[C(t)]. The value of dC/dt is taken from a concentration versus time curve produced by representing the batch reactor experimental data; its value at any particular point in time is the slope of the curve at that point. This is known as the differential method of analysis (Levenspiel, 1972). The principal drawback of this technique is that the differentiation of experimental data via graphical means introduces extra "noise" to the analysis due to the visual error in determining the slopes (Butt, 1980).

An alternative approach, known as the integral method of analysis (Levenspiel, 1972), tests individual rate equations of any order (except n = 1) until a suitable one is found. The analysis is based on the integration of equation 2-13, i.e.,

$$\left[\frac{C(t)}{C(0)} \right]^{1-n} - 1 = \left\{ \frac{n-1}{[C(0)]^{1-n}} \right\} Kt \tag{2-17}$$

or

$$C.F. = \left\{ \left[\frac{C(t)}{C(0)} \right]^{1-n} - 1 \right\} \frac{[C(0)]^{1-n}}{n-1} = Kt \text{ for } n \neq 1 \tag{2-18}$$

where C.F. is known as the concentration function. By plotting values of C.F. against values of t for the experimental data and a particular value of n, a determination can be made as to whether the proper value of n, the reaction order, has been selected. This is done by noting how well the values of C.F. and t are represented by a straight line. If they are, the value of n is correct; if not, another value must be selected and tested. Standard linear regression techniques can be used to determine the appropriateness of each value of n. An example of this method is shown later.

Reaction kinetics and environmental processes. Although reaction kinetics is primarily applied to chemical processes, it is also useful in environmental and sanitary engineering. The basic mathematical representation given by equation 2-13 is applicable to biological and physical phenomena as well as chemical reactions. For example, first-order kinetics is often used to represent the oxidation of substrate by microorganisms in the analysis and design of biological treatment units in sanitary waste facilities (Fair et al., 1968; Rich, 1973). The destruction of coliforms by disinfectants is also commonly treated as a first-order "reaction" (Chick, 1908; Collins et al., 1971). It is also useful in the analysis of storage/release systems where the settling of particles is often treated as a "reaction" (City of Milwaukee et al., 1975; Smith, 1975; Medina, 1976; Medina et al., 1981b).

An excellent example of how reaction kinetics can be applied to an environmental engineering problem is in settleability testing. Whipple and Hunter (1981) analyzed the settleability of several pollutants in urban runoff through the use of a 6-ft settling column (essentially a batch reactor without continuous mixing). The results for hydrocarbons are shown in Figure 2-4. Tests of several rate orders, using the integral method of analysis (equations 2-17 and 2-18), are shown in Figure 2-5. From these plots it is clear that the settling of hydrocarbons (at least for the data collected) is approximately a fourth-order "reaction." Of course, a more accurate representation might have been obtained at some non-integer value near 4. It is important to note that a settling column should be designed with a depth closely conforming to expected field conditions (i.e., the actual basin) in order for the rate constant and "reaction" order to be transferable.

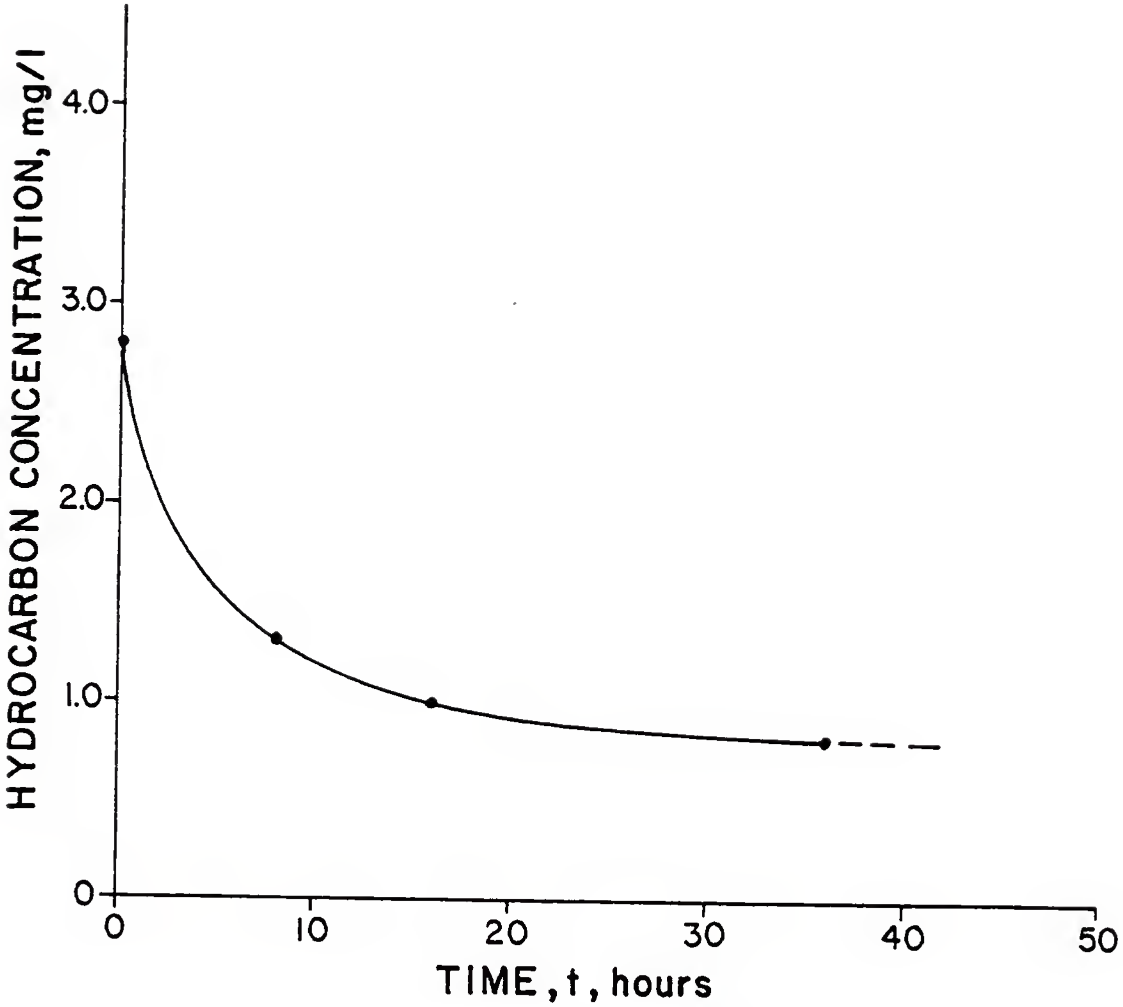


Figure 2-4. Results of Hydrocarbon Settling Test (Whipple and Hunter, 1981).

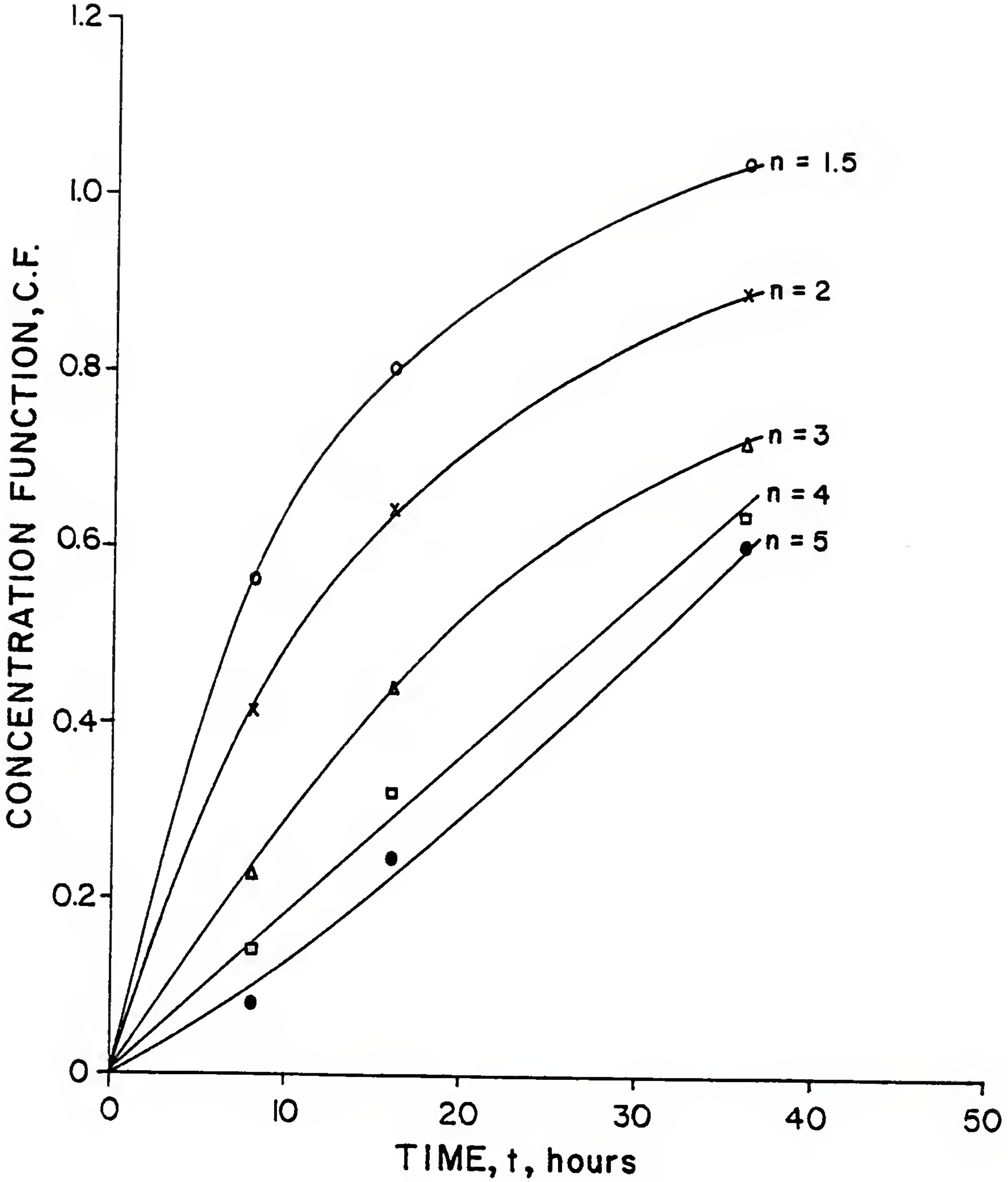


Figure 2-5. Integral Method of Estimating "Reaction" Order for Hydrocarbon Settling.

Flow reactors - general characteristics. Flow reactors recieve inflows and release outflows, whereas batch reactors do not. The fluid storage equation given by equation 2-1 governs the continuity of fluid mass, i.e.,

$$\frac{dV}{dt} = I(t) - O(t) \tag{2-19}$$

However, the behavior of the system as a reactor depends on how individual fluid parcels or elements are mixed and routed through the system. Unfortunately, equation 2-19 does not provide this information.

The two ideal-flow reactors represent the extreme levels of mixing. In the completely mixed reactor, all inflow elements are immediately, uniformly, and completely dispersed throughout the reactor. The plug-flow reactor queues the flow such that all fluid elements leave the reactor in the same order they entered. The extremes of mixing represented by these ideal reactors are better understood by investigating the age or detention time distributions of the fluid elements in the reactor and in the outflow. Intermediately mixed or arbitrary-flow reactors also have unique age distributions. These reactors experience mixing levels between the extremes of the completely mixed and plug-flow reactors.

Consider a steady-state reactor with constant and equal inflow and outflow rates, i.e., $I(t) = O(t) = Q$, and a constant fluid volume, i.e., $V(t) = \forall$. The mixing level is arbitrary. Assume that a conservative tracer is continuously injected with the inflow at a constant concentration, C_i, beginning at an arbitrary starting point, $t = 0$. The system response, as measured by the ratio of the outflow concentration, $C(t)$, to the inflow concentration, C_i, or $F(t) = C(t)/C_i$, has several possible forms. These are shown as functions of $(\frac{Q}{\forall}t)$ in Figure 2-6. Figure 2-6(a) shows the step function response associated with plug-flow reactors. Figure 2-6(b) shows the response of a completely mixed system. These responses will be demonstrated later. The response shown in Figure 2-6(c) is what might be expected for a reactor with an intermediate level of mixing.

The ages or detention times of the tracer elements in the reactor have a statistical distribution (Danckwerts, 1953; Butt, 1980). This distribution is defined by $\psi(t)$. The distribution of tracer ages in the outflow is defined by $\phi(t)$. In other words, the fraction of all tracer material in the reactor having ages between t and t + dt is $\psi(t)dt$. Similarly, for the outflow, the fraction is $\phi(t)dt$. Thus, the term F(t) can also be defined as the fraction of the fluid leaving the reactor that has resided in the reactor for less than t, i.e.,

$$F(t) = \int_0^t \phi(t')dt' \tag{2-20}$$

where t' = dummy variable of integration.

Also, by definition,

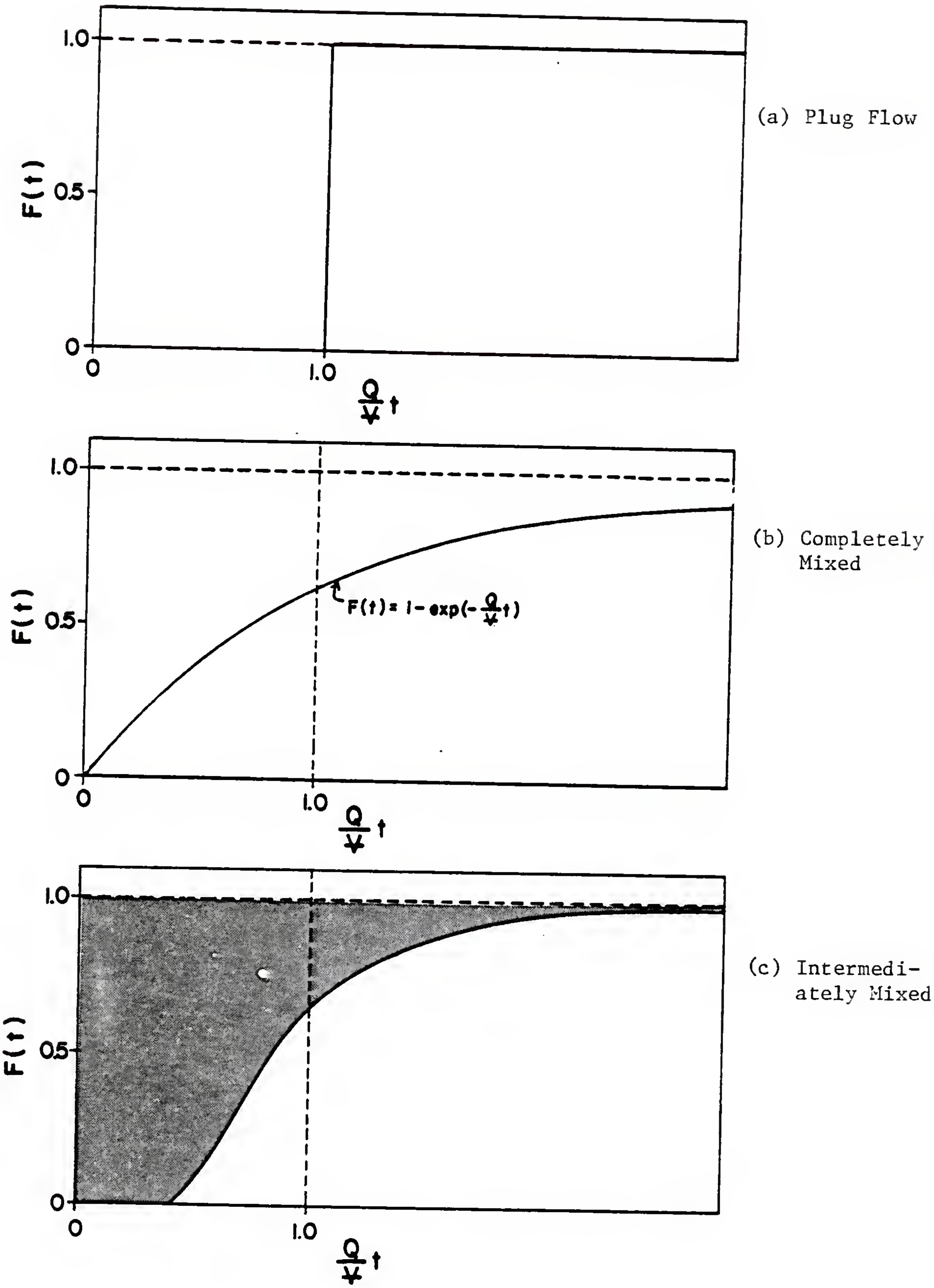
$$\int_0^\infty \psi(t)dt = \int_0^\infty \phi(t)dt \equiv 1 \tag{2-21}$$

Assuming $C_i = 1$ (the specific units are irrelevant), the "balance sheet" for the tracer at time t is as follows:

Entered reactor: Qt (2-22)

Still in reactor: $\forall \int_0^t \psi(t')dt'$ (2-23)

Left reactor: $Q \int_0^t \int_0^{t''} \phi(t')dt'\,dt''$ (2-24)



(a) Plug Flow

(b) Completely Mixed

(c) Intermediately Mixed

Figure 2-6. Tracer Response, Steady-State Flow Reactor (Butt, 1980).

The variables t' and t" are dummy variables of integration. The term $Q \int_0^{t''} \phi(t')dt'$ is the mass rate of tracer leaving the reactor at time t". The material balance at time t is

$$Qt = \forall \int_0^t \psi(t')dt' + Q \int_0^t \int_0^{t''} \phi(t')dt'dt'' \tag{2-25}$$

or

$$\frac{Q}{\forall}t = \int_0^t \psi(t')dt' + \frac{Q}{\forall} \int_0^t \int_0^{t''} \phi(t')dt'dt'' \tag{2-26}$$

Differentiating with respect to t and recalling equation 2-20 yields

$$\frac{Q}{\forall} = \psi(t) + \frac{Q}{\forall} F(t) \tag{2-27}$$

or

$$\psi(t) = \frac{Q}{\forall} (1 - F(t)) \tag{2-28}$$

From this result, the useful fact that the area between F(t) = 1 and F(t) (the shaded area in Figure 2-6(c)) is 1.0 emerges, i.e.,

$$1 \equiv \int_0^\infty \psi(t)dt = \int_0^\infty \frac{Q}{\forall} (1 - F(t))dt \tag{2-29}$$

The last integral is the shaded area shown in Figure 2-6(c).

The average age or detention time of all fluid elements leaving the reactor at any time t, θ, is given as follows:

$$\theta = \frac{\int_0^\infty t\phi(t)dt}{\int_0^\infty \phi(t)dt} = \int_0^\infty t\phi(t)dt \text{ since } \int_0^\infty \phi(t)dt = 1. \tag{2-30}$$

Differentiating equation 2-20 and substituting the result for ϕ(t) in equation 2-30 produces

$$\theta = \int_0^\infty t \frac{dF(t)}{dt} dt = \int_0^1 t\ dF(t) \tag{2-31}$$

Multiplying equation 2-31 by Q/¥ yields

$$\frac{Q}{¥}\ \theta = \int_0^1 \frac{Q}{¥}\ t\ dF(t) \tag{2-32}$$

The integral in equation 2-32 also defines the shade area shown in Figure 2-6(c); thus,

$$\frac{Q}{¥}\ \theta = 1 \tag{2-33}$$

or

$$\theta = \frac{¥}{Q} \tag{2-34}$$

Equation 2-34 represents the average or nominal detention time--it is not the detention time of all fluid elements passing through the system. This calculation is a familiar and useful tool, but it is limited to steady-state reactors or storage units (for any level of mixing). The average age of all fluid elements leaving the reactor up to time t, $\overline{\theta}$, is also ¥/Q.

When the steady-state reactor is completely mixed, the internal and external age distributions are equal and the tracer concentration in the outflow is equal to that in the tank. Thus, equation 2-28 can be written as

$$\phi(t) = \frac{Q}{¥} - \frac{Q}{¥}\ F(t) \tag{2-35}$$

Differentiating equation 2-20 and substituting the result for $\phi(t)$ in equation 2-35 produces the following:

$$\frac{dF(t)}{dt} + \frac{Q}{¥}\ F(t) = \frac{Q}{¥} \tag{2-36}$$

Solving for F(t) yields

$$F(t) = 1 - \exp(-\ \frac{Q}{¥}t) \tag{2-37}$$

Equation 2-37 is represented by the curve shown in Figure 2-6(b). The external age distribution is

$$\phi(t) = \frac{dF(t)}{dt} = \frac{Q}{\forall} \exp(-\frac{Q}{\forall}t) \quad (2\text{-}38)$$

When the steady-state reactor is operated in the plug-flow mode, the age of all fluid elements leaving at any time t is ∀/Q. Thus,

$$\begin{aligned} F(t) &= 0 \text{ for } t < \theta \\ F(t) &= 1 \text{ for } t \geqq \theta \end{aligned} \quad (2\text{-}40)$$

Equation 2-39 describes the step function shown in Figure 2-6(a).

For unsteady-state reactors (i.e., I(t), O(t), and V(t) are time variable), the average detention time of all fluid elements leaving the reactor at time t is

$$\theta(t) = \frac{\int_0^{\infty} t'\phi(t,t')dt'}{\int_0^{\infty} \phi(t,t')dt'} \quad (2\text{-}40)$$

This equation is valid for any level of mixing. However, the distribution of fluid element ages, $\phi(t,t')$, is complicated by the variable nature of the inflows and outflows and, thus, it changes with time t. As a result, equation 2-40 may be difficult or impossible to integrate directly. The average detention time for all flows leaving the reactor, up to time t, is

$$\bar{\theta}(t) = \frac{\int_0^{t} O(t')\theta(t')dt'}{\int_0^{t} O(t')dt'} \quad (2\text{-}41)$$

The concept of detention time is useful in understanding the behavior of various reactor types. Unfortunately, the term has been misunderstood by practitioners in sanitary and environmental engineering, especially those investigating stormwater storage/release systems. A common error is to assume that the flow leaving a storage unit has an average detention time of $V(t)/O(t)$. In steady-state units this is true but equation 2-40 clearly indicates that this is not so for unsteady-state conditions. In fact, it is possible for the outflow to have a variety of age distributions for the same value of $V(t)/O(t)$. The true average detention time at any time, t, must account for the history of the unit. This erroneous assumption has often been used to calculate the "detention time" (or related value, such as the overflow velocity) at various points in time. Another common error is to calculate the average detention time over a given period by dividing the average values of $V(t)$ and $O(t)$ or averaging the values of $V(t)/O(t)$. (At times, $[O(t) + I(t)]/2$ is substituted for $O(t)$.) Relationships between these parameters and the removal of particular pollutants have also been developed (Lager et al., 1977). These relationships might be of some predictive value but the term "detention time" should not be used. It would be preferable to use a term such as "detention parameter" in order to make it clear that such calculations are only indications of the detention ability of the storage basin.

Flow reactors--mathematical models. The previous subsection described the general mixing and detention characteristics of flow reactors. This subsection will describe mathematical models for plug-flow and completely mixed reactors. These models will be particularly useful in the development of a computer simulator in Chapter 4.

The mathematical models for plug-flow and completely mixed reactors are derivable from the one-dimensional advective-dispersive equation (Medina, 1976; Medina et al., 1981a), i.e.,

$$\frac{\partial C}{\partial t} = \frac{\partial}{\partial x}[E_L(x,t)\ \frac{\partial C}{\partial x} - U(x,t)C(x,t)] \pm \sum(S_o + S_i) \qquad (2\text{-}42)$$

where $C(x,t)$ = reactant concentration in the reactor, ML^{-3},

x = distance along flow axis, L,

$E_L(x,t)$ = longitudinal dispersion coefficient, L^2T^{-1},

$U(x,t)$ = longitudinal flow velocity, LT^{-1},

S_o = source of reactant, $ML^{-3}T^{-1}$, and

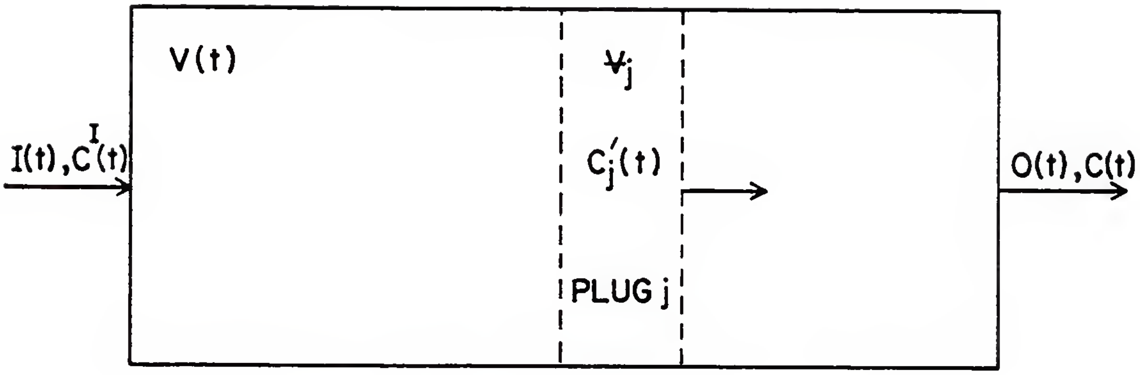
S_i = sink of reactant, $ML^{-3}T^{-1}$.

A plug-flow reactor is characterized by a regime in which flow through the reactor is queued; i.e., the fluid elements (or plugs) exit the reactor in the same order that they arrived. In other words, there is no dispersion ($E_L = 0$). Thus, with a reactive sink, equation 2-42 becomes

$$\frac{\partial C}{\partial t} = \frac{\partial}{\partial x}[- U(x,t)C(x,t)] - KC^n(x,t) \qquad (2\text{-}43)$$

Equation 2-43 is the pure advective form of equation 2-42. For practical purposes (e.g., computer simulation), equation 2-43 can be viewed as a series of discrete plugs, each acting as a batch reactor, moving along the flow axis. This representation of the plug-flow reactor is shown in Figure 2-7.

Completely mixed reactors immediately disperse all inflows (including reactants, tracers, pollutants, etc.) completely and uniformly throughout the reactor; i.e., $\partial C/\partial x = 0$. Therefore, the concentration of the reactant is the same in both the reactor and the outflow. A schematic of this type of reactor is shown in Figure 2-8. For a



NOTE: $\mathcal{V}_j$ and $C_j'(t)$ are the volume and reactant concentration of plug j, respectively.

Figure 2-7. Plug-Flow Reactor.

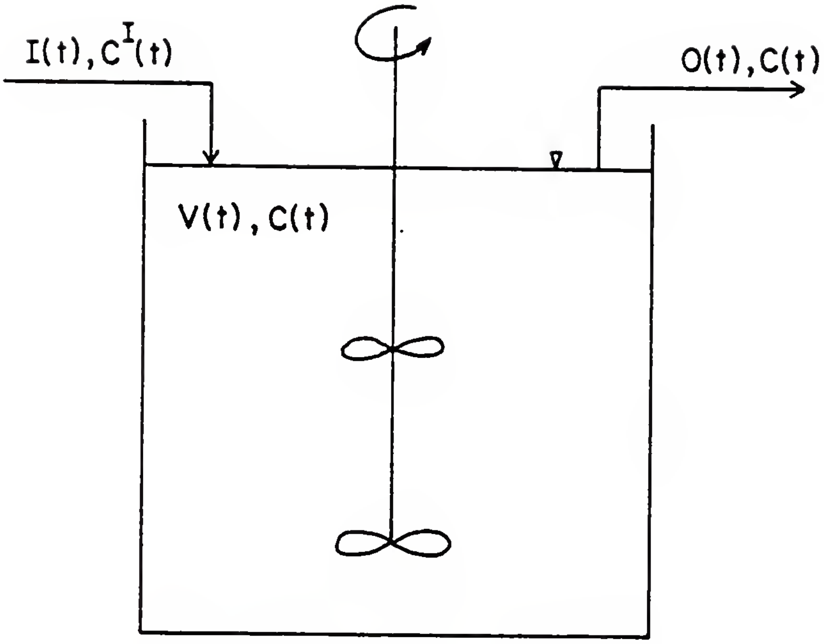


Figure 2-8. Completely Mixed Flow Reactor (Rich, 1973).

completely mixed flow reactor with an input source, an outflow sink, and a reactive sink, equation 2-42 becomes

$$\frac{d(CV)}{dt} = C^I(t)I(t) - C(t)O(t) - KC^n(t)V(t) \qquad (2\text{-}44)$$

where C(t) = reactant concentration in the reactor and the outflow, ML^{-3}, and

$C^I(t)$ = reactant concentration in the inflow, ML^{-3}.

Equation 2-44 can also be viewed as a version of the storage equation written in terms of reactant mass.

Unfortunately, the n-th order reaction provision in equation 2-44 makes the derivation of an analytical solution difficult. However, for first-order reactions (i.e., n = 1), equation 2-44 becomes

$$\frac{d(CV)}{dt} = C^I(t)I(t) - C(t)O(t) - KC(t)V(t) \qquad (2\text{-}45)$$

or

$$C(t)\frac{dV}{dt} + V(t)\frac{dC}{dt} = C^I(t)I(t) - C(t)O(t) - KC(t)V(t) \qquad (2\text{-}46)$$

Rearranging terms yields

$$\frac{dC}{dt} + \left[\frac{1}{V(t)}\frac{dV}{dt} + \frac{O(t)}{V(t)} + K\right] C(t) = \frac{I(t)}{V(t)} C^I(t) \qquad (2\text{-}47)$$

In terms of C(t), equation 2-47 is a simple first-order differential equation with the following solution (Ross, 1964):

$$C(t) = \exp\{\int_0^t [\frac{1}{V(t)}\frac{dV}{dt} + \frac{O(t)}{V(t)} + K]dt\} \; [\int_0^t \exp\{\int_0^t [\frac{1}{V(t)}\frac{dV}{dt} + \frac{O(t)}{V(t)} + K]dt\} \frac{I(t)}{V(t)} C^I(t)\, dt + C(0)] \qquad (2\text{-}48)$$

where C(0) = initial reactant concentration in the reactor and the outflow, ML^{-3}.

The integrals in this solution make it difficult to work with for most functions of V(t), O(t), and $C^I(t)$. In fact, it is only possible to evaluate equation 2-48 with the most trivial forcing functions (Medina, 1976; Medina et al., 1981a). Fortunately, numerical techniques can be used to approximate and evaluate equations that are not directly solvable.

Intermediately mixed flow is defined as any degree of fluid element mixing between plug and completely mixed flow (Metcalf and Eddy, Inc., 1972). Reactors with this type of mixing regime can be analyzed with the full advective-dispersive equation given by equation 2-42 (Medina, 1976; Butt, 1980; Medina et al., 1981a). Although most real reactors are, to some degree, arbitrary flow units, an attempt is usually made to achieve one of the extremes. For this reason, intermediately mixed flow reactors are not investigated in this study.

Particle Settling

The settling of particles is one of the most important mechanisms contributing to the removal of pollutants in storage units. Storage units used to settle particles are designed to closely resemble the ideal plug-flow reactor; in other words, queued quiescent flows. Obviously, these are more desirable conditions for settling than those found in completely-mixed reactors. Four types of settling are generally recognized (Fair et al., 1968; Clark et al., 1977):

1) discrete particles in a dilute suspension,
2) settling of flocculent suspensions,
3) hindered settling of discrete particles, and
4) compression.

The settling of discrete particles in a dilute suspension is assumed to be unhindered and a function only of fluid and particle properties.

The settling of flocculent suspensions is different from discrete particle settling because the particles are coalescing and, thus, changing their properties as they settle. Concentrated suspensions of discrete particles cause significant displacement of the supporting fluid and hinder settling. Compression occurs when the suspension becomes so concentrated that particles are in contact with each other and supported somewhat by the compacted mass. Unhindered discrete particle settling can be described in terms of simple dynamics. The other forms of settling are generally not adaptable to direct analysis by fundamental physical principles, although the hindered settling of discrete particles can be analyzed through theories developed for the backwashing of filter media (Fair et al., 1968). All forms of settling can be analyzed experimentally through the use of settling column or settleability tests (Clark et al., 1977). Such tests should always be conducted in situations where settling is a major pollutant removal mechanism.

Theoretical settling relationships, when available, can provide useful tools for understanding the behavior of storage/release systems and developing mathematical models. The theory governing the unhindered settling of discrete particles is briefly discussed below. Fortunately, this concept of settling probably mirrors the settling of particles in stormwater more closely than the other types of settling.

A discrete particle is one that does not change its shape, weight, or size while settling (Fair et al., 1968). In a dilute suspension, such a particle will accelerate until the drag or frictional resistance of the fluid equals the effective weight of the particle. The effective weight is given by

$$F_p = (\rho_s - \rho)gV_p \qquad (2\text{-}49)$$

where F_p = effective weight of the particle, MLT^{-2},

ρ_s = mass density of the particle, ML^{-3},

ρ = mass density of the fluid, ML^{-3},

g = gravitational constant, LT^{-2}, and

V_p = volume of the particle, L^3.

A dimensionally derived relationship for the frictional drag exerted by the fluid is given by Fair et al. (1968):

$$F_D = C_D A_c \rho v_s^2/2 \qquad (2\text{-}50)$$

where F_D = drag force of the fluid, MLT^{-2},

C_D = coefficient of drag,

A_c = cross-sectional area of the particle, L^2, and

v_s = terminal settling velocity of the particle, LT^{-1}.

When the particle is no longer accelerating, the drag force and the effective weight are equal, i.e.,

$$(\rho_s - \rho)gV_p = C_D A_c \rho v_s^2/2 \qquad (2\text{-}51)$$

and the terminal settling velocity can be calculated as

$$v_s = [\frac{2g}{C_D} \frac{(\rho_s - \rho)}{\rho} \frac{V_p}{A_c}]^{1/2} \qquad (2\text{-}52)$$

For spherical particles of diameter d, $V_p = (\pi/6)d^3$ and $A_c = (\pi/4)d^2$. Equation 2-52 can be written as

$$v_s = [\frac{4}{3} \frac{g}{C_D} \frac{(\rho_s - \rho)}{\rho} d]^{1/2} \qquad (2\text{-}53)$$

The coefficient of drag, C_D, varies with the Reynolds number, N_R. The Reynolds number is a dimensionless measure of the effect of fluid viscosity in fluid systems. For spherical particles, C_D, is approximated by

$$C_D = \frac{24}{N_R} \text{ for } N_R < 0.5; \qquad (2\text{-}53)$$

or

$$C_D = \frac{24}{N_R} + \frac{3}{N_R^{1/2}} + 0.34 \text{ for } 0.5 \leqq N_R \leqq 10^4; \qquad (2\text{-}55)$$

or

$$C_D = 0.4 \text{ for } N_R > 10^4 \qquad (2\text{-}56)$$

The value of N_R is calculated as

$$N_R = v_s d/\nu \qquad (2\text{-}57)$$

where ν = kinematic viscosity of the fluid, L^2T^{-1}.

Kinematic viscosity is a function of fluid temperature. When C_D is specified by equation 2-55, an iterative technique is required to solve for v_s (Sonnen, 1977).

The vertical travel length of a discrete particle (or any particle) during the particle's residence time in the storage unit is the deciding factor in the removal of the particle from the fluid stream. If resuspension is ignored it can be assumed that a particle reaching the bottom of a storage unit is removed. The ability of a particle to reach the bottom is, of course, a function of the settling velocity, v_s. In a steady-state, plug-flow system in which the incoming particles are uniformly distributed along the depth of a rectangular basin with a horizontal bottom, the removal fraction of particles with velocity v_s is v_s/v_c. The time it takes the particles to reach their terminal velocity is ignored. The variable v_c is the settling velocity required for a particle entering at the top of the settling zone to reach the bottom, i.e.,

$$v_c = \frac{d_e}{\theta} \qquad (2\text{-}58)$$

where v_c = critical particle settling velocity, LT^{-1},

d_e = depth of fluid in the storage basin, L, and

θ = detention time of all fluid elements leaving the basin, T.

Obviously, if $v_s > v_c$, all particles with a velocity v_s are removed. When unsteady-state conditions are encountered, the analysis is not as straightforward. One approach to this problem is discussed in Chapter 4.

Evaluation Techniques

The theoretical presentation given in the previous section serves as a backdrop to a series of practical tools for analyzing the response of urban stormwater storage/release systems. It was readily apparent that the basic differential equations governing the behavior of storage units are not directly solvable with anything but the most trivial input functions. Thus, it becomes necessary to develop and use alternative techniques to evaluate the system's response in some reasonable manner.

The basic techniques are empirical methods, numerical analysis, statistical analysis, and computer simulation. Empirical methods use a cross-section of data from a number of systems to develop relationships between long-term pollutant removal and design and operating parameters. Numerical analysis essentially approximates the original differential equation with a simpler representation. These methods are sometimes clumsy and, by definition, subject to some level of error. However, they are generally well adapted to computerization. Statistical analysis, as the name implies, relies on a set of statistical parameters describing the inflows to the system and a relatively simple representation of the storage/release system to perform the analysis. Statistical techniques can be powerful tools, especially in preliminary analyses. They do suffer from several limiting assumptions and they lack the ability to generate a time series of system responses to a time series of incoming flows and pollutant loads. However, once the required influent statistics are obtained, the long-term response of a variety of system sizes or capacities can be rapidly evaluated--without computer aids. This is an impressive advantage but the price it extracts is a lack of flexibility in specific system designs and operating policies. Computer simulators, for the most part, automate the numerical solution techniques discussed above. The power of a detailed simulator is flexibility and its ability to produce time-series responses to serial stimuli. These are important features; just as the ease of application of a statistical technique is a potentially valuable trait. Some critics of computer simulators have implied that the use of a computer is a liability. This is not as compelling an argument as it once was because of the recent proliferation of a new generation of powerful and relatively inexpensive computers.

Computer simulators and statistical techniques have complementary traits. This fact will be exploited in Chapter 7 to develop long-term performance information at different levels of detail. One obvious but necessary caveat is that neither class of techniques is without flaws--both are nothing but mathematical models of a physical system. Such representations must be calibrated and verified with measured data in order to insure their accuracy. The need for calibration is an indictment of model assumptions and implies, a priori, imperfection. It is sometimes useful to remember that all man-made systems designed to measure or analyze natural systems are imperfect models. Measuring devices (e.g., dissolved oxygen meters) are nothing more than electrical models coupled to a measurable, and often indirectly related, quantity. Laboratory methods are also models which make certain assumptions about the quantities to be measured. Thus, before discounting computer simulators or statistical techniques, it is wise to recall the ubiquitous nature of models. The only real evil of simulators and statistical methods is that they are one more step removed from the "real" world, but they can open up a broader understanding of the system without requiring extensive (and expensive) monitoring programs. Mathematical models and monitoring programs should be viewed as mutually necessary and beneficial.

Empirical Techniques

Empirical techniques use a cross-section of data from a variety of storage/release systems to develop relationships between design and operating parameters and long-term pollutant removal. These relationships are developed from a wide range of system designs and are generally based on the storage basin volume, annual inflows and/or the drainage area. Relationships for single systems are also possible but these are usually developed for individual storm characteristics (Davis et al., 1978; McCuen, 1980). By their nature, single-system relationships are not as applicable to other proposed or in-place systems as their cross-sectional counterparts. Nevertheless, the cross-sectional relationships should only be used for very preliminary estimates.

Summaries of cross-sectional relationships have been prepared by Chen (1975), Ward et al. (1977), and Nix et al. (1981). Most of the cited work is taken from the literature dealing with the sediment retention capability of reservoirs. Most of these cross-sectional relationships are limited to normally ponded reservoirs (i.e., wet basins). A prominent example is provided by Brune's sediment trap efficiency curves (Brune, 1953). These curves are shown in Figure 2-9. Each curve represents a different type of sediment (i.e., fine, coarse, and "median") and each is a function of the basin capacity-to-annual inflow ratio. Brune's curves are widely used in reservoir design and, because of their ease of application, they have been used in the analysis of small sediment retention basins (Chen, 1975). Meta Systems, Inc. (1979) presents a summary of additional relationships for the removal of sediment, nitrogen, phosphorus, and other pollutants in agricultural detention basins.

Numerical Solution of Governing Equations

The differential equations defining the continuity of mass in storage/release systems are not amenable to direct analytical solutions except under very restrictive conditions. Fortunately, numerical techniques are available to circumvent this problem. Numerical solutions are generally obtained by (Hornbeck, 1975)

1) the direct numerical solution of the differential equation in question; or
2) numerically evaluating the integrals found in analytical solutions where the integrand is not readily integrable.

The primary advantage of numerical techniques is that they can be readily applied to a wide range of differential equations or integral-laden analytical solutions. This is not to say, however, that the solution is always easy to attain. Numerical techniques are

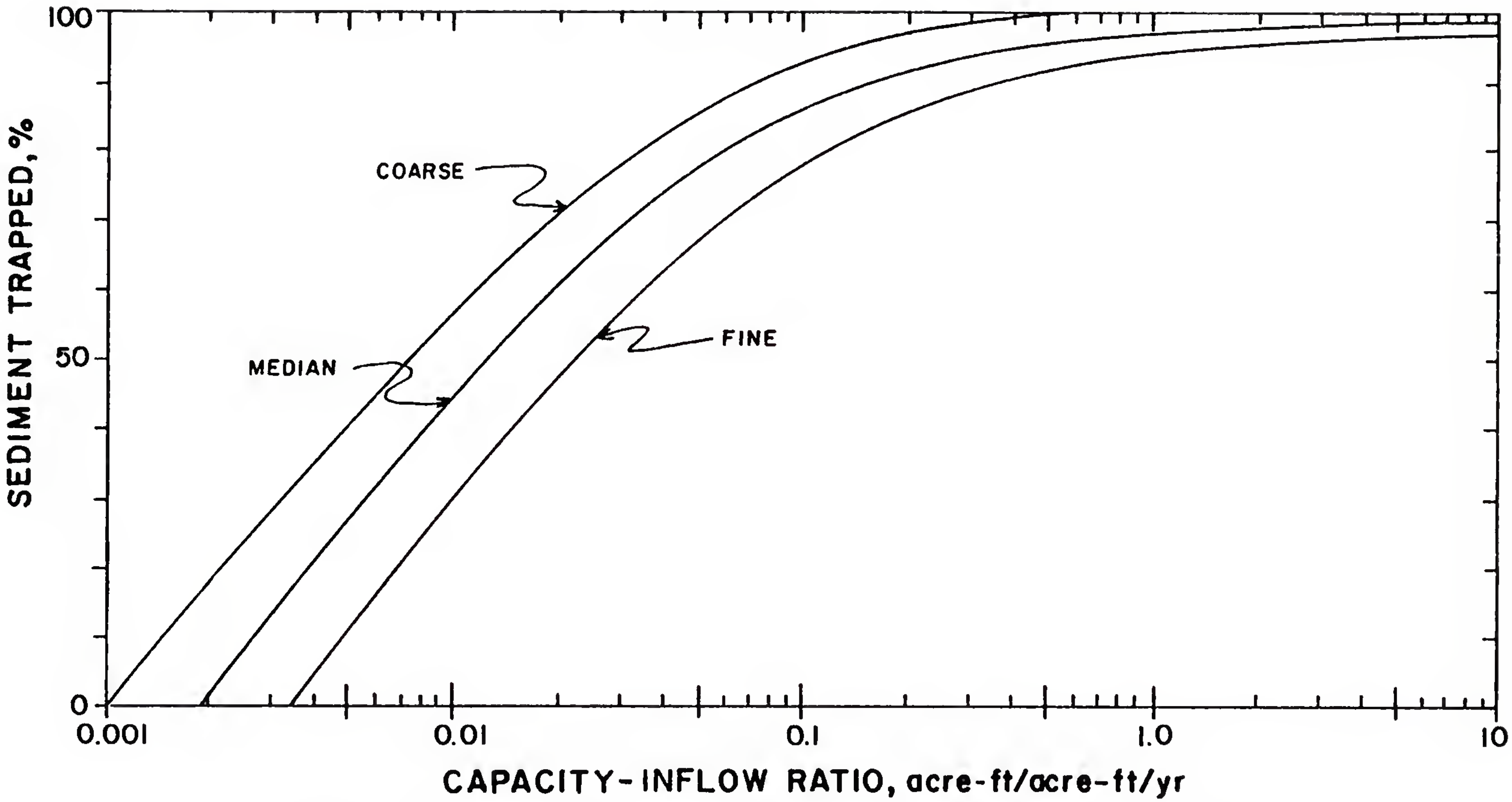


Figure 2-9. Brune's Sediment Trap Efficiency Curves (Brune, 1953).

approximators and, by definition, are flawed and subject to convergence and stability problems. Nevertheless, many problems can be solved with the judicious selection of a solution technique.

Numerical solution of ordinary differential equations. A numerical solution to an ordinary, initial-valued differential equation can be obtained through one of several techniques. Some of the more important classes of techniques are (Hornbeck, 1975)

1) multistep methods,
2) Runge-Kutta formulae, and
3) predictor-corrector methods.

Each of these classes has its own advantages and disadvantages. A specific group of multistep methods and a specific member of the Runge-Kutta family will be discussed below to illustrate their applicability to the differential equations describing storage units.

The closed Adams formulae are a group of multistep methods in which truncated, backward Taylor series expansions are used to approximate the differential equation. For example, the second-order Adams formula approximates the differential equation

$$\frac{dy}{dx} = g(y,x) \tag{2-59}$$

by

$$y_{j+1} = y_j + \frac{\Delta x}{2} [g_{j+1} + g_j] \tag{2-60}$$

where Δx = constant interval of x,

y_j, x_j = values of y and x at the beginning of interval j;

y_{j+1}, x_{j+1} = values of y and x at the end of interval j;

$g_j = g(y_j, x_j)$; and

$g_{j+1} = g(y_{j+1}, x_{j+1})$.

This numerical method can be used to approximate the storage equation in the following manner:

$$V_{j+1} = V_j + \frac{\Delta t}{2} [(I_{j+1} - O_{j+1}) + (I_j - O_j)] \qquad (2\text{-}61)$$

where Δt = constant time interval or step, T;

V_j, I_j, O_j = values of V(t), I(t), and O(t) at the beginning of time step j; and

$V_{j+1}, I_{j+1}, O_{j+1}$ = values of V(t), I(t), and O(t) at the end of time step j.

For any time step, it is assumed that the values of I_j, O_j, V_j, and I_{j+1} are known. Thus, the terms in equation 2-61 are rearranged to write the unknown variables, O_{j+1} and V_{j+1}, in terms of the known quantities, i.e.,

$$V_{j+1} + O_{j+1} \frac{\Delta t}{2} = V_j + (I_{j+1} + I_j - O_j) \frac{\Delta t}{2} \qquad (2\text{-}62)$$

Unfortunately, V_{j+1} and O_{j+1} cannot be determined until a relationship is developed between them, i.e.,

$$O(t) = f[V(t)] \qquad (2\text{-}63)$$

With this storage-outflow or storage-discharge relationship in hand, the right-hand side of equation 2-62 is evaluated and the actual values of V_{j+1} and O_{j+1} determined by substituting equation 2-63 for O_{j+1} and solving for V_{j+1} (and in turn, O_{j+1}). The method is repeated for the next time step with the j+1 values of this time step becoming the j-subscripted values of the next time step. This method (also known as the Puls method) is presented in more detail in Chapter 4.

The differential equation representing the routing of reactants or pollutants in a completely mixed reactor or storage basin can also be analyzed in a manner similar to that shown above. This application is also discussed in Chapter 4.

A well known method of the Runge-Kutta family is the "fourth-order Runge-Kutta formula" (Hornbeck, 1975). The formula approximates equation 2-59 by

$$y_{j+1} = y_j + \Delta x[\frac{1}{6} g(y_j, x_j) + \frac{1}{3} g(y'_{j+1/2}, x_{j+1/2}) + \frac{1}{3} g(y''_{j+1/2}, x_{j+1/2}) + \frac{1}{6} g(y'_{j+1}, x_{j+1})] \qquad (2\text{-}64)$$

where Δx = constant interval of x;

y_j, x_j = the values of y and x at the beginning of interval j;

y_{j+1}, x_{j+1} = the values of y and x at the end of interval j;

$x_{j+1/2} = x_j + \frac{\Delta x}{2}$;

$y'_{j+1/2} = y_j + \frac{\Delta x}{2} g(y_j, x_j)$;

$y''_{j+1/2} = y_j + \frac{\Delta x}{2} g(y'_{j+1/2}, x_{j+1/2})$; and

$y'_{j+1} = y_j + \Delta x\, g(y''_{j+1/2}, x_{j+1/2})$.

As an example, the following form of the storage equation is set up with the fourth order Runge-Kutta formula:

$$\frac{dV}{dt} = I(t) - f[V(t)] \qquad (2\text{-}65)$$

The resulting Runge-Kutta approximation is

$$V_{j+1} = V_j + \Delta t[\frac{1}{6} g(V_j, t_j) + \frac{1}{3} g(V'_{j+1/2}, t_{j+1/2}) + \frac{1}{3} g(V''_{j+1/2}, t_{j+1/2}) + \frac{1}{6} g(V'_{j+1}, t_{j+1})] \qquad (2\text{-}66)$$

where $g(V,t) = I(t) - f(V(t))$;

Δt = constant time interval or step, T;

V_j, t_j = values of V(t) and t at the beginning of time step j;

V_{j+1}, t_{j+1} = values of V(t) and t at the end of time step j;

$$t_{j+1/2} = t + \frac{\Delta t}{2};$$

$$V'_{j+1/2} = V_j + \frac{\Delta t}{2}\,[I(t_j) - f(V_j)];$$

$$V''_{j+1/2} = V_j + \frac{\Delta t}{2}\,[I(t_{j+1/2}) - f(V'_{j+1/2})];\ \text{and}$$

$$V'_{j+1} = V_j + \Delta t\,[I(t_{j+1/2}) - f(V''_{j+1/2})].$$

Runge-Kutta formulae are probably the most commonly used of the numerical methods (Hornbeck, 1975). They are easily programmed and offer good stability. It can be seen in equation 2-66 that the form of I(t) or f[V(t)] poses no obvious problem for the technique, but stability problems are always possible.

Numerical integration. Numerical integration can be used to complete the solution of analytical solutions containing integrals that are not readily evaluated. Essentially, numerical integration approximates the complicated integrand with a simple function over relatively short intervals. There are several techniques, including the following (Hornbeck, 1975):

1) trapezoidal rule,
2) Simpson's rule,
3) Romberg integration, and
4) Gauss quadrature.

The trapezoidal rule approximates the function to be integrated, f(x), with a series of straight lines over a finite number of equal intervals, Δx. This piecewise linear approximation results in a series of trapezoids for which the area is easily evaluated and, thus, the approximate value of the integral obtained. Simpson's rule achieves the same goal with parabolic arcs instead of straight lines. Romberg integration uses the trapezoidal rule with improved error analysis. Gauss quadrature employs unequally spaced intervals and approximates f(x) with orthogonal polynomials.

As an example, consider the following form of the storage equation:

$$\frac{dV}{dt} = I(t) - aV(t) \qquad (2\text{-}67)$$

where the outflow, O(t), is a linear function of V(t). The solution to the equation at time t is (Ross, 1964)

$$V(t) = \exp(-at) \int_0^t I(t)\exp(at)dt \qquad (2\text{-}68)$$

when the initial condition, V(0), is zero. Obviously, the inflow (or forcing) function can take on a complex form and be impossible to solve analytically. However, V(t) can be estimated by using the trapezoidal rule:

$$V(t) = \exp(-at)[I(0)\frac{\Delta t}{2} + 2I(t_1)\exp(at_1)\frac{\Delta t}{2} + \ldots$$

$$+ 2I(t_{n-1})\exp(at_{n-1})\frac{\Delta t}{2} + I(t_n)\exp(at_n)\frac{\Delta t}{2}] \qquad (2\text{-}69)$$

where Δt is the constant time step or interval, and the terms in brackets represent the trapezoidal approximation of $I(t)e^{at}$ from t = 0 to t_n (=t). The approximation is made over n equally spaced time steps or intervals. In general, the accuracy is dictated by the size of the time step. For highly fluctuating functions, smaller time steps are desirable. For smoother functions, a straight line approximation is often reasonably accurate with larger time steps.

Discrete inputs. A third approach, not listed earlier, approximates the input or forcing function (e.g., I(t)) by a series of discrete, equally spaced inputs such as that shown in Figure 2-10 (Medina, 1976; Medina et al., 1981a). This technique completes the integral-laden solutions for simpler versions of the storage equation and equations describing the various reactor models by substituting discrete inputs and evaluating the integrals over each time interval. Unfortunately, the fact that the technique requires an analytical solution limits its usefulness. In a general sense, the direct numerical evaluation of the differential equation is probably more useful.

Statistical Techniques

Statistical techniques have been developed to analyze stormwater runoff and the response of general storage/release systems (Howard, 1976; DiToro and Small, 1979; Hydroscience, Inc., 1979; Charles Howard and Assoc., 1979; Howard et al., 1979), constant-volume, completely mixed equalization basins (DiToro, 1975) and the effects of pollutant loads on receiving waters (Hydroscience, Inc., 1979; E.D. Driscoll and Assoc., 1981). Each method requires and/or develops a set of statistics describing the long-term characteristics of the incoming flows and is simple to apply to the analysis of storage/release systems once these statistics are developed. The result is a set of statistics describing the long-term performance of the system. However, these methods are limited by their inability to produce the transient response of a storage/release system.

Equalization basins. Equalization basins are designed to smooth out variations in the influent flow rate and/or pollutant loads. DiToro (1975) developed a statistical approach to the analysis of pollutant

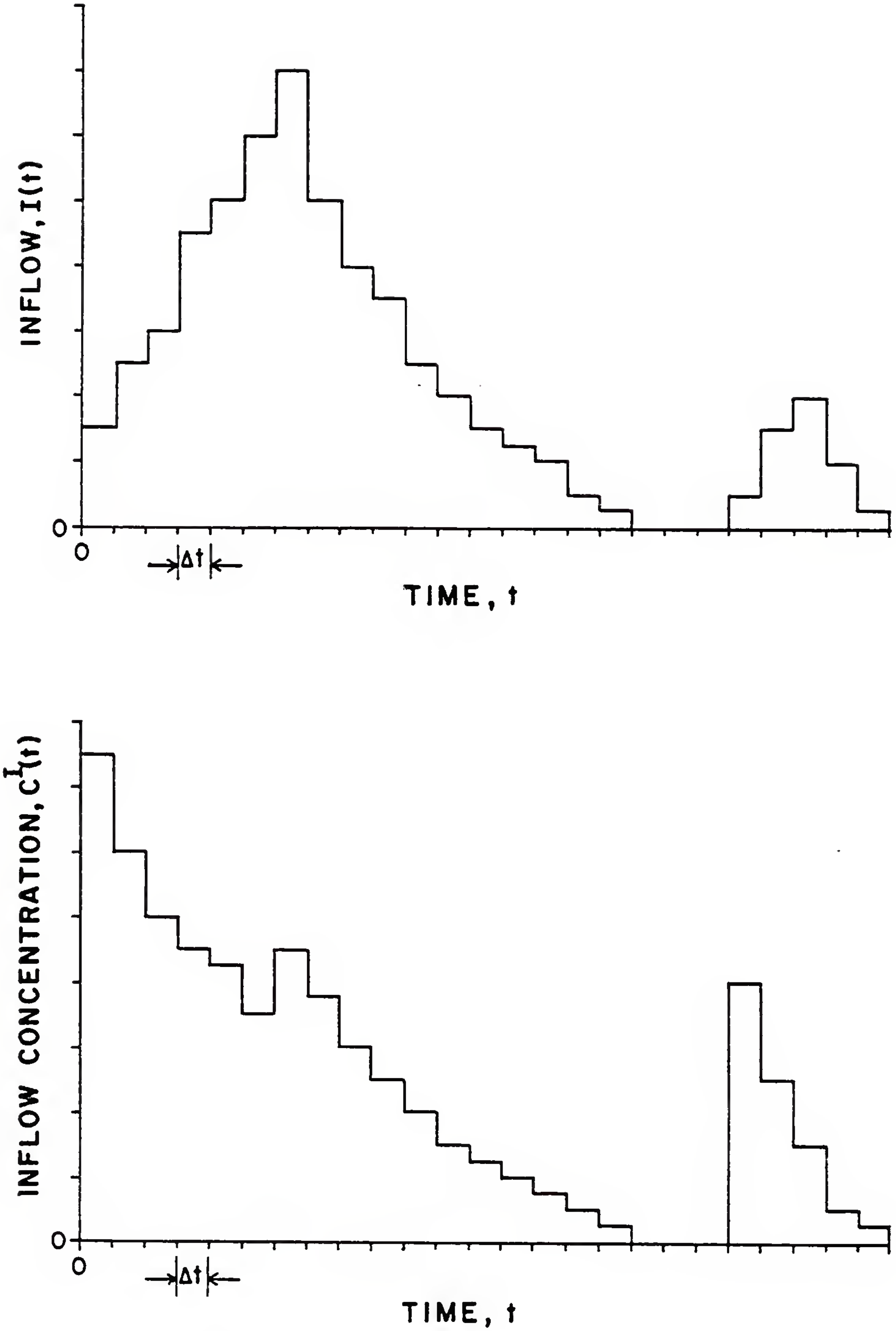


Figure 2-10. Discrete Inputs of the Inflow Rate and Pollutant Concentration at Equal Time Intervals (Medina, 1976).

loads in a constant-volume, completely mixed basin. Several parameters were used to describe the probabilistic nature of the input pollutant loads and flow. These parameters were then used to develop parameters describing the effluent pollutant loads. A first-order reaction characterized the behavior of pollutants within the basin. The procedure was compared with a simple simulator to verify the statistical derivations. Medina (1976) also compared DiToro's method with his constant-volume, completely mixed basin simulator (see later discussion). In both cases, the results were very similar.

Howard's statistical method. Howard (1976) developed and subsequently modified (Charles Howard and Assoc., 1979; Howard et al., 1979) a statistical method to obtain expressions for the probability distributions of runoff from a watershed and analyze the performance of storage/release or storage/treatment systems. The storage/release system is modeled as an in-line configuration. A schematic is shown in Figure 2-11.

The principal input to the model is a long-term hourly rainfall record. The rainfall record is transformed into a runoff record with a runoff coefficient after considering the effects of natural and man-made depression storage. Rainfall or storm events are defined by specifying the minimum dry period which separates independent storms or events. An autocorrelation analysis is suggested to determine this value. A more complete discussion of the definition of an independent event is presented in Chapter 3. The duration of each event is easily obtained from the record, as is the depth or volume. The average intensity of each event is its volume divided by the duration. Thus, the rainfall record becomes

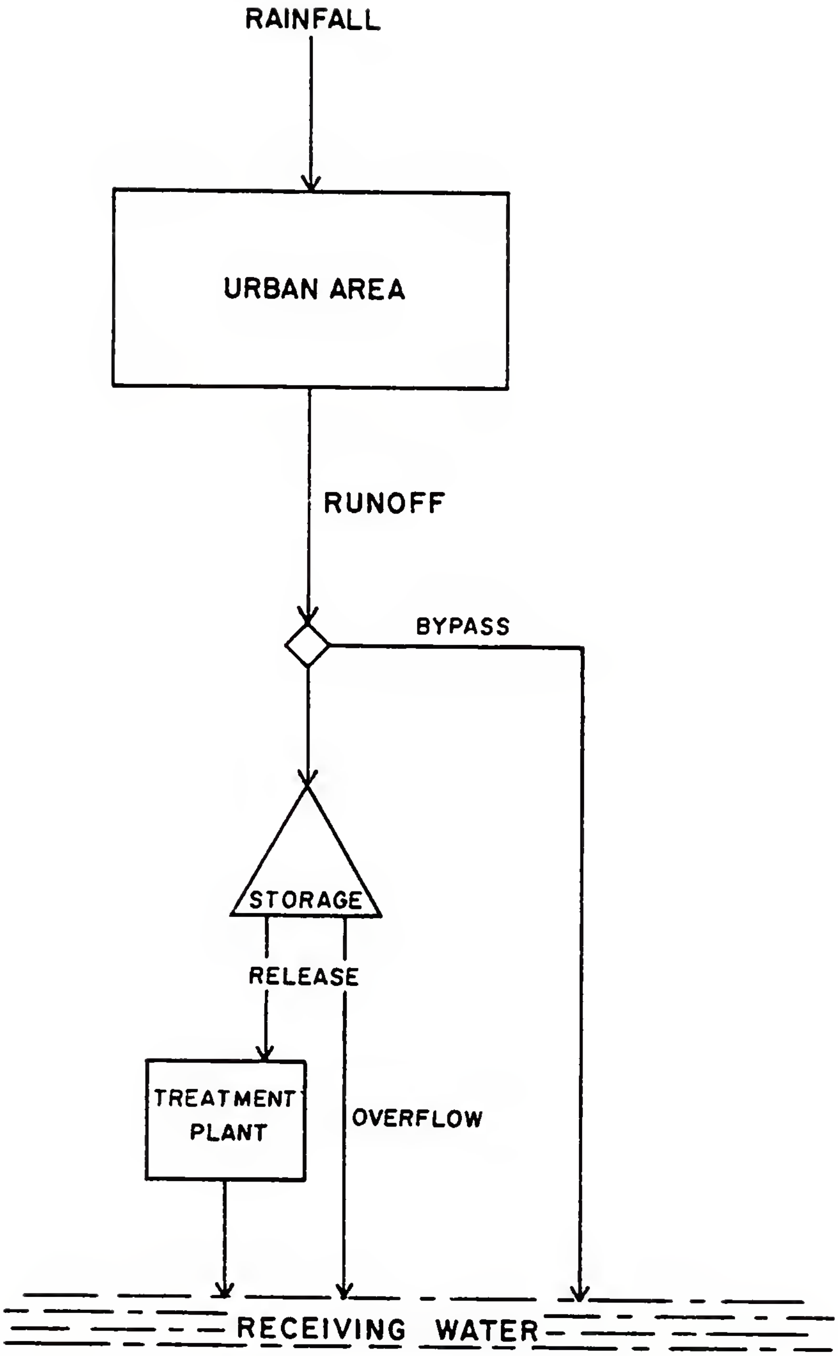


Figure 2-11. Storage/Release System, Howard's Statistical Method (Howard et al., 1979).

a series of rectangular hydrographs. With the selected event definition, the time between individual events is readily calculated.

With the rainfall record segregated into independent events, the average values for event duration, intensity, volume, and time between events times can be determined. It is assumed that each of these rainfall characteristics has a probability distribution which can be specified by the mean through a single-parameter exponential function;

$$p(x) = (1/\bar{x})\exp(-x/\bar{x}) \tag{2-70}$$

where x = random variable, $x \geq 0$,

$p(x)$ = probability density or distribution function of x, and

$\bar{x}$ = mean value of x.

It is also assumed that event intensities and durations are independent of each other. A similar assumption is made between event volumes and the times between events.

Based on several simplifying assumptions, exponential distributions are derived for runoff event intensities, depths, durations and the times between events with parameters that are derived from the rainfall parameters (i.e., mean values of flow, volume, duration and the time between events). An alternative approach would be to derive these statistics from the results generated by a rainfall-runoff simulator or from actual data. Pollutant concentrations in runoff are assumed to be constant and independent of all event parameters.

The storage/release or storage/treatment analysis is based on the following assertions (see Figure 2-11):

1) The treatment plant operates at a constant flow rate, Ω, as long as water is in the storage basin. This rate can be interpreted in several ways. It can be the flow capacity of a

wet-weather treatment facility or the excess capacity available in a dry-weather or sanitary sewage facility. It can also be defined as a constant storage basin release rate.

2) For the analysis of pollution control, the efficiency of the treatment unit, η_Ω, is assumed to be constant. The storage basin also "treats" flows passing through it by settling or some other mechanism related to detention time. The effect of various storage capacities and release rates is approximated by

$$\eta_V = a \log_{10}(DT) + b \tag{2-71}$$

where η_V = average removal efficiency of the storage basin, $0 \leqq \eta_V \leqq 1.0$,

DT = detention parameter, T, $DT \leqq DTMIN$,

DTMIN = minimum value of DT for which equation 2-71 is valid, T, and

a,b = coefficients.

The detention parameter is so named to distinguish it from detention time. It is defined as

$$DT = \frac{V_B}{2\Omega} \tag{2-72}$$

where V_B = storage basin capacity, L^3.

Howard et al. (1979) refer to DT as the average detention time over the period of interest. This is a misnomer and should be discouraged; it is only an indicator of the detention ability of the basin (see earlier discussion). Basins with higher values of DT tend to hold water longer. The variable DT provides a convenient parameter upon which to estimate η_V but the equation must be calibrated either to actual field data or simulation results. It is not correct to equate DT with holding times in settleability or treatability tests conducted in the laboratory. This would falsely assume that all water parcels passing through storage have the same detention time.

3) The bypassed flows receive no treatment, and, thus, enter the receiving water with the original pollutant concentration.

4) The storage basin is assumed to be full at the end of each storm. This requirement is reasonable for small basins but can be a problem for larger basins.

With these assumptions and the runoff statistics and distributions, it becomes a relatively simple task to estimate the expected number of overflow events (events exceeding the storage capacity) per year, the average annual volume of overflows, and the average annual runoff and pollutant control efficiency.

Hydroscience statistical method. Hydroscience, Inc. (1979) has presented a statistical method capable of analyzing urban stormwater runoff, storage/release systems, wet-weather treatment devices, and receiving water impacts. Much of the runoff analysis work appears in the Areawide Assessment Procedures Manual (Municipal Environmental Research Laboratory, 1976). The analyses of storage/release systems and wet-weather treatment devices were developed by DiToro and Small (1979) and Small and DiToro (1979), respectively, and were included in the Hydroscience report.

The Hydroscience method develops a set of rainfall statistics with a method similar to that used by Howard (1976). However, in this case, the minimum dry period that drives the coefficient of variation (the standard deviation divided by the mean) of the time between events (i.e., interevent times) to unity is used to separate events. The reason for this criterion revolves around the use of the exponential distribution to characterize the time between events. A fundamental characteristic of the exponential distribution is that the coefficient of variation is unity.

The Hydroscience method assumes that event durations and intensities are gamma distributed. The gamma distribution is a two-parameter distribution requiring the mean and coefficient of variation. In general, because the gamma distribution accounts for different variability patterns (through the coefficient of variation), it is more adept at representing these characteristics than the exponential distribution (Howard et al., 1979).

The Hydroscience method includes a simple method to obtain runoff event statistics from rainfall statistics. Additionally, it is assumed that the gamma distribution describes the runoff event flows and volumes. An alternative, and superior, way to derive runoff event statistics is to directly analyze a long-term runoff record. Unfortunately, such long-term records are usually not available. However, a simulator could provide the necessary data.

The Hydroscience method is capable of analyzing the two basic system configurations (shown in Figure 2-1) with three restrictions. One restriction is that the storage unit is operated with the bypass mode in place. Secondly, all releases from the storage unit are at a constant rate. The third restriction is that pollutant removal is handled by assuming constant removal efficiencies. Based on several simplifying assumptions, several sets of nomographic curves depicting the long-term performance of the system were developed. Performance is defined in terms of what portion of the runoff volume or pollutant load does not appear as a bypassed quantity.

Receiving water impacts are also analyzed by the Hydroscience method. This feature is potentially useful in relating the storage/release system design and operation to impacts. Unfortunately, this application is limited because the issue of how the storage/release system affects runoff statistics is not sufficiently addressed.

The Hydroscience method will be discussed in more detail in Chapter 3. In general, this method is more flexible and has a longer history of use and application (Municipal Environmental Research Laboratory, 1976; Hydroscience, Inc., 1979, E.D. Driscoll and Assoc., 1981) than Howard's method. Thus, it will be used, along with a simulator developed in Chapter 4, to analyze the long-term performance of storage/release systems.

Computer Simulation

Computer simulators have become common tools in scientific and engineering endeavors. The advent of computers with large memories and data handling capacities has allowed the development of sophisticated environmental system simulators capable of detailed analyses. The recent revolution in computer accessibility has brought this capacity to most analysts. The need to analyze urban stormwater storage/release systems has spawned a small family of computer simulators over the last decade, ranging from very simple models with limited capabilities to more complex models capable of simulating a wide range of designs and operating policies. Simulators have the advantage (over the statistical methods) of being able to produce the transient response of these systems. Several of these models are briefly reviewed below.

City of Milwaukee detention tank model. In a study done for the City of Milwaukee concerning the design and operation of the Humboldt Avenue combined sewer overflow detention basin, a simple model was developed to aid in the analysis (City of Milwaukee et al., 1975). This model treats the detention basin as a plug-flow reactor and assumes a constant volume once the basin is filled to the overflow depth. Pollutants are assumed to be removed under first-order conditions. The model also has a simple surface runoff module which develops a series of hourly runoff values from an hourly rainfall record. The volume remaining in the basin at the end of each storm is drained at a constant rate and all retained pollutants, including the removed quantities, are thoroughly mixed for release with this flow. This option was included to simulate the return of the basin contents to an interceptor sewer and, in turn, to a sanitary sewage treatment facility. The model was used to study the effectiveness of varying basin sizes and dry-weather drawdown rates. The results were also coupled with a receiving water model to estimate the effect of the basin on water quality in local waterways. The program was written in FORTRAN and is displayed in the report cited above.

University of Kentucky sediment detention basin model (DEPOSITS). DEPOSITS is a moderately sophisticated model developed by Ward et al. (1977) to simulate sediment detention facilities. It models these storage/release systems as variable-volume, plug-flow reactors. However, in this case, the pollutants (i.e., sediment loads) are removed by simulating the settling of particles. Because it is designed to simulate sediment detention basins, the model also calculates the effect of sediment deposition on the storage capacity of the basin. The model was verified on a number of surface mine sedimentation ponds. DEPOSITS requires particle size distributions and a complete hydraulic description of the basin and outlet structure. The model is primarily designed to be run on single storm event hydrographs but it can be adapted for multi-storm simulations.

Medina's storage/treatment model. Medina (1976; 1981_a_ with others) constructed a model that treats urban stormwater detention basins as completely or intermediately mixed reactors. Pollutants are treated as first-order reactants. The differential equations governing this system are evaluated using the discrete-input method described in an earlier section. Unfortunately, the model is limited to a linear relationship between outflow and basin volume. This model was also successfully applied to the Humboldt Avenue detention basin (Medina et al., 1981_b_) and linked with a receiving water model to study the impact of the stormwater detention basins on water quality (Medina, 1976; Medina et al., 1981_b_).

Corps of Engineers Storage, Treatment, Overflow, Runoff Model (STORM). The Corps of Engineers STORM model is a widely used model in the field of urban and rural stormwater runoff modeling (Hydrologic Engineering Center, 1977). The model was designed to generate long-term runoff and pollutant load records with a continuous long-term hourly rainfall record. The hourly runoff record is, in turn, routed to a storage/treatment (or release) system. The system is depicted in Figure 2-12. Runoff exceeding the maximum treatment rate is stored for release at a later time. If the storage capacity is exceeded, the excess overflows directly to the receiving water. When runoff eases to the point where the treatment rate is no longer exceeded, the storage unit is drained at the rate equivalent to the difference between the maximum treatment rate and the runoff rate. The STORM model is written in FORTRAN and is specifically designed to process decades of hourly rainfall data. The model also provides simple statistics for runoff quantity and quality before and after encountering the storage/release system. In addition, the program accounts for the age of runoff held in the storage unit (on a plug-flow basis). However, it does not simulate pollutant reduction due to particle settling or any other mechanism. The term "treatment" is used in the STORM literature but no pollutant removal is simulated. The term is more precisely linked with the mainstream capacity associated with the off-line storage/release configuration. Flows remaining in the mainstream or returning to it are often routed to a treatment facility of some type and, thus, the term "treatment" is commonly applied.

EPA Storm Water Management Model (SWMM), Storage/Treatment Block. The author has developed the Storage/Treatment Block as part of this study and the comprehensive Storm Water Management Model (Huber et al., 1981). The S/T Block is a flexible, FORTRAN simulator capable of modeling several types of wastewater storage and/or treatment units and system configurations, including storage/release systems. The S/T Block

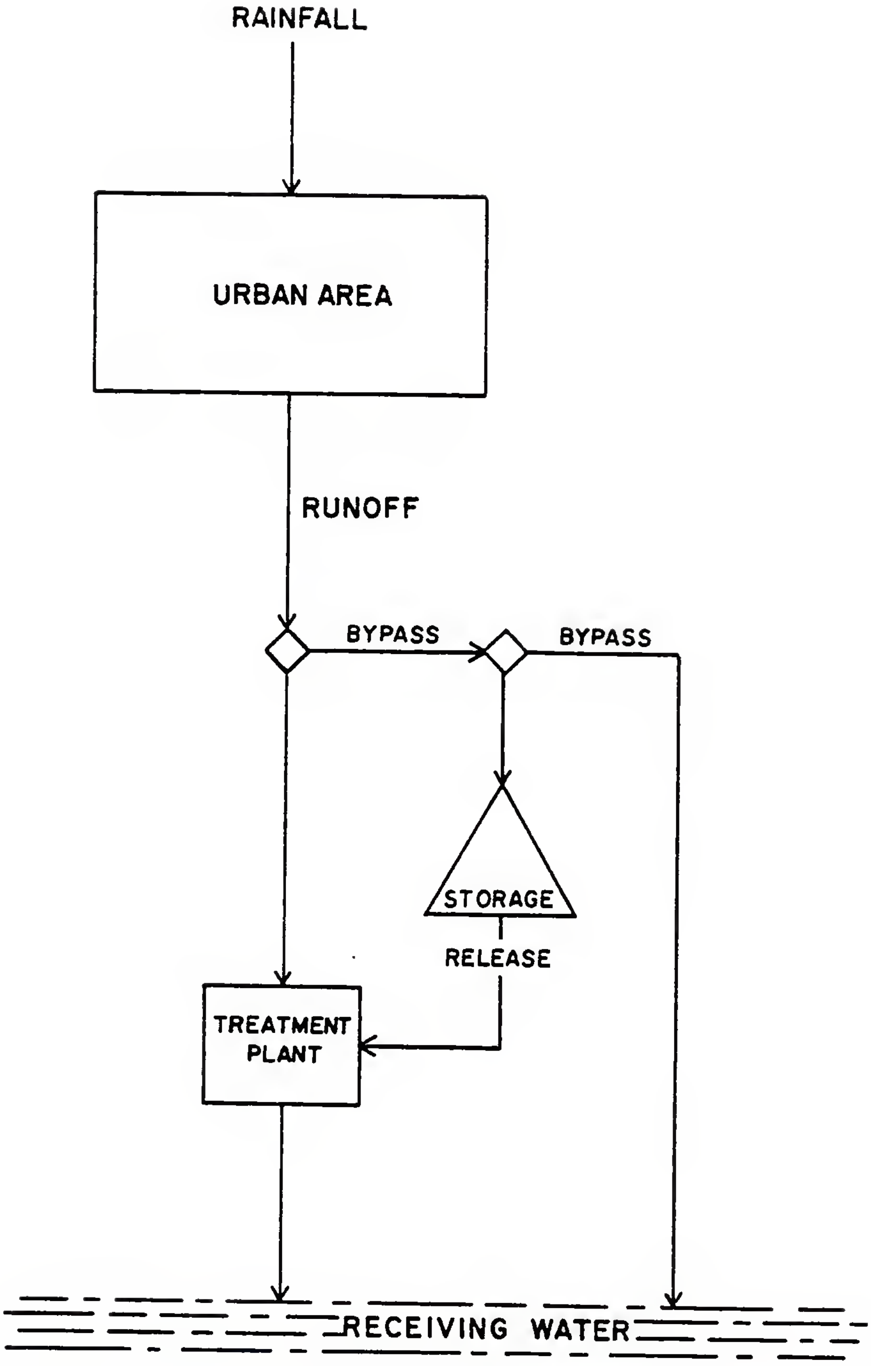


Figure 2-12. Storage/Release System, STORM (Hydrologic Engineering Center, 1977).

is readily coupled with the other portions of the SWMM model. The other modules simulate the urban runoff process, the routing of flows and pollutants through collection systems, and the impacts of urban runoff on receiving waters. Among the more relevant and interesting features of the S/T Block are

1) the ability to model a wide variety of storage basin geometries and outlet structures;
2) the capability to simulate the dry-weather release of the basin contents;
3) the ability to characterize pollutants by particle size/specific gravity or settling velocity distributions and to simulate particle settling;
4) a provision for modeling storage units as plug-flow or completely mixed reactors;
5) the ability to use a wide variety of pollutant removal equations in the plug-flow mode; and
6) the modeling of pollutants as first-order reactants in the completely mixed mode.

The Storage/Treatment Block is probably the most versatile model of its kind. It will be used in later chapters of this study to provide the necessary simulation capacity to develop long-term production functions. The details of the model algorithms are discussed in Chapter 4.

Measure of Performance--Reliability

The information provided by the various evaluation techniques can be expressed in terms of reliability. Reliability is a widely used engineering tool that provides a performance measure for systems operating on stochastic processes. Kritskiy and Menkel (1952, indirect ref. from Klemeš, 1981) recognized three reliability measures for use in the design analysis of water supply reservoirs. These measures are discussed below and modified for application to urban stormwater quality problems.

Annual reliability, R_a, is defined in water supply parlance as the probability that no failure in water supply will occur within a year. Failure is defined as not being able to meet a predetermined draft from the reservoir. The value of R_a is estimated by

$$R_a = \frac{N - N_f}{N} \tag{2-73}$$

where N_f = number of failure years, and

N = total number of years.

This measure of reliability is also known as "occurrence-based" reliability and this term is preferred here. The non-catastrophic nature of stormwater quality problems and economic considerations require a different definition. Thus, occurrence-based reliability may be redefined to denote the probability that a failure will not occur more than a specified number of times per year (or other period). Alternatively, annual reliability could be defined as the average number of occurrences or failures per year (or other period). This measure of reliability has been used in stormwater quality management to indicate the annual number of times a storage/release system overflows or is bypassed (Roesner et al., 1974; Labadie and Grigg, 1976; Charles Howard and Assoc., 1979; Howard et al., 1979) and the reduction in the annual number of beach closings due to the control of combined sewer overflows with a storage/release system (E.D. Driscoll and Assoc., 1981).

Time reliability, R_t, is defined in the water supply context as the portion of the time that the water demand or specified draft was satisfied, i.e.,

$$R_t = \frac{T_d}{T_t} \qquad (2\text{-}74)$$

where T_d = time period that demand was satisfied, T, and

T_t = total operating period, T.

Equation 2-74 is useful for the purposes herein when the term "demand" is replaced with "water quality objective." This modified definition is the essence of the pollutant load- or concentration-frequency relationships found in several studies on the effects of stormwater pollution control on receiving water quality (Medina, 1976, 1979; Heaney et al., 1977; Scholl and Wycoff, 1981; E.D. Driscoll and Assoc., 1981; Medina et al., 1981b).

Volume reliability, R_v, is defined for water supply reservoirs as the portion of the total demand volume actually supplied during the operating period, i.e.,

$$R_v = 1 - \frac{\int\limits_{q_s<q_d} (q_d - q_s)\, dt}{\int_0^{T_t} (q_d)\, dt} \qquad (2\text{-}75)$$

where q_d = water demand rate, L^3T^{-1}, and

q_s = water supply rate, L^3T^{-1}.

This measure of reliability can be applied to stormwater storage/release systems by allowing the numerator to represent the total volume and/or pollutant load not captured by the system. Correspondingly, the denominator represents the total volume and/or load entering the system. This is a popular measure of performance in stormwater quality studies and it is often reported as the average performance level (Heaney et al., 1976, 1977, 1979; Hydroscience, Inc., 1979; DiToro and Small, 1979; Nix et al., 1981).

The use of reliability in the analysis of urban stormwater quality management and, specifically, storage/release systems is not new, but the term is rarely used. This is unfortunate because it is a descriptive term and entirely appropriate to storage/release systems operating under such conditions. The classification scheme described above allows a broad framework with which to evaluate stormwater quality problems. Not all of the evaluation techniques are capable of providing the information needed to estimate each form of reliability. Obviously, the selection of a technique must fit the nuances of the problem.

Summary

This chapter has established the basic theoretical framework for analyzing storage/release systems and briefly reviewed several useful evaluation techniques. These techniques range from simple empirical relationships to sophisticated computer simulations. Examples of two of the more useful techniques will be investigated in detail in the next two chapters. The concept of reliability was introduced as a unifying framework for quantifying the long-term performance of storage/release systems as stormwater quality control devices.

CHAPTER 3

STATISTICAL ANALYSIS OF STORAGE/RELEASE SYSTEMS

Introduction

Knowledge of the probability distributions governing the inflow stream to a storage/release system provides the basis for a set of powerful statistical analysis procedures. Hydroscience, Inc. (1979) developed a methodology to analyze long-term hourly rainfall records and to develop a set of statistics describing the resulting stormwater runoff characteristics of urban areas. Given the set of runoff statistics and governing distributions, the control effectiveness of various treatment, interception and storage/release devices can be determined through techniques proposed by Small and DiToro (1979) and DiToro and Small (1979). This material was also included in the Hydroscience work (1979). These techniques were later used by the author and others in a report describing a methodology to analyze combined sewer overflows, their control, and impact in receiving waters (E. D. Driscoll and Assoc., 1981). The techniques are described in some detail in the remainder of this chapter.

System Conceptualization

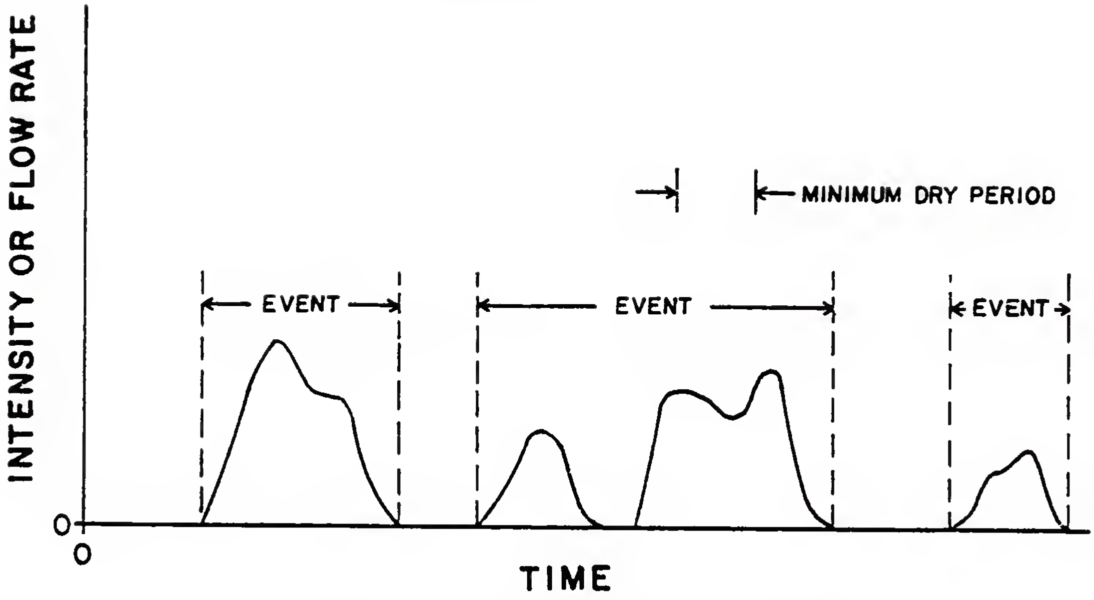
The basic storage/release system configurations discussed in Chapter 2 are amenable to the techniques forming the Hydroscience statistical method. However, some simplications were necessary to avoid overburdening the methodology with theoretical details that would probably prove to be debilitating. Foremost among the simplifications is the assumption that the storage basin is operated with the bypass mode in place. The second simplification requires the interevent (i.e. dry weather) releases from the storage unit to be at a constant rate. The third simplification assumes that pollutant removal is determined by constant removal efficiencies. This particular restriction is probably the most untenable of the system simplifications.

Characterization of Rainfall and Runoff Events

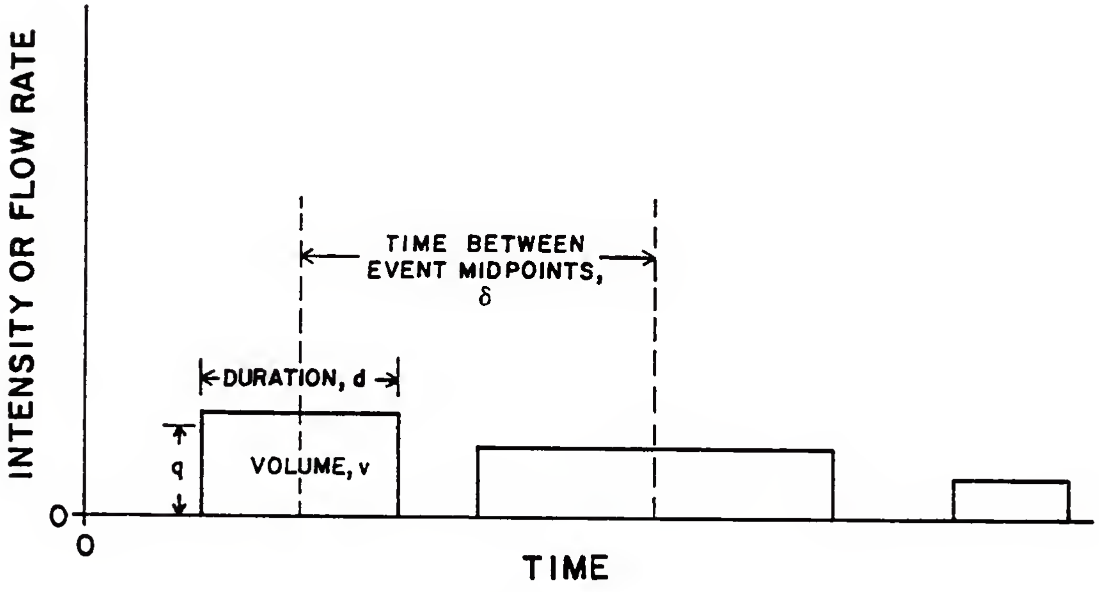
The stochastic rainfall or runoff process is segregated into a series of independent, randomly occurring events as shown in Figure 3-1(a). This representation is further simplified by characterizing each event by a uniform, rectangular hydrograph (see Figure 3-1(b)). Each event is described by its duration, volume, average flow rate or intensity, and the elapsed time since the last event or interevent time. The interevent time is measured between event midpoints. The discussion that follows is valid for rainfall and runoff events.

Statistical Properties of Event Characteristics

The characteristics describing each rainfall or runoff event are random variables. Statistics describing these characteristics can be estimated from historical records. The complete set of statistics required for this method is given in Table 3-1. The mean and coefficient of variation are used to describe each random variable. The sample mean is given by



(a) Actual Record and Event Delineation



(b) Simplified Representation

Figure 3-1. Simplified Representation of Independent Rainfall or Runoff Events (Hydroscience, Inc., 1979).

Table 3-1. Rainfall/Runoff Event Parameters and Statistics

Parameter[a]	For each event[b]	Mean[b]	Coefficient of variation[b]
Intensity or flow rate, L^3T^{-1}	q	Q	ν_q
Duration, T	d	D	ν_d
Volume, L^3	v	V	ν_v
Time between event midpoints (interevent time), T	δ	Δ	ν_δ

a. Event intensities/flow rates and volumes are often normalized over the catchment area. When this is done, the units become LT^{-1} and L, respectively.

b. The subscripts "P" and "R" are used to denote a rainfall or runoff event parameter or statistic, respectively.

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \qquad (3\text{-}1)$$

where $\bar{x}$ = mean of the sample

x_i = i^{th} sample, and

n = total sample size.

The sample coefficient of variation is

$$\nu = s/\bar{x} \qquad (3\text{-}2)$$

where ν = sample coefficient of variation, dimensionless, and

$s = \frac{1}{n}\sum_{i=1}^{n} (x_i - \bar{x})^n$ = sample standard deviation.

In effect, the first two moments of the data are used to describe the event characteristics. The mean describes the central tendency and the coefficient of variation measures the variance around the mean. The coefficient of variation is used in place of the standard deviation because it frees the measure of variability from dependence on specific dimensions. High values of ν reflect greater variability in the random sample; low values reflect lesser variability. Of course, sample statistics only approximate the true statistics; they become more accurate as the sample population increases.

The sample statistics allow the assignment of one- or two-parameter probability density functions to each of the event characteristics. Rainfall and runoff event flows and durations appear to be well represented by the gamma distribution (Hydroscience, Inc., 1979). Such a distribution requires an estimate of the mean and coefficient of variation and is given as

$$p(x) = (\kappa/\bar{x})^{\kappa} \frac{x^{\kappa-1}}{\Gamma(\kappa)} \exp(-\kappa x/\bar{x}) \qquad (3\text{-}3)$$

where $p(x)$ = gamma probability distribution function of x,

$\kappa = 1/\nu^2$, and

$\Gamma(\kappa)$ = gamma function with argument κ.

The probability distribution function for the time between rainfall and runoff event midpoints is well represented by an exponential distribution (Hydroscience, Inc., 1979). However, the exponential distribution is simply a special case of the gamma distribution in which the coefficient of variation is 1.0, i.e.,

$$p(x) = (1/\bar{x})\ \exp(-x/\bar{x}) \qquad (3\text{-}4)$$

The cumulative gamma distribution for several values of ν is shown in Figure 3-2. The cumulative plots allow the extraction of valuable frequency information. For example, if the mean runoff event flow was determined to be 0.10 inches/hour and the coefficient of variation found to be 1.5, it could be stated that 8 percent of all events exceed 0.30 inches/hour. If the average number of events is known to be 100 per year, one could conclude that 8 events per year exceed 0.30 inches/hour and the return period for these events is 1.50 months.

Throughout the methodology, it is assumed that event flows and durations and interevent times are independent. This is probably not a valid assumption. For example, areas that experience short intense storm cells in the summer and long, less intense storms in the winter would show some level of dependence between flows and durations. However, this assumption allows some interesting and relatively simple analytical procedures to be developed and it appears to be adequate for the objectives of the methodology.

The gamma and exponential distributions have often been used in the analyses of rainfall data (Thom, 1951, 1958; Eagleson, 1970; Chow and Yen, 1976; Howard, 1976; Yen, 1977; Howard et al., 1979; Hydroscience, Inc., 1979). However, it may not always be the most appropriate.

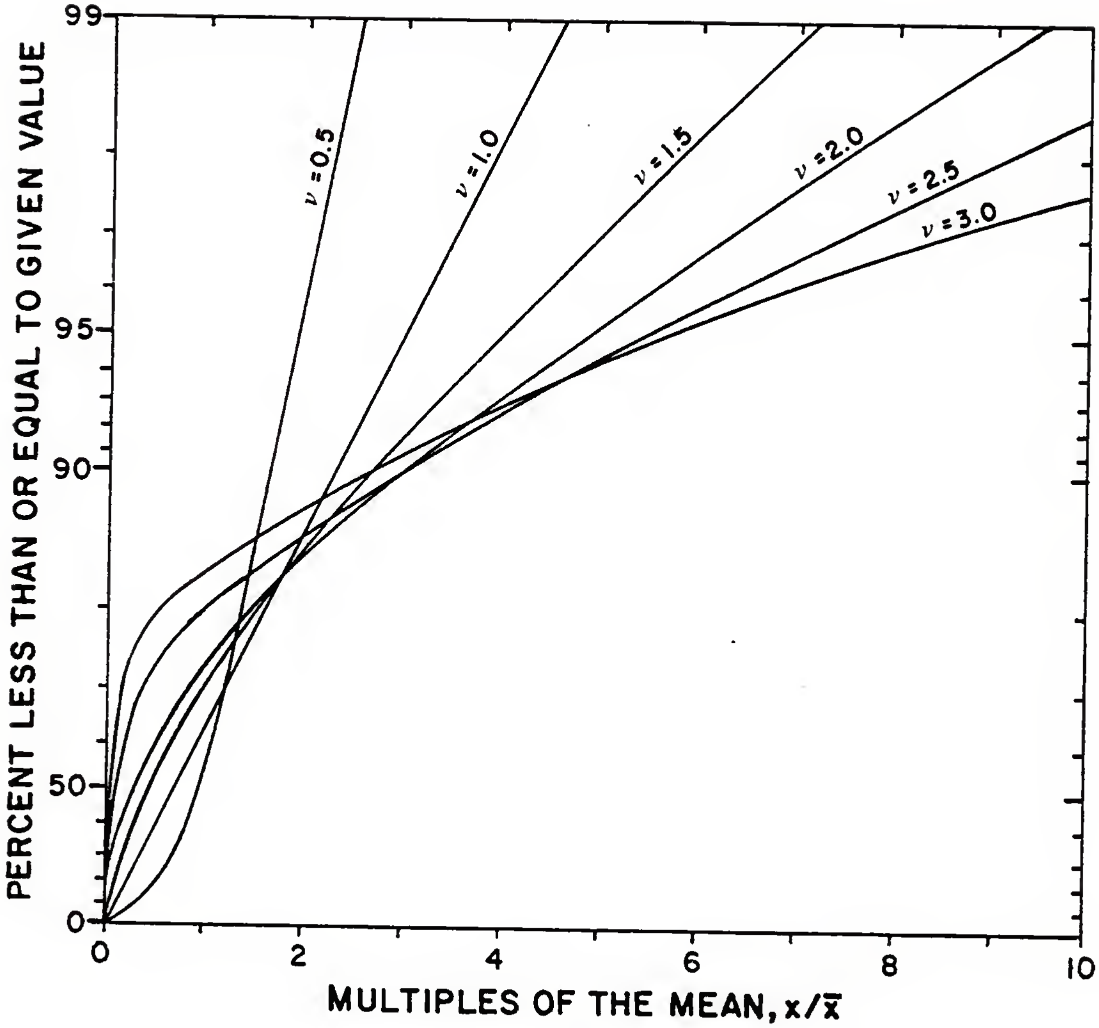


Figure 3-2. Cumulative Gamma Distributions (Hydroscience, Inc., 1979).

Distributions of rainfall event characteristics are skewed; i.e., there are many small observations and few large ones. Other distributions, including the lognormal and Weibull functions, are also capable of representing such populations. The advantage of the gamma distribution is that it appears to be reasonably accurate and simple enough to determine analytical solutions for the performance of storage/release systems.

Definition of an Independent Event

The statistical method is based on the analysis of independent or uncorrelated events. Up to this point, no definition of an independent event has been given. The most widely accepted approach is to select the minimum amount of "dry" time which must occur between two separate independent events (Heaney et al., 1977; Hydroscience, Inc., 1979). Any activity not preceded by this minimum is not a separate, independent event. The "dry" time is not the interevent time; the latter is calculated between event midpoints after the events are defined with the former. The appropriate value will usually be different for rainfall and runoff records because of the effects of the catchment system.

Several methods have been proposed to determine the minimum dry period. Medina (1976), Howard (1976), and Heaney et al. (1977) recommend that an open-series autocorrelation analysis be performed on hourly rainfall records to determine the time lag at which no significant autocorrelation is present (as measured by the autocorrelation coefficient). An autocorrelogram for the 1968 hourly rainfall record in Des Moines, Iowa, is shown in Figure 3-3 (Medina, 1976). From this analysis it was determined that the appropriate minimum dry period is 10 hours. The tolerance limit (defined at the 95-percent probability level) was

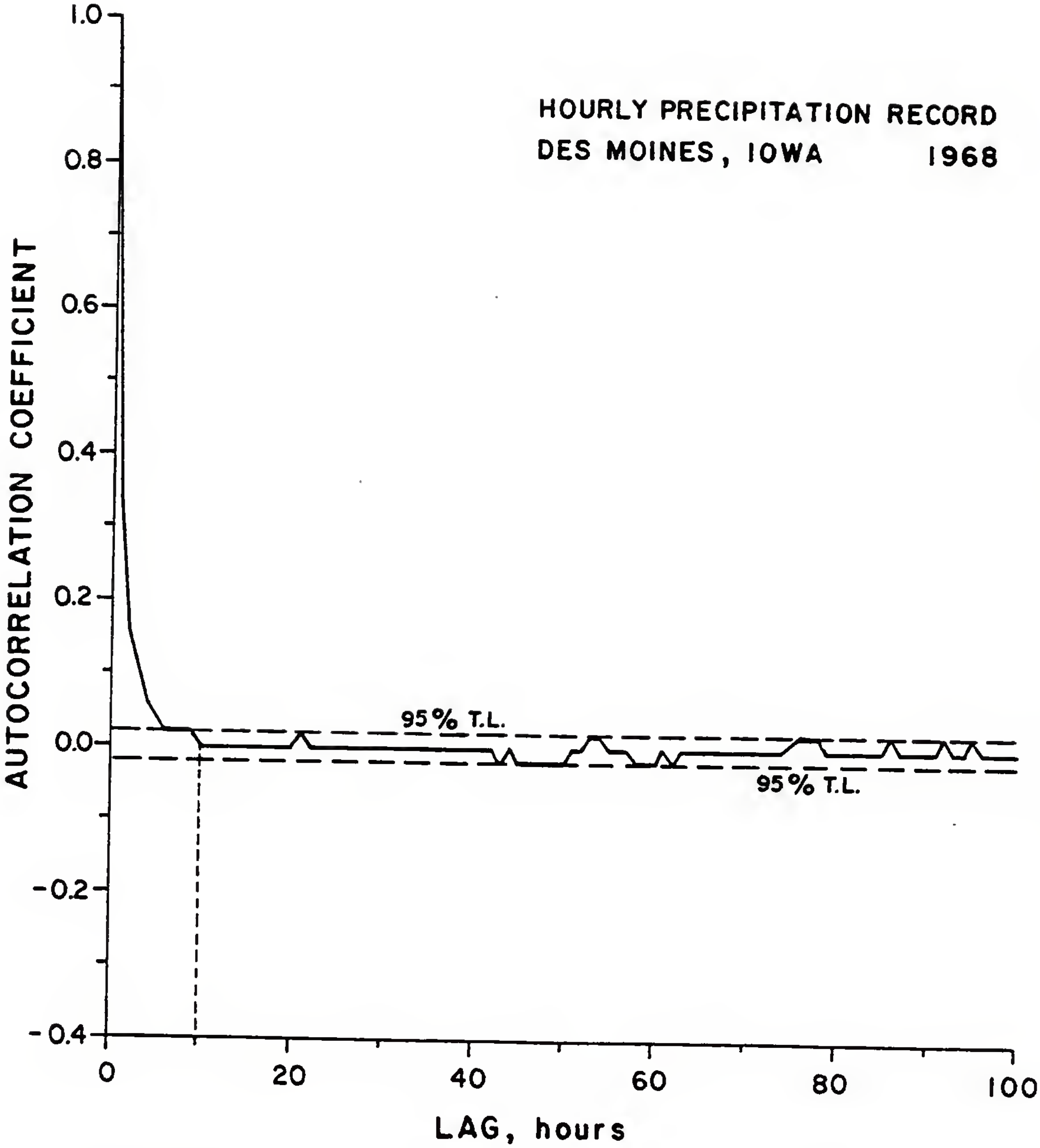


Figure 3-3. Autocorrelation Function for the Hourly Precipitation Record of Des Moines, Iowa, 1968 (Medina, 1976).

first reached at 7 hours but it was recommended that the point where the autocorrelation coefficient first reaches zero be selected (i.e., 10 hours). This rainfall record was used to generate a runoff record with the computer simulator STORM. The generated data set was analyzed in the same manner; the resulting autocorrelogram is shown in Figure 3-4 (Medina, 1976). In this case, the dry period time separating independent events is 9 hours. The differences between the two autocorrelograms are due to the fact that STORM first satisfies depression storage and evaporation before producing runoff as a simple product of a runoff coefficient and the rainfall rate. Hydroscience, Inc. (1979) has noted that this technique is biased at shorter lag times because the autocorrelation coefficient is inflated due to high correlations within events. They suggest that it is better to perform the autocorrelation analysis on the events resulting from different minimum dry periods.

A second approach investigates the relationship between the minimum dry period and the number of events per year (Medina, 1976; Heaney et al., 1977). Several such relationships are plotted in Figure 3-5. The point where an increase in the minimum dry time no longer causes a significant decrease in the number of events determines the proper value. Such an approach is somewhat more subjective than the autocorrelation technique; as one can see in Figure 3-5, a well-defined breakpoint is not always evident.

The third approach, advocated by Hydroscience, Inc. (1979), assumes that the time between independent rainfall or runoff events is exponentially distributed. The exponential distribution and its discrete counterpart, the Poisson distribution, are commonly used to describe times between independent, probabilistic events of several kinds (Benjamin

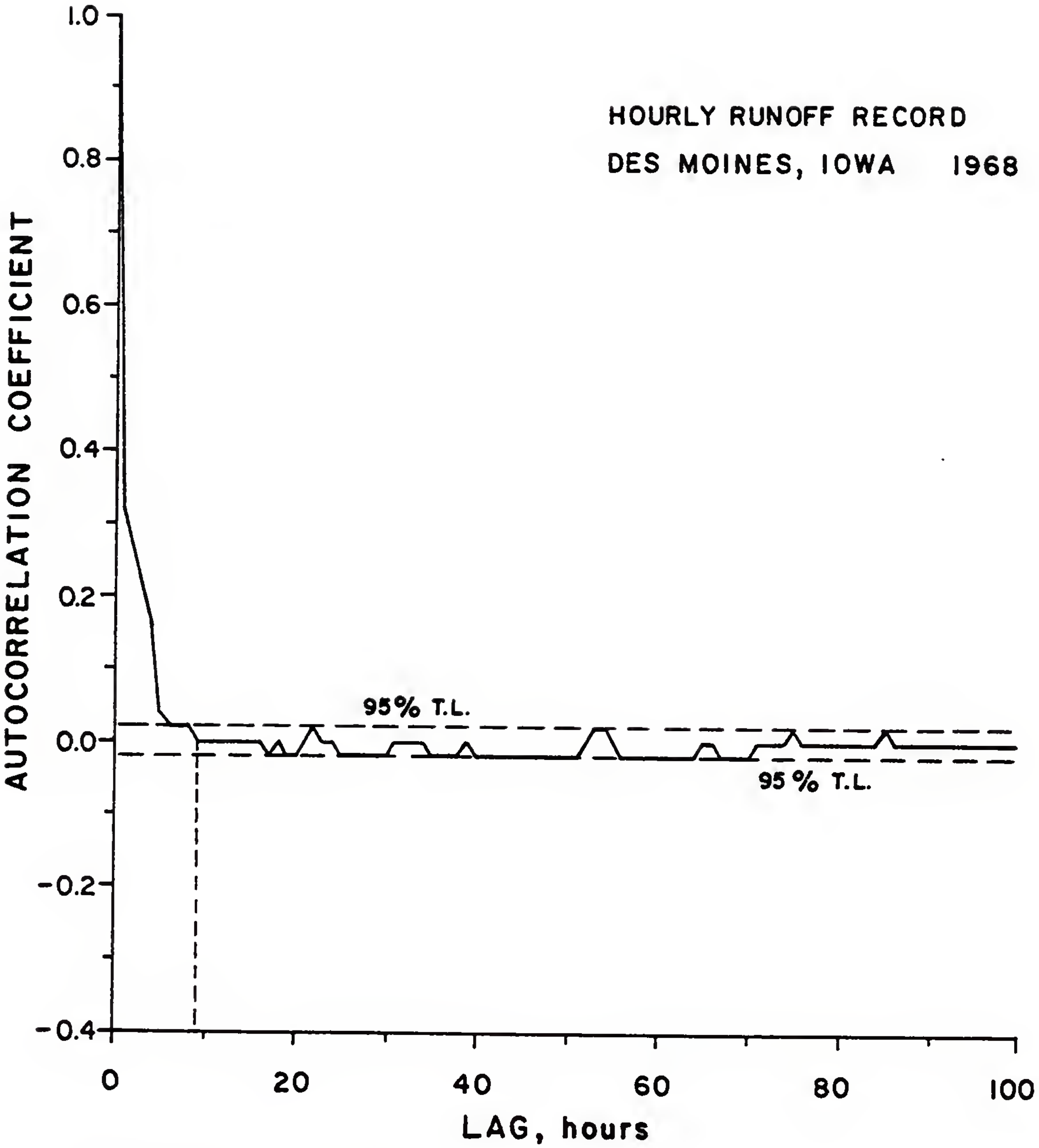


Figure 3-4. Autocorrelation Function for the Hourly Runoff Record (Generated by STORM) of Des Moines, Iowa, 1968 (Medina, 1976).

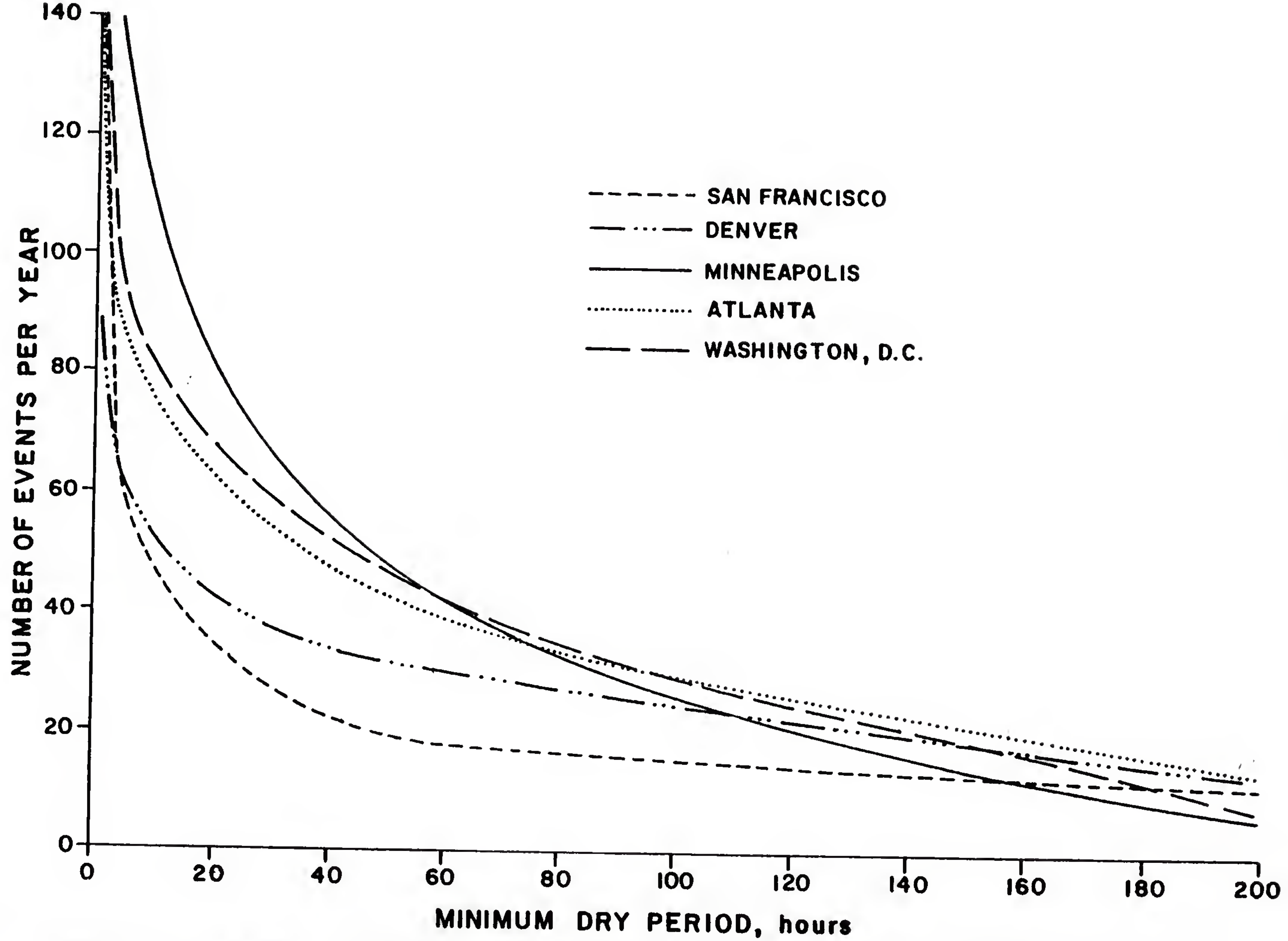


Figure 3-5. Number of Events per Year Versus the Minimum Dry Period (Heaney et al., 1977).

and Cornell, 1970). Given this assumption, the minimum dry period is adjusted until the coefficient of variation of interevent times (determined from a sufficiently long record) is 1.0. Recall that an exponential distribution is a special case of the gamma distribution in which $\nu = 1.0$. This is certainly not an absolute event definition but it provides a convenient framework for characterizing the distribution of interevent times. Based on this approach, analyses performed by Hydroscience, Inc. (1979) and E. D. Driscoll and Assoc. (1981) indicate that most areas of the United States have minimum dry periods of 3 to 15 hours.

An alternative to the selection of a minimum dry period to separate independent events investigates the local meteorological patterns to determine independent events based on the storm structure or scale (Eagleson, 1970, Heaney et al., 1977). For example, two periods of rainfall or runoff may come from two separate, independent convective cells or the same frontal system. This approach involves considerable study and is probably not warranted for the purposes of the Hydroscience method.

Synoptic Analysis Program--SYNOP

A computer program, named SYNOP, has been developed to analyze long-term rainfall records (Municipal Environmental Research Laboratory, 1976). Its primary function is to generate the array of statistics listed in Table 3-1. These statistics are provided on a monthly basis, an annual basis, and for all storms over the entire record. The monthly values are particularly useful where wide seasonal differences occur. Although designed to analyze rainfall records (specifically in the form provided by the National Climatic Center, NOAA), the program can also analyze long-term runoff records. These records may be generated by a computer simulator, by a monitoring program, or both.

Runoff Statistics from Rainfall Statistics

Long-term hourly rainfall records, extending over many years, are more commonly available than equally detailed runoff records. A large network of rainfall gauges has long been maintained by the National Weather Service, whereas runoff records are somewhat sporadic and relatively short. The Hydroscience statistical method includes techniques to estimate runoff statistics from rainfall event statistics. The techniques were derived to allow preliminary analyses from little more than the rainfall record.

Quantity statistics.

Two extremely simple relationships are used to derive the mean runoff event volume and flow from the corresponding rainfall statistics. For volume,

$$V_R = R_c V_P \tag{3-5}$$

where V_R = mean runoff event volume, L^3,

R_c = average runoff coefficient, and

V_P = mean rainfall event volume, L^3.

The runoff coefficient, R_c, represents the average runoff-to-rainfall ratio. Of course, this ratio varies from storm to storm, but for preliminary analyses the estimate is probably adequate. The value of R_V can be estimated by an analysis of local rainfall/runoff data or estimated from one of several simple techniques (Miller and Viessman, 1972; Hydrological Engineering Center, 1977; Hydroscience, Inc., 1979).

The mean runoff flow is calculated as

$$Q_R = R_c Q_P \frac{D_P}{D_R} \tag{3-6}$$

where Q_R = mean runoff event flow rate, L^3T^{-1},

Q_P = mean rainfall event intensity, L^3T^{-1},

D_P = mean rainfall event duration, T, and

D_R = mean runoff event duration, T.

The ratio D_P/D_R is included to account for runoff continuing after the rainfall event has subsided. This is particularly useful in large catchments where travel times are lengthy. The value of D_R is estimated by a unit hydrograph procedure developed by Brater and Sherill (1975). The details are presented in the Hydroscience report (1979).

The mean interevent time for runoff events, Δ_R, is assumed to equal the rainfall value, Δ_P. Given the simple coefficient method of estimating V_R, this is reasonable. Actually, many small rainfall events produce no runoff because initial storages or abstractions are not satisfied. This would cause the value of Δ_R to be greater than Δ_P. The coefficients of variation for runoff event flows, ν_{qR}, and volumes, ν_{vR}, are also assumed to equal their rainfall counterparts.

The above measures are crude. It would be preferable to drive a runoff simulator with the hourly rainfall record and analyze the results with SYNOP. Any available runoff data could be used to calibrate the model. Obviously, the best method would analyze a long series of observed runoff flows but such records are scarce.

Quality statistics. The mean pollutant load for all runoff events is determined by the mean pollutant concentration and the mean runoff event volume:

$$M_R = C_R V_R \tag{3-7}$$

where M_R = mean runoff event pollutant load, M, and

C_R = mean runoff event pollutant concentration, ML^{-3}.

This simple equation assumes that the pollutant concentrations are independent of runoff volumes. This is probably not true, but is assumed to be adequate for most preliminary analyses. A similar relationship was developed for the mean pollutant load rate, i.e.,

$$W_R = C_R Q_R \tag{3-8}$$

where W_R = mean runoff event pollutant load rate, MT^{-1}.

Again, the assumption of independence between pollutant concentrations and runoff flows is made. If the independence assumptions are inadequate, the following corrections can be made:

$$M_R = C_R V_R \ (1 + \nu_c \nu_{vR} \rho_{cv}) \tag{3-9}$$

$$W_R = C_R Q_R \ (1 + \nu_c \nu_{qR} \rho_{cq}) \tag{3-10}$$

where ν_c = coefficient of variation for runoff event pollutant concentrations,

ρ_{cv} = linear correlation coefficient between pollutant concentrations and runoff volumes, $-1 \leq \rho_{cv} \leq 1$, and

ρ_{cq} = linear correlation coefficient between pollutant concentrations and runoff flow rates, $-1 \leq \rho_{cq} \leq 1$.

A positive value of ρ_{cv} or ρ_{cq} would indicate that higher flows or volumes produce higher concentrations. A negative value would indicate that the dilution effect of large runoff events is dominant. Unfortunately, deriving statistically significant estimates for ρ_{cv} and ρ_{cq} requires a large amount of data.

From the assumptions associated with equations 3-7 and 3-8, it is also assumed that

$$\nu_m = \nu_{vR} \qquad (3\text{-}11)$$

and

$$\nu_w = \nu_{qR} \qquad (3\text{-}12)$$

where ν_m = coefficient of variation of runoff pollutant loads, and

ν_w = coefficient of variation of runoff pollutant load rates.

It also seems reasonable to expect the distribution of loads and load rates to be similar to the probability distributions for runoff volumes and flows (i.e., gamma distributed).

Once again, it should be emphasized that it would be more appropriate to analyze the results of a long-term simulator (calibrated against any available local data) or an actual long-term record.

Analysis of Storage/Release Systems

Direct analysis of the long-term behavior of storage/release systems under the stimulus of a series of random runoff events is possible if the probability distributions governing the event characteristics are known (DiToro and Small, 1979; Hydroscience, Inc., 1979). The gamma and exponential distributions were established in the pertinent literature and in the previous section as applicable to the characteristics of independent rainfall or runoff events. This knowledge and the system conceptualization, presented earlier, form the basis of the analytical procedures developed by DiToro and Small (1979), and presented by Hydroscience, Inc. (1979). Their work is summarized below. The subscript "R" is dropped from most of the runoff event parameters and statistics to avoid unnecessary clutter.

Storage (In-Line Configuration)

A storage basin captures, from a particular event, up to its currently available or effective capacity, V_e. If the volume of the event exceeds V_e, the excess is bypassed. The only time the entire storage basin capacity, V_B, is available is when the basin is totally emptied of water captured from previous storms. The basin is emptied between events at a constant release rate, Ω. It is readily seen that the average effective storage capacity, V_E, and the release rate, Ω, determine the basin's long-term performance.

Unfortunately, the effective storage capacity is a stochastic process with a memory, i.e., it is at least a first-order Markov or autoregressive process. In other words, the current value of V_e is a function of previous events. The analysis of processes such as these can be difficult and unwieldy. In order to facilitate the development and application of a representative model of the storage/release system, the interaction between any two events is assumed to be as shown in Figure 3-6. Event 1 begins with the mean effective storage capacity, V_E, available to store runoff. The volume associated with an arbitrary runoff event (denoted here as event 1), v, fills the basin to $(V_B - V_E) + v$. Between events 1 and 2, the basin is emptied at a constant release rate, Ω. At the beginning of event 2, the basin has an available capacity of V_e. The value of V_e is a function of v and the time between events, δ:

$$V_e = V_B \text{ for } v \geqq V_E,\ \delta \geqq \frac{V_B}{\Omega} \qquad (3\text{-}13)$$

$$V_e = V_B \text{ for } v \leqq V_E,\ \delta \geqq \frac{v + V_B - V_E}{\Omega} \qquad (3\text{-}14)$$

$$V_e = \delta\Omega \text{ for } v \geqq V_E,\ \delta \leqq \frac{V_B}{\Omega} \qquad (3\text{-}15)$$

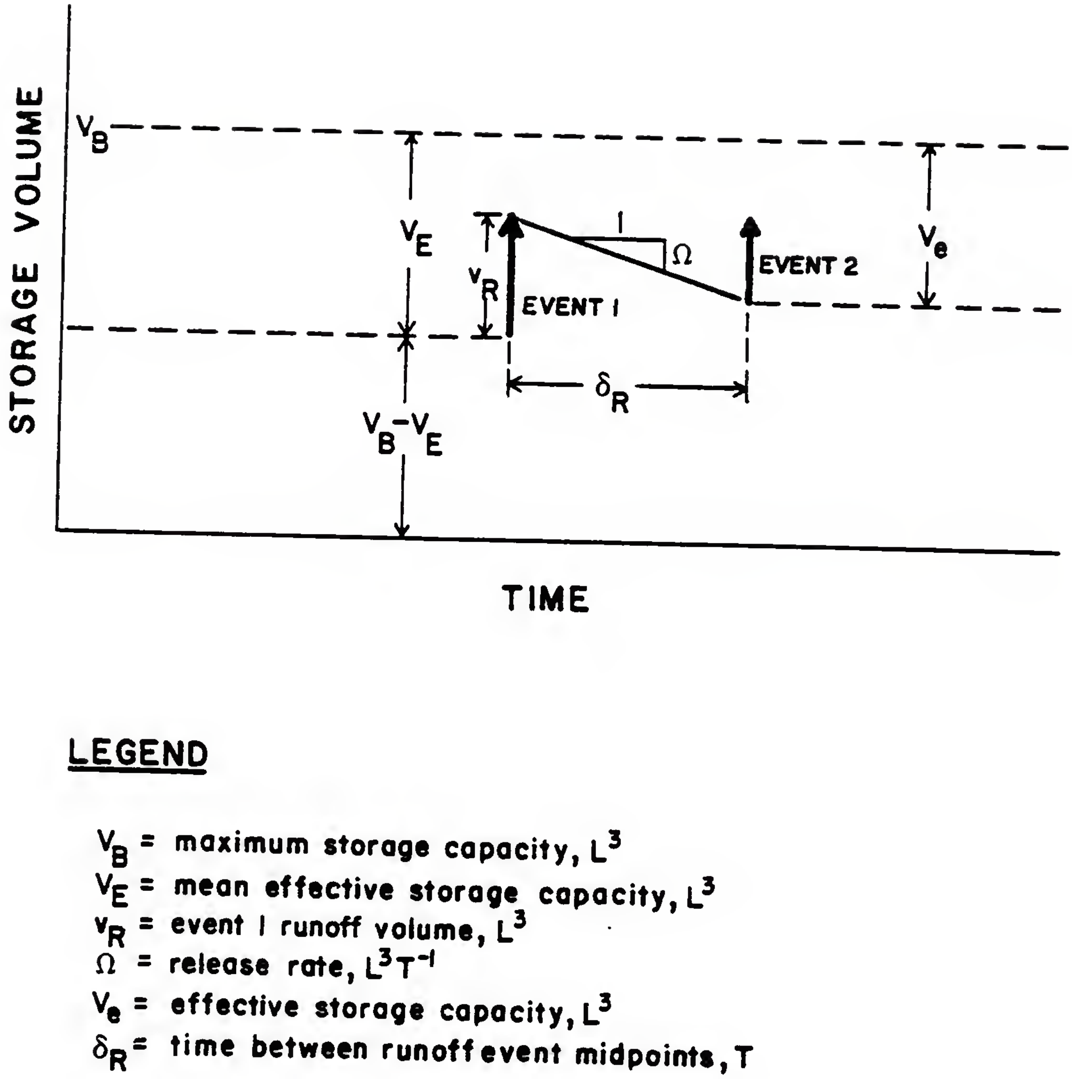

STORAGE VOLUME
V_B
V_E
v_R
1
Ω
EVENT 1
EVENT 2
V_e
$V_B - V_E$
δ_R
TIME
LEGEND
V_B = maximum storage capacity, L^3
V_E = mean effective storage capacity, L^3
v_R = event 1 runoff volume, L^3
Ω = release rate, L^3T^{-1}
V_e = effective storage capacity, L^3
δ_R = time between runoff event midpoints, T

Figure 3-6. Determination of Effective Storage Capacity, V_e (DiToro and Small, 1979; Hydroscience, Inc., 1979).

$$V_e = \delta\Omega + V_E - v \text{ for } v \leqq V_E,\ \delta \leqq \frac{v + V_B - V_E}{\Omega} \qquad (3\text{-}16)$$

The expected value of V_e, V_E, is given as

$$V_E = \int_\delta \int_v V_e\ p(\delta)\ p(v)\ dv\ d\delta \qquad (3\text{-}17)$$

where $p(\delta)$ = probability distribution function for interevent times, and

$p(v)$ = probability distribution function for runoff event volumes.

Equation 3-17 was evaluated with an exponential distribution for interevent times and the following distribution function for runoff event volumes:

$$p(v) = \frac{2}{V} K_0[2(\frac{v}{V})^{1/2}] \qquad (3\text{-}18)$$

where V = mean runoff event volume, L^3, and

K_0 = modified Bessel function of the second kind.

Equation 3-18 was derived from the assumption that event flows and durations are exponentially distributed ($\nu_q = \nu_d = 1.0$) and independent (thus, $V = QD$). This is accomplished by constructing the corresponding cumulative distribution function for runoff event volumes, i.e.,

$$P(v) = \frac{1}{QD} \int_q \int_d \exp(-\frac{q}{Q})\ \exp(-\frac{d}{D})\ dd\ dq \qquad (3\text{-}19)$$

or

$$P(v) = 1 - 2(\frac{v}{QD})\ K_1[2(\frac{v}{QD})^{1/2}] \qquad (3\text{-}20)$$

where $P(v)$ = cumulative distribution function for runoff event volumes,

Q = mean runoff event flow rate, L^3T^{-1},

D = mean runoff event duration, T, and

K_1 = modified Bessel function of the second kind of order one.

Differentiating equation 3-20 yields equation 3-18. It can also be shown that the coefficient of variation for event volumes under these circumstances is $\sqrt{3}$ (Hydroscience, Inc., 1979), i.e.,

$$\nu_v = \nu_q \nu_d [1 + 1/\nu_q^2 + 1/\nu_d^2]^{1/2} \qquad (3\text{-}21)$$

$$= (1)(1)[1+1+1]^{1/2} = \sqrt{3}$$

With equation 3-18 and probability of V_e occurring as described by each of equations 3-13 through 3-16, equation 3-17 can be written as

$$V_E = 2\Delta\Omega \left[1 - \exp\left(\frac{-V_B}{\Delta\Omega}\right)\right] \left(\frac{V_E}{V}\right)^{1/2} K_1 \left[2\left(\frac{V_E}{V}\right)^{1/2}\right]$$

$$+ \frac{2}{V} \int_{v=0}^{V_E} \left[-(\Delta\Omega)\exp\left(- \frac{v+V_B-V_E}{\Delta\Omega}\right) - v + V_E + \Delta\Omega\right]$$

$$K_0[2(\tfrac{v}{V})^{1/2}]\ dv \qquad (3\text{-}22)$$

where Δ = mean time between runoff event midpoints (interevent time, T.

The solution to this nonlinear equation is obtained numerically and presented in Figure 3-7. These curves show that V_E is approximately equal to V_B when $\Delta_R\Omega/V_R > 2$. This is not surprising if one views the term $\Delta_R\Omega$ as the "average" (not precisely) volume released from storage between events. DiToro and Small (1979) produce results from a simple simulator confirming that the simplifications given above do not significantly degrade the analysis, at least when compared with the simulator.

The determination of V_E opens the door for the analysis of the storage/release system performance in terms of stormwater volume and pollutant load control. With the assumption of independence between event flows and durations, the long-term fraction of the runoff volume bypassing the storage basin is calculated as the mean runoff event

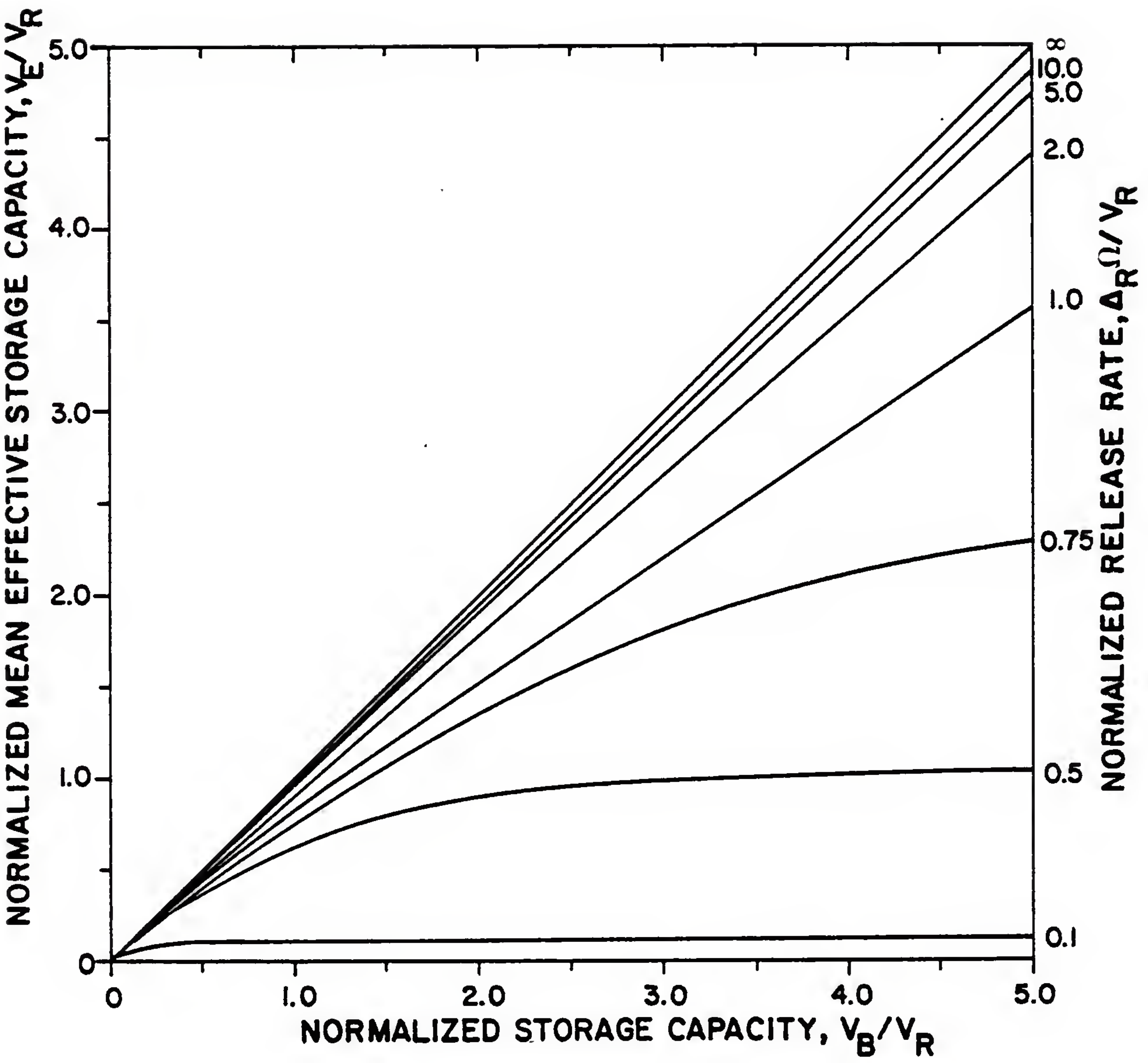


Figure 3-7. Determination of Mean Effective Storage Capacity, V_E (DiToro and Small, 1979; Hydroscience, Inc., 1979).

volume bypassing the storage basin, V', divided by the mean runoff volume, V, i.e.,

$$f_V = \frac{V'}{V} = \frac{\int_{q=0}^{\infty} \int_{d=V_E/q}^{\infty} q\left(d - \frac{V_E}{q}\right) p(d)\ p(q)\ dd\ dq}{QD} \tag{3-23}$$

where p(d) = probability distribution function for runoff event durations, and

p(q) = probability distribution function for runoff event flow rates.

The assumption of independence between event flows, durations, and pollutant concentrations allows f_V to also represent the pollutant load fraction bypassing the storage basin, i.e.,

$$f_V = \frac{C_R V'}{C_R V} \tag{3-24}$$

where C_R = mean runoff event pollutant concentration, ML^{-3}.

Note that $1\text{-}f_V$ is equivalent to a volume (or load) reliability measure (see Chapter 2).

If event flows and durations are exponentially distributed (i.e., $\nu_q = \nu_d = 1.0$) and independent, the solution to equation 3-23 becomes

$$f_V = \frac{2V_E}{V} K_2[2(\frac{V_E}{V})^{1/2}] \tag{3-25}$$

where K_2 = modified Bessel function.

For other situations in which ν_q or ν_d does not equal 1.0 (i.e., flows and durations are gamma distributed), equation 3-23 may be evaluated numerically. The results for several values of ν_v (calculated from ν_q or ν_d, see equation 3-21) are shown in Figure 3-8. DiToro and Small (1979) provide results from a simulator to demonstrate that the simplifications and assumptions used in deriving Figure 3-8 are reasonable.

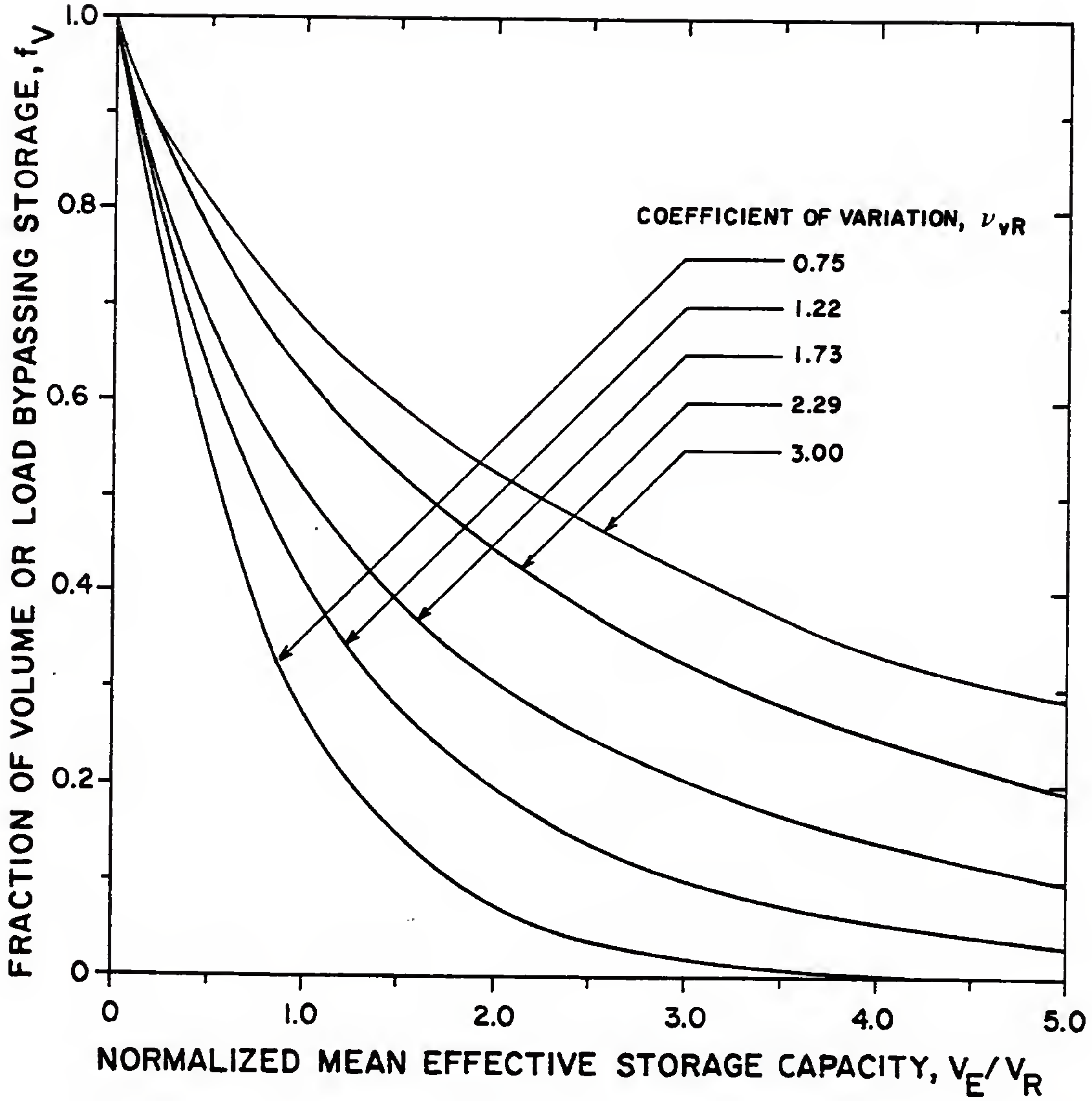


Figure 3-8. Determination of the Long-Term Fraction of the Total Pollutant Load or Runoff Volume Not Captured by the Storage Basin (DiToro and Small, 1979; Hydroscience, Inc., 1979).

Pollutant removal can occur in storage. In order to account for this, the Hydroscience method simply assigns a constant removal efficiency, η_V, to all flows leaving the storage basin. Of course, the use of a constant removal coefficient is a gross simplification of the removal ability of the basin. One possible alternative is to use the method proposed by Howard et al. (1979) and discussed in Chapter 2.

Interception and Storage (Off-Line Configuration)

The previous discussion dealt with the in-line storage/release system configuration. DiToro and Small (1979) also analyze the performance of interceptors. An interceptor is normally used in combined sewer systems to divert normal sanitary sewage flows to a treatment plant while allowing storm surges to be relieved from the system. This scenario is identical to the mainstream of the off-line configuration discussed in Chapter 2. The mainstream bypasses all flows exceeding a specified level. This type of mechanism is readily analyzed with statistical tools.

An interceptor bypasses flows when its wet-weather capacity is exceeded. The wet-weather capacity is the capacity available beyond that normally taken by sanitary sewage or dry-weather flows. The long-term fraction of the runoff volume bypassing the interceptor, f_I, is calculated as the mean runoff event volume bypassing the interceptor, V', divided by the total mean runoff event volume, V, i.e.,

$$f_I = \frac{V'}{V} = \frac{\int_{q=Q_I}^{\infty} \int_{d=0}^{\infty} (q-Q_I)d\ p(d)\ p(q)\ dd\ dq}{QD} \tag{3-26}$$

where Q_I = wet-weather capacity of the interceptor, L^3T^{-1}. With the assumption of independence between event flows, durations, and pollutant concentrations, f_I also becomes the long-term fraction of the pollutant load bypassing the interceptor, i.e.,

$$f_I = \frac{C_R V'}{C_R V} \tag{3-27}$$

The equation for f_I can also be written as

$$f_I = \frac{\kappa^\kappa \exp(-\kappa Q_I/Q)}{\Gamma(\kappa)} \int_0^\infty \zeta \, (\zeta + \frac{Q_I}{Q})^{\kappa-1} \exp(-\kappa\zeta) \, d\zeta \tag{3-28}$$

where $\kappa = 1/\nu_q^2$, and

$\zeta = (q - Q_I)/Q$.

When the runoff flows are exponentially distributed (i.e., $\nu_q = 1$),

$$f_I = \exp(- Q_I/Q) \tag{3-29}$$

However, when ν_q or ν_d is not equal to 1.0, numerical evaluation of equation 3-28 is required. The results are shown in Figure 3-9.

When an interceptor and storage basin are operated together, as in the off-line configuration, the long-term fraction of the runoff volume bypassing the interceptor and the storage basin, f_{IV}, is given by

$$f_{IV} = \frac{V'}{V}$$

$$= \frac{\int_{q=Q_I}^{\infty} \int_{d=V_E/(q-Q_I)}^{\infty} \left(q-Q_I\right) \left(d - \frac{V_E}{q-Q_I}\right) p(d)\, p(q)\, dd\, dq}{QD} \tag{3-30}$$

Once again, independence between event flows, durations, and pollutant concentrations allows f_{IV} to represent the analogous pollutant load fraction:

$$f_{IV} = \frac{C_R V'}{C_R V} \tag{3-31}$$

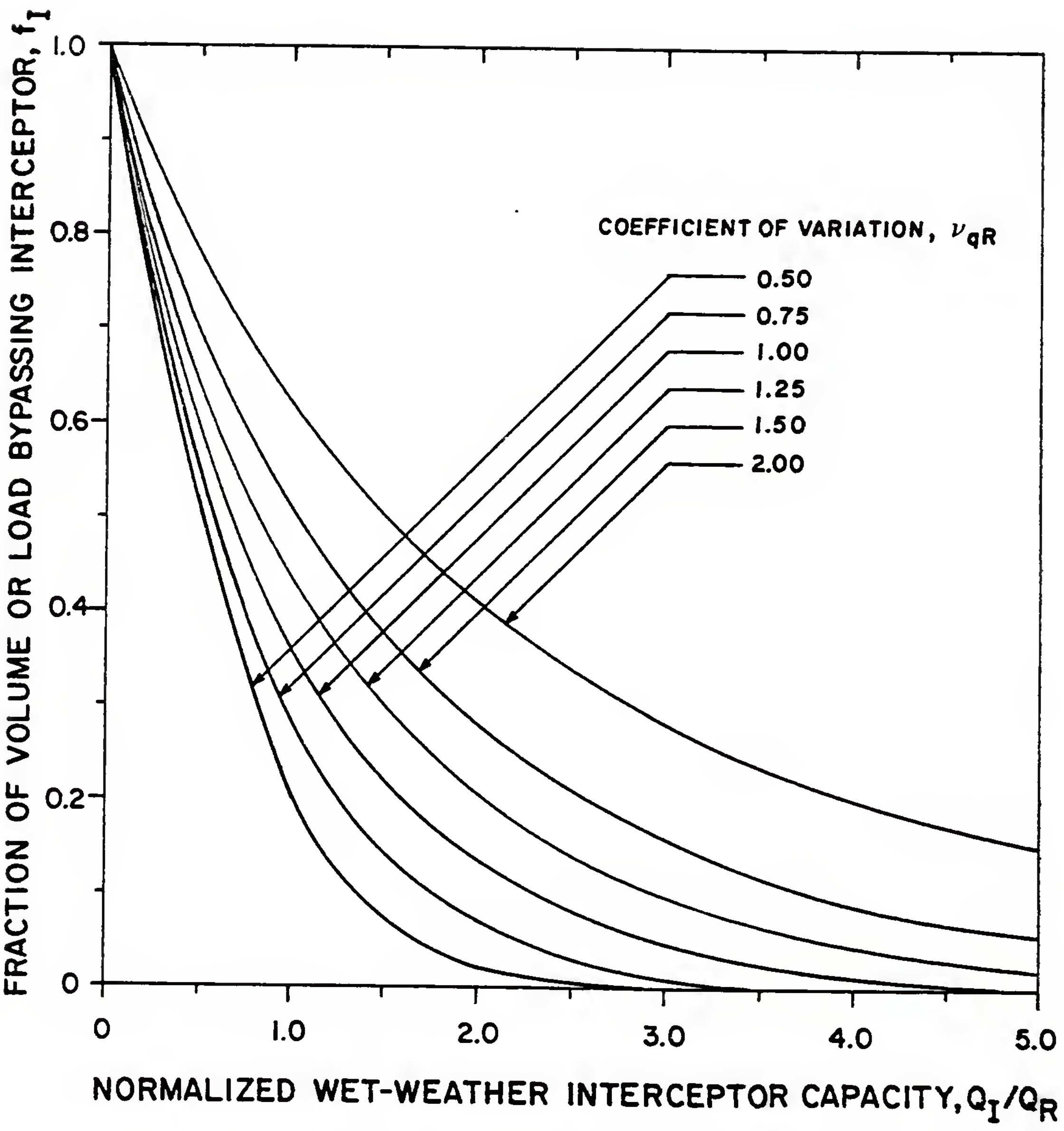


Figure 3-9. Determination of the Long-Term Fraction of the Total Pollutant Load or Runoff Volume Bypassing the Interceptor or Mainstream (DiToro and Small, 1979; Hydroscience, Inc., 1979).

DiToro and Small (1979) have evaluated equation 3-30 numerically and have noted that the results are well estimated by

$$f_{IV} = f_I f_V \tag{3-32}$$

When $\nu_q = 1$, equation 3-32 produces results identical to that of equation 3-30. Equation 3-32 slightly underestimates equation 3-30 when $\nu_q > 1$ and slightly overestimates when $\nu_q < 1$. Figures 3-7, 3-8, and 3-9 can be used to generate values of f_{IV} by allowing the release rate, Ω, to equal the wet-weather capacity of the interceptor, Q_I.

Summary and Critique

The procedures developed by DiToro and Small (1979) and Hydroscience, Inc. (1979) appear to be useful for "first-cut" or preliminary analyses of stormwater storage/release systems. They are best applied to the determination of the level of volume or pollutant control (i.e., amount entering storage) provided by a particular system. These results can then be used to provide an "operational" range for a more detailed analysis by a simulator.

Some of the integrals in the previous section are difficult or impossible to analyze under some conditions. However, it is desirable and gratifying to develop analytical solutions or integral forms that are amenable to numerical solution. In this quest, a significant amount of flexibility and some accuracy was sacrificed by the assumptions used to analyze storage/release systems. Since the objective of their work appears to be the construction of a set of nomographic curves, it would follow that analytical elegance could have been sacrificed to develop a more useful and accurate set of curves.

One possible alternative is a technique known as Monte Carlo simulation. This technique could be used to produce a series of stormwater runoff data (i.e., flows, durations, and times between events) through a random gamma variate generator. These "data" could then be processed through a very simple simulator modeling the storage/release system in the manner idealized by DiToro and Small (1979). One possible result would be several sets of curves relating runoff and/or pollutant control to the normalized basin capacity, V_B/V_R, and the normalized release rate, $\Omega\Delta_R/V_R$, for various coefficients of variation (similar to Figure 5-21). A procedure outlined by Haan (1977) could be used to determine the necessary length of the random gamma variate record to insure an acceptable level of error in reproducing the required gamma distributions. Haan's method develops a distribution of errors as a function of the artificially generated sample size and thereby allows the selection of the sample size necessary to maintain an acceptable error level.

CHAPTER 4

COMPUTER SIMULATION OF STORAGE/RELEASE SYSTEMS: THE STORM WATER MANAGEMENT MODEL STORAGE/TREATMENT BLOCK

Introduction

The EPA Stormwater Management Model (SWMM) is a relatively large, complex model, capable of simulating the movement of precipitation from the surface through pipe and channel networks, treatment units, and finally to receiving waters. The model may be run for a single event or on a continuous basis for extended periods of time. It is divided into several blocks and each is designed to handle a separate phase and stand alone as a model in its own right. SWMM has been released under several different versions, each superseding the previous version (Metcalf and Eddy, Inc. et al., 1971; Huber et al., 1975, 1977, 1981).

SWMM has included, since its introduction in 1971, a Storage/Treatment (S/T) block capable of simulating wet weather storage/treatment units. Versions I and II simulated various units on the basis of limited empirical data and operating experience. The user was required to do little more than select, from a limited number of options, a string of units and the model did the rest. However, because the data and assumptions upon which the model was built were tenuous, the S/T Block was one of the weaker links in SWMM. A new S/T Block was developed for inclusion in this study and to enhance SWMM. Its versatility and range are more than required here but it will be described in total. The ability of the model to simulate storage/release systems will become evident.

The following considerations were instrumental in creating the new model:

1) There should be a high degree of flexibility in the simulation of individual storage/treatment units and the interaction among units.
2) In addition to simulating the mass of pollutants, it is important to account for the physical characteristics (i.e., particle size and specific gravity distribution) of each pollutant.
3) Sludge handling is an important part of any wastewater treatment scheme and should be simulated.
4) All costing routines should be as flexible as the performance algorithms.
5) The model should be capable of modeling integrated wet- and dry-weather systems.

The most important feature of the new S/T Block is flexibility. However, increased flexibility implies greater user input and knowledge of the processes to be simulated. In other words, the model has become "user intensive" rather than "program intensive." The user is responsible for providing the program with a storage/treatment or storage/release scheme and the operating characteristics of each unit. This approach was taken for two reasons. First, there is a general lack of data concerning the behavior of storage/treatment units under wet-weather conditions. Secondly, the user is ultimately responsible for the model output. Therefore, it is important that the user carefully select the parameter estimates for the model.

Model Structure

The Storage/Treatment Block is approximately 2000 FORTRAN statements in length and consists of eight subroutines. A user's manual for this block and the rest of SWMM has been prepared and is readily available (Huber et al., 1981). The program listing for the S/T Block can be found with the SWMM source listing that accompanies this manual (on magnetic tape). The program structure is depicted in Figure 4-1. Each subroutine handles a well-defined task. The routing of flow and pollutants through the entire block is controlled by subroutine STRT which is called from the Executive Block of SWMM. STRT also provides the main driving loop for the model and generally acts as the central coordinating subroutine. Subroutine STRDAT reads the input data. Subroutine CONTRL is called each time step from the main driving loop in STRT. CONTRL directs flow and pollutants from one unit to another as prescribed by the desired scheme and coordinates the majority of the printed output. Subroutine UNIT is called from CONTRL for each unit modeled and is the heart of the Storage/Treatment Block. It contains the necessary flexibility and capability to model most storage/treatment processes (units). Subroutine EQUATE is used by UNIT to provide several forms of pollutant removal equations. Subroutine INTERP is employed by UNIT for linear interpolation. Subroutine PLUGS is used by UNIT to simulate plug flow in a detention unit. Subroutine STCOST is called from STRT to determine capital and operation and maintenance costs.

Modeling Techniques

Basic Framework

The S/T Block can accept a flow rate and three pollutant concentrations for an arbitrary number of equally sized time steps. The model

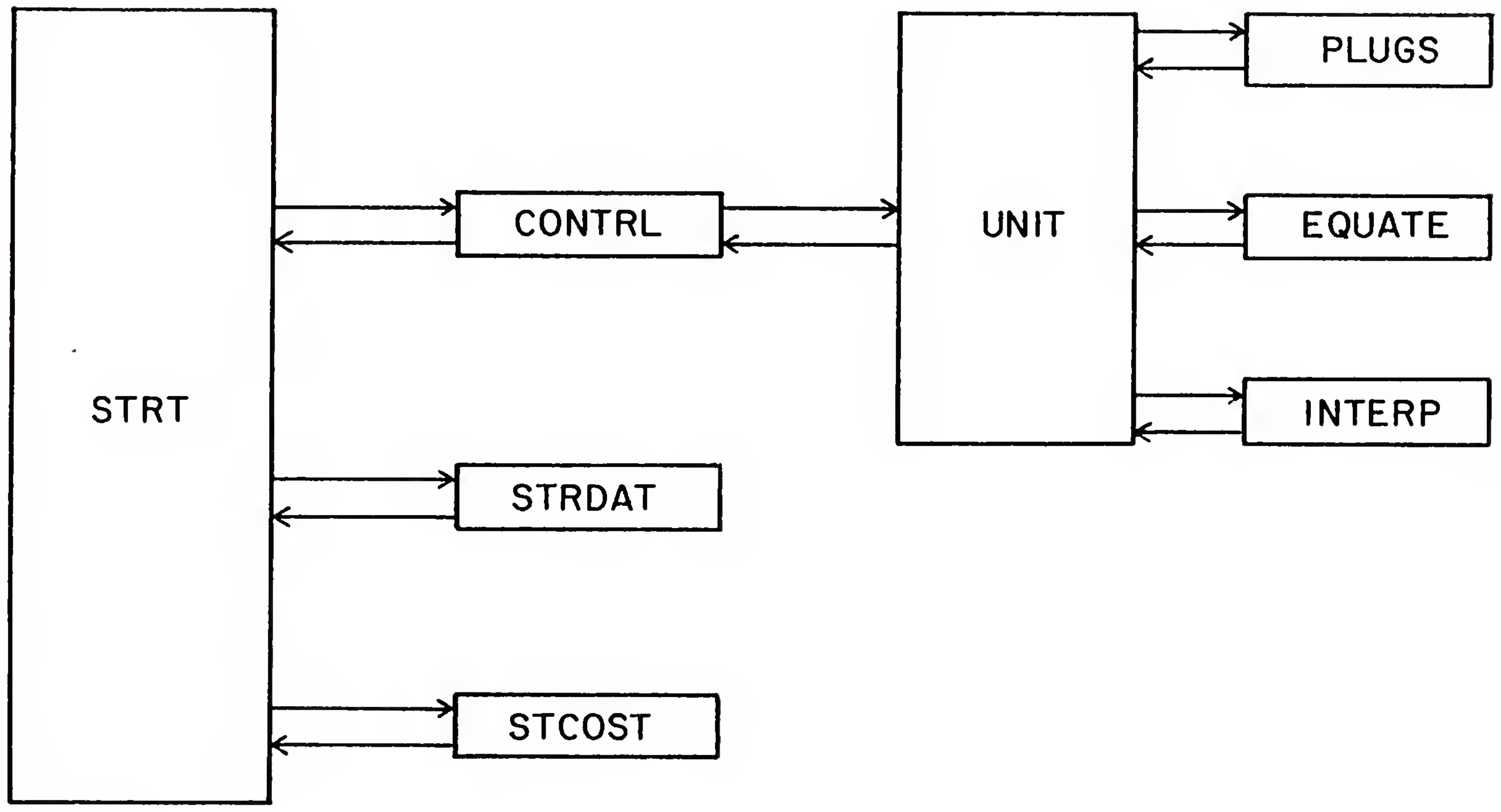


Figure 4-1. SWMM Storage/Treatment Block.

assumes that these input data are instantaneous values occurring at the end of each time step. The flow and pollutant input data may be taken from the output of any other SWMM Block or any other model with the proper interfacing. The user may also feed these data directly to the model. When both sources are used, they are combined to form a single stream.

Flow and pollutants are routed through a network of as many as five storage and/or treatment units. The flows into, through and out of a unit are shown in Figure 4-2. The units may be arranged in any fashion, restricted only by the following: 1) inflow to the plant enters at only one unit and 2) the products (treated outflow, residual or sludge flow, and bypass flow) from each unit must be directed to units with a larger identifying number. Units may or may not have a detention capability. Pollutants or sludges may be represented by mass only (i.e., concentration) or further characterized by a particle size or settling velocity distribution. A unit may remove pollutants (or concentrate sludges) through a generalized removal equation or through particle settling and/or obstruction.

The following sections describe techniques which allow the user to model several types of storage/treatment units. All dimensions are expressed in U.S. customary units, but the model has the capability to operate with metric units.

Pollutant Characterization

Pollutants are characterized by their concentration and, if desired, by particle size and specific gravity distributions or terminal settling velocity distributions. Describing pollutants by particle size

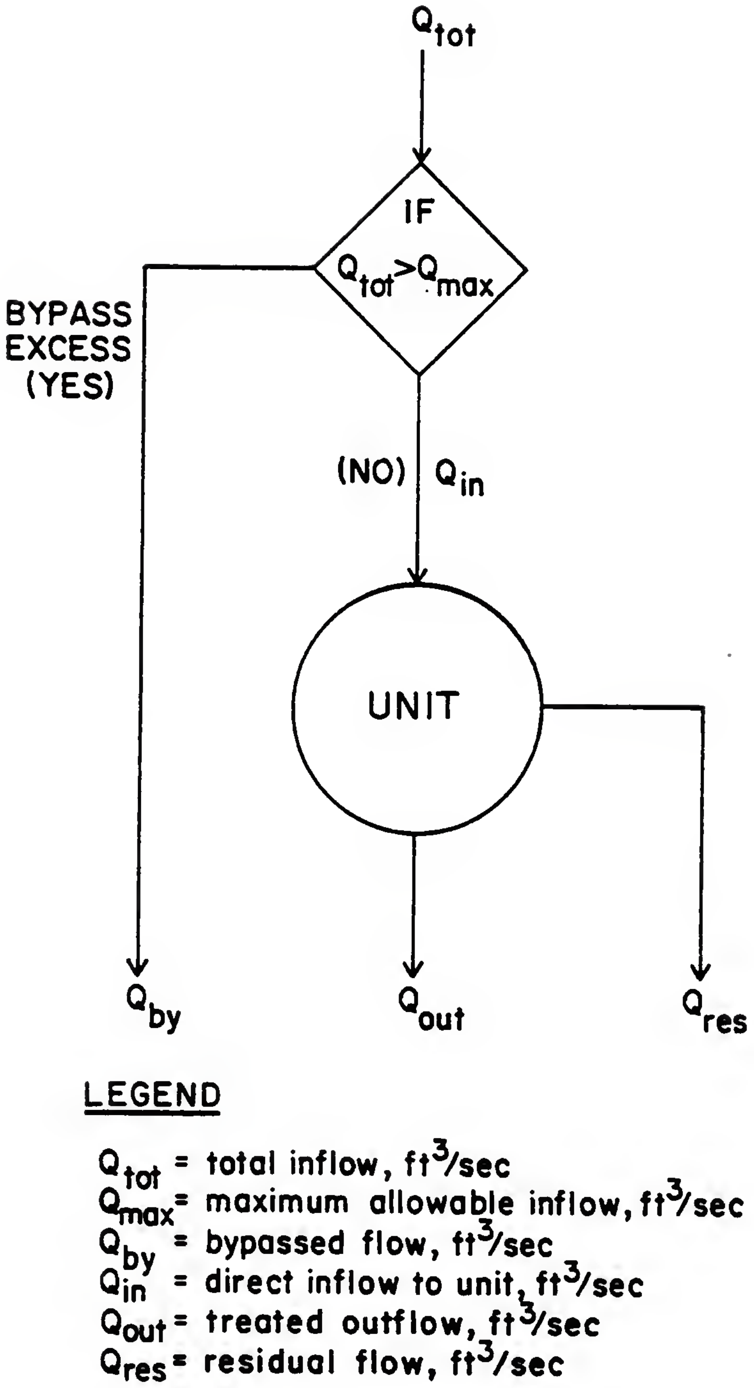

Q_{tot}
IF
$Q_{tot} > Q_{max}$
BYPASS EXCESS (YES)
(NO)
Q_{in}
UNIT
Q_{by}
Q_{out}
Q_{res}
LEGEND
Q_{tot} = total inflow, ft^3/sec
Q_{max} = maximum allowable inflow, ft^3/sec
Q_{by} = bypassed flow, ft^3/sec
Q_{in} = direct inflow to unit, ft^3/sec
Q_{out} = treated outflow, ft^3/sec
Q_{res} = residual flow, ft^3/sec

Figure 4-2. Storage/Treatment Unit.

or settling velocity distributions is especially appropriate when settling is the dominant removal mechanism and potentially important when several storage/treatment units are operated in series. For example, if the influent pollutant load is primarily sand and grit, then a sedimentation basin would be very effective; if clay and silt predominate, sedimentation may be of little use. Also, if several units are operated in series, the first units may remove a certain range of particle sizes, thus affecting the performance of downsteam units. The need for describing pollutants in this manner is fairly obvious.

If a pollutant is characterized by its particle size/specific gravity or terminal settling velocity distribution, it is apportioned over several (up to ten) particle size/specific gravity ranges (e.g., ten percent of suspended solids are found in the range from 10 to 50 microns) or settling velocity ranges. Each range is preset by the user and assigned an upper and lower bound on the particle diameter and a value for specific gravity. If a size/specific gravity distribution is specified the model estimates the average settling velocity for each range. Alternatively, a representative set of settling velocities may be specified. Also specified is the distribution of the pollutant over the various ranges of particle sizes or settling velocities as it enters the first unit. This distribution is modified as it passes through the storage/treatment plant. The distribution entered at the first unit must remain constant over time since the other blocks of SWMM do not provide a time varying particle size or settling velocity distribution.

Non-Detention Units

Units modeled as non-detention processes are treated as instantaneous flow-through devices in which the inflow during any time step leaves that same step less the residual flow (i.e., there is no storage capability). The residual flow is calculated as a constant fraction of the inflow. Screens of all types are examples of non-detention units.

Pollutants are also routed instantaneously through non-detention units. In other words, the pollutants arriving during a time step leave the same time step with the treated outflow and residual flow. Removed pollutants are routed along with the residual flow. The amount removed is determined by a user-supplied removal equation if the pollutant is characterized by concentration only or by a critical particle size or settling velocity if the pollutant is characterized by concentration and a particle size or settling velocity distribution.

If pollutants are characterized only by their concentrations, then the model improves the quality of the waste stream by removal equations. Removal of any pollutant may be simulated as a function of its influent concentration, the influent concentrations of other pollutants, the inflow rate, and/or the removal fraction of other pollutants. This selection is made by the user. A single, generalized equation is provided by the program to construct the desired removal equation:

$$\begin{aligned} R = \{ & a_{12}\exp[a_1x_1]x_2^{a_2} + a_{13}\exp[a_3x_3]x_4^{a_4} + a_{14}\exp[a_5x_5]x_6^{a_6} \\ & + a_{15}\exp[a_7x_7 + a_8x_8]x_9^{a_9}x_{10}^{a_{10}}x_{11}^{a_{11}}\}^{a_{16}} \end{aligned} \qquad (4\text{-}1)$$

where x_i = removal equation variable,

a_k = coefficient, and

R = removal fraction, $0 \leq R \leq 1.0$.

The user assigns the removal equation variables, x_i, to the variables discussed above (inflow concentration, inflow flow rate, etc.). The removal equation constructed with equation 4-1 is applied to the inflow each time step. If an equation variable is not assigned, it is set equal to 1.0 for the duration of the simulation. There is considerable flexibility contained in equation 4-1 and, with a judicious selection of coefficients and assignment of variables, a broad variety of removal equations can be formed. Equation 4-1 is also used in the pollutant removal algorithms of detention units.

An example application of equation 4-1 is taken from a study by Lager et al. (1977) in which several curves for suspended solids removal from microstrainers with a variety of aperture sizes are presented. Fitting a power function to the curve representing a 35-micron microstrainer yields

$$R_{TSS} = 0.0963\ C_{TSS}^{0.286} \tag{4-2}$$

where R_{TSS} = total suspended solids removal fraction, $0 \leqq R_{TSS} \leqq 1.0$, and

C_{TSS} = influent total suspended solids concentration, mg/l.

Equation 4-1 can be used to duplicate this removal equation by setting $a_{12} = 0.0963$, $a_2 = 0.286$, $a_{16} = 1.0$, and x_2 = influent suspended solids concentration, C_{TSS}. All other a_k are zero.

Whenever a non-detention unit is encountered and a pollutant is characterized by a size or settling velocity distribution, the user specifies a "critical" size or settling velocity and any particles with a greater size or settling velocity are removed (and, in turn, so are the associated pollutants). The program operation is simple, but the interpretation of the "critical" size or settling velocity is more complex. The primary intent of including this mechanism in the model is to simulate screens. Pollutant removal by screens is a result of two actions: straining by the screens, and the additional filtration provided by the mat produced by the initial screening (Maher, 1974).

Screens vary widely in the size of the aperture and the manner in which the wastewater flows through them. It may be adequate to assume that the removal of particles is explicitly related to size of the aperture. In other words, a particular screen size will remove only those particles larger than that size and filtration by the mat is ignored. If a settling velocity distribution is employed, the user must specify a corresponding settling velocity. This approach is not entirely accurate, of course, but the result is a conservative removal estimate that may be accurate in cases where backwashing is at a relatively high rate. In fact, a study by Maher (1974) indicates that this simplifying assumption may be reasonable. In this case, a microstrainer with a Mark "0" screen (aperture of 23 microns) was installed in a residential area of Philadelphia, Pennsylvania. The analysis of the backwash material for two storms (one in which a coagulant was used) revealed that, by particle count, 88 to 96 percent of the particles were indeed smaller than 23 microns. However, by weight, over 99 percent of the material was found in particles greater than 23 microns. Although Maher did not report the distribution in terms of weight, an estimate was made by assuming a reasonable specific gravity.

Detention Units

Detention units include, among others, reservoirs, storage basins, sedimentation tanks, dissolved air flotation units, and chlorination basins. Treated outflows and residual flows are handled in the same manner, with the user supplying a depth-discharge relationship for each. The user also provides data describing the depth-surface area-volume relationship. The timing of the residual (e.g., sludge) flow is determined by the user, with the restriction that they are only permitted during dry periods (i.e., no inflow or treated outflow).

Flow routing. The rate of change of storage in a detention unit is described by the storage equation, i.e.,

$$\frac{dV(t)}{dt} = I(t) - O(t) \tag{4-3}$$

where $V(t)$ = volume of water in the detention unit, ft^3,

$I(t)$ = inflow rate to the detention unit, ft^3/sec,

$O(t)$ = outflow rate from the detention unit, ft^3/sec, and

t = time, sec.

Analytical solutions to equation 4-3 are especially difficult to derive under unsteady-state conditions (e.g., stormwater runoff flows). Thus, it is approximated by the second-order closed Adams formula as follows (see Chapter 2):

$$V_{j+1} = V_j + \frac{\Delta t}{2}[(I_{j+1} - O_{j+1}) + (I_j - O_j)] \tag{4-4}$$

where Δt = constant time interval or step, sec;

V_j, I_j, O_j = values of $V(t)$, $I(t)$, and $O(t)$ at the beginning of time step j; and

V_{j+1}, I_{j+1}, O_{j+1} = values of $V(t)$, $I(t)$, and $O(t)$ at the end of time step j.

For a given time step, I_j, I_{j+1}, O_j, and V_j are known and O_{j+1} and V_{j+1} need to be determined. Grouping the unknowns on the left hand side of the equation and rearranging terms yields

$$O_{j+1}\frac{\Delta t}{2} + V_{j+1} = (I_j + I_{j+1})\frac{\Delta t}{2} + (V_j - O_j\frac{\Delta t}{2}) \tag{4-5}$$

The values of O_{j+1} and V_{j+1} are found by knowing the relationship between volume and outflow. The procedure is illustrated in the following example.

Table 4-1 presents geometric and routing data for a hypothetical reservoir with a base elevation of 343.0 ft and a maximum pool elevation of 353.0 ft. The corresponding depths are shown in column 3. Surface area, as a function of depth, is presented in column 4. If the reservoir has an irregular geometry, the surface area can be measured from a topographic map. The discharge data (column 5) may be input directly by assigning values to each depth, generated by a depth-discharge equation (e.g., weir equation) or provided by a pumping scheme. The volume associated with each depth (column 6) may be specified directly or calculated by the model from the depth-area data. The model estimates volume by averaging the surface area between adjacent values of depth, multiplying by the difference in depth, and adding the incremental volume to the accumulated volume. The depth-area data pairs are also used to estimate the volume lost from the reservoir due to evaporation.

Recalling equation 4-5, the objective is to find a relationship between O_{j+1} and V_{j+1}. This can be expressed as

$$O_{j+1} \frac{\Delta t}{2} = f(O_{j+1} \frac{\Delta t}{2} + V_{j+1}) \qquad (4\text{-}6)$$

The data in Table 4-1 give O_{j+1} and V_{j+1} as functions of depth. In this case, discharge or outflow occurs only if the reservoir depth exceeds 8.0 ft. The model uses these data to calculate the values of $O_{j+1} \frac{\Delta t}{2}$ (column 7) and $O_{j+1} \frac{\Delta t}{2} + V_{j+1}$ (column 8) for each depth. Thus, the relationship described by equation 4-6 is generated indirectly. During the simulation, the current value of $O_{j+1} \frac{\Delta t}{2} + V_{j+1}$ is calculated by equation 4-5 and the corresponding value of $O_{j+1} \frac{\Delta t}{2}$ is found by linear interpolation among the previously generated set of values. The values of O_{j+1} and V_{j+1} are subsequently calculated. This procedure is repeated each time step with the value of O_{j+1} and V_{j+1} becoming the

Table 4-1. Geometric and Hydraulic Data for Hypothetical Reservoir

(1)	Elevation d_h ft (2)	Depth d_e ft (3)	Surface Area A_s 1000 ft^2 (4)	Discharge O_{j+1} ft^3/sec (5)	Volume V_{j+1} 1000 ft^3 (6)	$O_{j+1} \frac{\Delta t}{2}$ 1000 ft^3 (7)	$O_{j+1} \frac{\Delta t}{2} + V_{j+1}$ 1000 ft^3 (8)
1	343.0[a]	0.0	0.	0.	0.	0.	0.
2	344.0	1.0	3.	0.	2.	0.	2.
3	345.0	2.0	15.	0.	10.	0.	10.
4	346.0	3.0	45.	0.	40.	0.	40.
5	347.0	4.0	121.	0.	120.	0.	120.
6	348.0	5.0	225.	0.	300.	0.	300.
7	349.0	6.0	365.	0.	590.	0.	590.
8	350.0	7.0	550.	0.	1050.	0.	1050.
9	351.0[b]	8.0	790.	0.	1720.	0.	1720.
10	351.5	8.5	910.	30.	2140.	324.	2464.
11	352.0	9.0	1080.	65.	2650.	702.	3352.
12	352.2	9.2	1130.	80.	2900.	864.	3764.
13	352.4	9.4	1190.	105.	3100.	1134.	4234.
14	352.6	9.6	1270.	130.	3400.	1404.	4804.
15	352.8	9.8	1350.	165.	3700.	1782.	5482.
16	353.0[c]	10.0	1440.	200.	3900.	2160.	6060.

Column:

(1) Counter

(2) Elevation from topographic map

(3) Depth = d_h - 343.0

(4) Measured from topographic map or may be calculated if geometry is regular

(5) Measured data or calculated from discharge formulas

(6) Measured data or calculated

(7) Calculated from column 5, Δt = 21,600 sec

(8) Calculated from columns 6 and 7

Notes:

a. Base of reservoir

b. Weir elevation

c. Maximum pool

values of O_j and V_j for use in equation 4-5 during the next time step. In an actual simulation the outflow, O_{j+1}, can represent treated outflow or residual flow.

Evaporation losses from the detention unit are accounted for at the end of each time step. The loss rate is computed by

$$e_v = \frac{A_s \, e_d}{1036800} \qquad (4\text{-}7)$$

where e_v = evaporation loss from the detention unit, ft^3/sec,

A_s = water surface area in the detention unit, ft^2,

e_d = evaporation rate, inches/day, and

1036800 = conversion factor, inches/day per ft/sec.

The values of e_d are supplied with the model input for each month of the simulation period.

In summary, the S/T Block is provided with or develops the required relationships between depth, surface area, treated outflow, residual flow, and storage volume through as many as sixteen data sets. This approach permits the user to select the data points which best approximate the hydraulic relationships. This approach was felt to be preferable to adding more complexity to the model in order to handle a wide variety of specific basin geometries and operating policies.

An excellent description of this level-surface routing procedure (known as the Puls method) is presented in Viessman et al. (1977). However, the approximation of the storage equation is developed in a different manner. Sound engineering judgment is essential in setting up this routing procedure. The input data and associated assumptions should be checked carefully.

Pollutant routing and removal. Pollutants are routed through a detention unit by one of two modes: complete mixing or plug flow. For complete mixing, the concentration of the pollutant in the unit is assumed to equal the effluent concentration. The model algorithm assumes that the removal of pollutants is represented by a first-order reaction. The differential equation governing the completely mixed, variable-volume detention unit (or reactor) shown in Figure 4-3 is discussed in Chapter 2 and repeated below.

$$\frac{d(CV)}{dt} = C^I(t)I(t) - C(t)O(t) - KC(t)V(t) \qquad (4\text{-}8)$$

where $C(t)$ = pollutant concentration in the detention unit and the outflow, mg/l,

$C^I(t)$ = pollutant concentration in the inflow, mg/l,

$V(t)$ = volume of water in the detention unit, ft^3,

$I(t)$ = inflow rate to the detention unit, ft^3/sec,

$O(t)$ = outflow rate from the detention unit, ft^3/sec,

t = time, sec, and

K = rate constant, sec^{-1}.

Equation 4-8 is very difficult to work with directly. Therefore, the S/T Block approximates it with the second-order closed Adams formula (see Chapter 2), i.e.,

$$C_{j+1}V_{j+1} = C_jV_j + \frac{\Delta t}{2}[(C^I_{j+1}I_{j+1} - C_{j+1}O_{j+1} - KC_{j+1}V_{j+1}) + (C^I_jI_j - C_jO_j - KC_jV_j)] \qquad (4\text{-}9)$$

where C_j, C^I_j = values of $C(t)$ and $C^I(t)$ at the beginning of time step j; and

C_{j+1}, C^I_{j+1} = values of $C(t)$ and $C^I(t)$ at the end of time step j.

From the flow routing procedure, the variables I_j, I_{j+1}, O_j, O_{j+1}, V_j, and V_{j+1} are known. The concentration in the reservoir at the

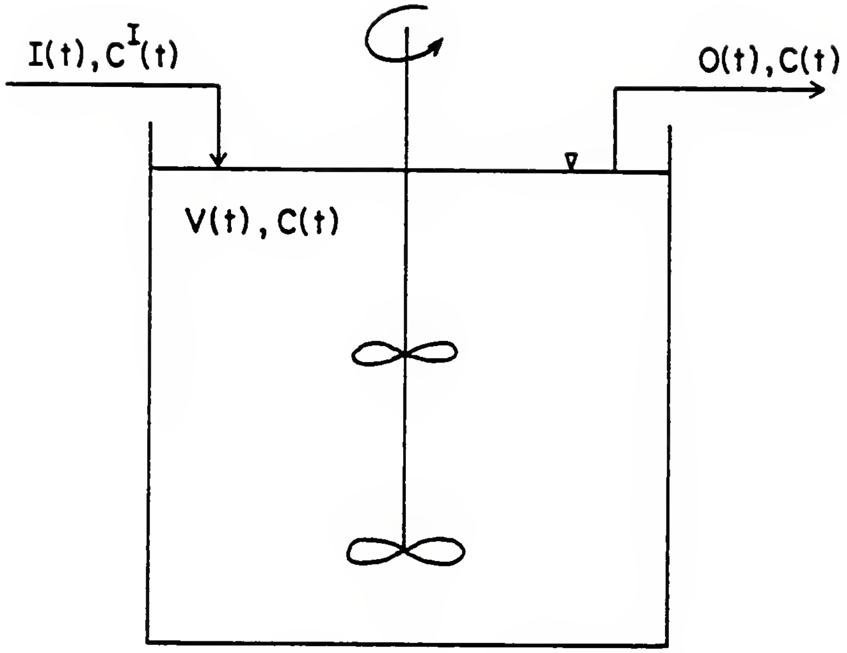


Figure 4-3. Completely Mixed, Variable-Volume Detention Unit (Rich, 1973).

beginning of the time step, C_j, and the influent concentrations, C^I_j and C^I_{j+1}, are also known, as are the rate constant, K, and the time step, Δt. Thus, the only unknown, the concentration at the end of time step, C_{j+1}, can be found by rearranging equation 4-9 to yield

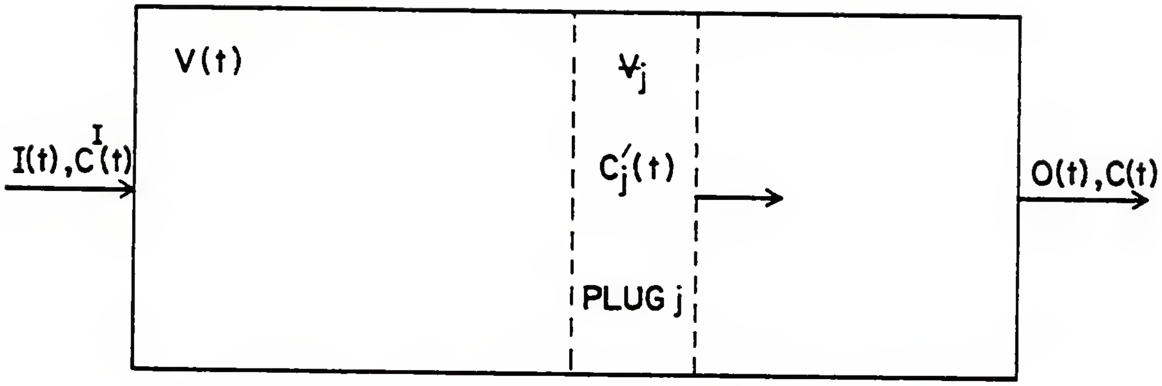
$$C_{j+1} = \frac{C_j V_j + (C^I_j I_j + C^I_{j+1} I_{j+1} - C_j O_j - K C_j V_j)\frac{\Delta t}{2}}{V_{j+1} + (K + O_{j+1})\frac{\Delta t}{2}} \qquad (4\text{-}10)$$

Equation 4-10 is the basis for the complete mixing mode of pollutant routing through a detention unit.

Equations 4-8, 4-9, and 4-10 assume that pollutants are removed at a rate proportional to the concentration present in the unit. In other words, a first-order reaction is assumed. The coefficient K is the rate constant. The product of K and Δt is controlled through the use of equation 4-1, i.e., $R = K\Delta t$. This simple relationship can be created from equation 4-1 by letting $x_2 = \Delta t$, $a_2 = 1.0$, $a_{12} = K$, and $a_{16} = 1.0$.

Pollutants are not allowed to settle in a completely mixed detention unit. Strictly speaking, pollutants cannot settle under such conditions. Therefore, the residual stream is effectively another route for treated outflow. Additionally, particle size or settling velocity distributions are not used when completely mixed detention units are present.

If the plug-flow mode is selected, the inflow during each time step, herein called a plug, is labeled and queued through the detention unit (see Figure 4-4). Transfer of pollutants between plugs is not permitted. The treated outflow for any time step is comprised of the oldest plugs, and/or fractions thereof, present in the unit. This is accomplished by satisfying continuity for the present outflow volume (determined from the flow routing procedure):



NOTE: V_j and $C'_j(t)$ are the volume and reactant concentration of plug j, respectively.

Figure 4-4. Plug-Flow Detention Unit.

$$\sum_{j=JP}^{LP} \forall_j \cdot f_j = V_{out} \qquad (4\text{-}11)$$

where V_{out} = treated outflow volume leaving the detention unit during the present time step, ft^3,

$\forall_j$ = volume entering the detention unit during the j^{th} time step (i.e., plug j), ft^3,

f_j = fraction of plug j that must leave the detention unit to satisfy continuity with V_{out}, $0 \leqq f_j \leqq 1$,

JP = time step number of the oldest plug in the detention unit, and

LP = time step number of the youngest plug in the detention unit required to satisfy continuity with V_{out}.

Each plug is treated as a batch reactor (see Chapter 2) in which pollutants are removed by a specified removal equation created from equation 4-1 or by particle settling if the pollutant is characterized by a particle size or settling velocity distribution (see later discussion). The user can create a removal equation for a particular pollutant from its influent concentration, the influent concentration of other pollutants, the removal fraction of other pollutants and/or the detention time of the plug. Obviously, most removal equations for plug-flow units will involve detention time. The detention time for each plug is calculated as

$$\theta_j = (KKDT - j)\Delta t \qquad (4\text{-}12)$$

where θ_j = detention time of plug j (plug entering the detention unit at time step j), sec, and

KKDT = present time step number.

For computational convenience, pollutants are removed from a plug when it leaves the unit. In reality, pollutants are constantly removed from the waste stream but the model assumption is adequate for most applications.

An example of the application of equation 4-1 as a removal equation is taken from an earlier version of the S/T Block (Huber et al., 1977). The following equation was used to remove total suspended solids (through settling) in plug-flow storage basins:

$$R_{TSS} = R_{max}[1- \exp(-K\theta_j)] \qquad (4\text{-}13)$$

where R_{TSS} = total suspended solids removal fraction, $0 \leq R_{TSS} \leq R_{max}$,

R_{max} = maximum removal fraction,

θ_j = detention time of plug j, sec, and

K = rate constant, sec^{-1}.

This same equation could be built from equation 4-1 by setting $a_{12} = R_{max}$, $a_{13} = -R_{max}$, $a_3 = K$, $a_{16} = 1.0$, and letting x_3 = detention time, θ_j. All other coefficients, a_k, would equal zero. Equation 4-13 is simply the solution to the first-order batch reactor equation (equation 2-14) written in terms of the fraction removed and applied only to the portion of the total suspended solids load that is settleable (i.e., R_{max}).

Removed pollutant quantities accumulate in a plug-flow unit until drawn off with the residual flow. The accumulated pollutants are not allowed to affect the amount of available storage and are assumed to be conservative (i.e., no decay). The former assumption is reasonable for well maintained units. When a residual flow occurs, the entire contents of the unit (including the removed pollutant quantities) are mixed and drawn off until the unit is empty or wet weather continues--recall that residual flows occur only during dry weather. If wet weather occurs before the unit is empty, the contents are aggregated into one plug for further routing. In order to cause the residual flow to resemble an actual sludge flow, suspended solids should be routed through the unit. However, the dry-weather mixing assumption may create a "sludge" of low solids content if the treated outflow regime leaves a large volume of water in the unit at the end of an event.

When a detention unit is modeled as a plug-flow reactor and a pollutant is characterized by concentration and a particle size/specific gravity or terminal settling velocity distribution, the pollutant is removed via discrete particle settling. The procedure is based on an adaptation of Camp's sediment trap efficiency curves (Dobbins, 1944; Camp, 1946; Brown, 1950) as described by Chen (1975). These curves were developed as a complex function of particle settling velocity and several other parameters, i.e.,

$$E = f\left(\frac{v_s d_e}{2\varepsilon}, \frac{v_s}{v_c}\right) \tag{4-14}$$

where E = particle removal efficiency, $0 \leq E \leq 1$,

ε = vertical turbulent diffusivity or mixing coefficient, ft^2/sec,

v_s = terminal settling velocity of the particle, ft/sec,

$v_c = d_e/\theta_j$ = critical particle settling velocity, ft/sec,

d_e = depth of water in the detention unit, ft, and

θ_j = detention time of plug j, sec.

The first term within the function accounts for the effects of turbulence on settling. The second term, v_s/v_c, is the removal fraction under quiescent conditions. Camp's technique was developed for rectangular basins operating under constant flow or steady state conditions. It is modified for use under the conditions encountered by urban stormwater basins by applying it to each plug and using the average water depth during the plug's residence within the detention unit, $\bar{d}_e$, instead of d_e.

Camp (1946) solves for the functional form of equation 4-14 by assuming a uniform horizontal velocity distribution and constant diffusivity, ε. A form of the advective-diffusive equation then results in which local changes in concentration at any vertical elevation are equal to the net effect of settling from above and diffusion from below. The diffusivity will be constant if the horizontal velocity is assumed to have a parabolic distribution; although this assumption is clearly at variance with the uniform velocity distribution above. For the parabolic distribution, ε is then found from

$$\varepsilon = 0.075\, d_e \sqrt{\tau_o/\rho} \qquad (4\text{-}15)$$

where τ_o = boundary shear stress, lbs/ft^2, and

ρ = mass density of water $\cong$ 1.94 slugs/ft^3 (1.00 g/cm^3).

The shear velocity term, $\sqrt{\tau_o/\rho}$, can be evaluated using Manning's equation for open channel flow (Brown, 1950), i.e.,

$$u_* = \sqrt{\tau_o/\rho} = \frac{v_t n \sqrt{g}}{1.49\, d_e^{1/6}} \qquad (4\text{-}16)$$

where u_* = shear velocity, ft/sec,

v_t = flow-through velocity, ft/sec, and

n = Manning's roughness coefficient.

The flow-through ("horizontal") velocity, v_t, is given by

$$v_t = \frac{L_t}{\theta_j} \qquad (4\text{-}17)$$

where L_t = travel length of the detention unit, ft.

Equations 4-15 and 4-16 are then used to convert $v_s d_e/2\varepsilon$, which herein is called the turbulence factor, to a more usable form, i.e.,

$$\sigma = 0.1\,\frac{v_s d_e}{2\varepsilon} = \frac{v_s}{1.5 u_*} = \frac{1.49\, v_s d_e^{1/6}}{1.5\, v_t n \sqrt{g}} \cong \frac{v_s d_e^{1/6}}{v_t n \sqrt{g}} \qquad (4\text{-}18)$$

where σ = turbulence factor, dimensionless when all parameters are in units of feet and seconds.

At low values of σ, turbulence is hindering settling: at higher values, settling is more closely linked to the quiescent case.

The algorithm used for finding the terminal particle settling velocity, v_s, was developed by Sonnen (1977). The procedure is based on the theory given in Chapter 2. The average of the high and low ends of each particle size range is used as the representative particle size for use in the well-known discrete particle settling relationships. If a terminal settling velocity distribution is provided by the user, then the v_s calculations are omitted.

Camp's sedimentation trap efficiency curves (Dobbins, 1944; Camp, 1946; Brown, 1950; Chen, 1975), shown in Figure 4-5 as a function of σ and v_s/v_c, represent the numerical solution to equation 4-14. Ideally, these curves should be included directly in the model, but their representation is not straightforward from a programming standpoint. Thus, a linear approximation (with respect to ln σ) of the curves is used. The particle removal efficiency under highly turbulent conditions, E_T, can be estimated by (Vetter, 1940; Chen, 1975)

$$E_T = [1 - \exp(-v_s/v_c)] \text{ for } 0 \leq E_T \leq 1. \qquad (4\text{-}19)$$

This equation accurately represents the values of E at $\sigma = 0.01$. Under quiescent conditions, the particle removal efficiency, E_Q, is computed as

$$E_Q = v_s/v_c \text{ for } 0 \leq E_Q \leq 1. \qquad (4\text{-}20)$$

If equation 4-20 is allowed to approximate the situation at $\sigma = 1.0$, the following linear approximation of the curves in Figure 4-5 is possible:

$$E = E_T + \frac{\ln \sigma - \ln 0.01}{\ln 1 - \ln 0.01} (E_Q - E_T)$$

$$= E_Q + \frac{\ln \sigma}{4.605} (E_Q - E_T) \qquad (4\text{-}21)$$

where E = particle removal efficiency, $0 \leq E \leq 1$.

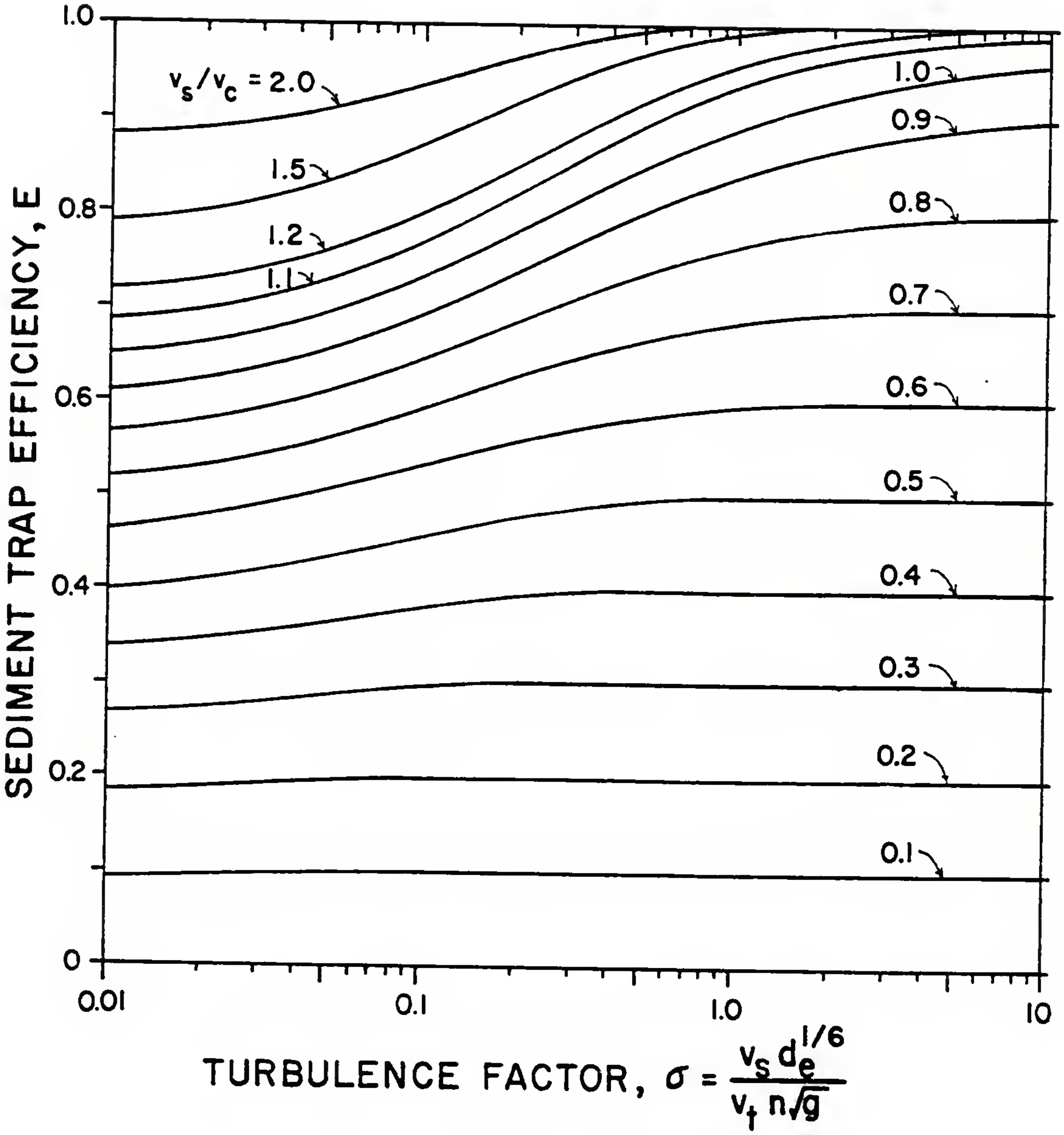


Figure 4-5. Camp's Sediment Trap Efficiency Curves (Camp, 1946; Brown, 1950).

Within the program, values of the turbulence factor are limited to $0.01 \leq \sigma \leq 1.0$. The procedures outlined above are applied to each of the representative settling velocities in each plug leaving the detention unit. Thus, the effect of settling is accounted for at the end of each plug's residence in the basin. Pollutants are removed by their association with various particle size or settling velocity ranges.

In a normal simulation, several plugs leave the detention unit in any given time step. The effluent is all or part of a number of plugs depending on the required outflow (as determined by the storage routing techniques discussed earlier). Thus, the effluent particle size or settling velocity distribution associated with any pollutant may be a composite of several plugs. This composite distribution is determined by taking a weighted average (by pollutant weight in each plug) over the effluent plugs. This distribution is then routed downstream for release or further treatment. The particles that were removed from each plug are also aggregated and used to characterize the accumulated pollutant quantities.

Cost Estimation

The capital cost for each unit is computed as

$$Z_{cap} = c_1 X^{\alpha_1} \tag{4-22}$$

where Z_{cap} = capital cost, dollars,

X = maximum allowable inflow, ft^3/sec, or maximum allowable storage volume, ft^3, or maximum inflow encountered during a simulation, ft^3/sec, or maximum storage volume encountered during a simulation, ft^3, and

c_1, α_1 = coefficients.

The selection of the parameter represented by the variable X and the values of c_1 and α_1 are provided as model input.

Operation and maintenance costs are calculated as a function of the variables listed above and the total operating time (calculated from the number of time steps with flows to or from the unit), i.e.,

$$Z_{om} = c_2 X^{\alpha_2} + c_3 OT \qquad (4\text{-}23)$$

where Z_{om} = operation and maintenance costs, dollars,

OT = operating time during the simulation period, hours, and

c_2, α_2, c_3 = coefficients.

Again, the selection of the parameter represented by the variable X and the values of c_2, α_2 and c_3 are provided as model input. The costs computed by equation 4-23 are for the simulation period only and the results should be converted to an annual basis.

Application to a Storage/Release System

The use of this model as a storage/release simulator was demonstrated by its application to an actual study. Data from the Humboldt Avenue Combined Sewer Overflow Detention Tank in Milwaukee, Wisconsin (collected as part of a demonstration project) were chosen for this purpose (City of Milwaukee et al., 1975).

The Humboldt Avenue detention facility serves a 570-acre combined sewer area. Flows bypassing an interceptor are routed to a storage tank. The tank is operated as a settling basin during wet-weather periods with the treated overflows discharging to the Milwaukee River. During dry-weather periods the tank contents are mixed and routed to the Jones Island Wastewater Treatment Plant. Pertinent design details are shown in Figure 4-6. The system is identical to the off-line configuration in which the release mechanism consists of both the overflow weir and the dry-weather routing of the tank contents back to the interceptor.

Although the monitoring program was relatively extensive, only four storms (occurring over the period of September 12 to September 21, 1972) had concurrent influent/effluent quantity and quality data. The results of the quantity simulation are shown in Figure 4-7 along with the measured overflows. The S/T Block appears to reasonably mimic the observed data. The apparent one-hour lag between the observed and computed results is due to the use of averaged inflow data (provided in the original project report) instead of the instantaneous data required by the model.

The quality simulation was performed under the assumption of plug flow. The removal of total suspended solids was estimated by the exponential relationship given by equation 4-13. The values of R_{max} and K were adjusted to 0.50 and 0.00003 sec^{-1}, respectively, to calibrate the model. The simulation results were plotted against observed data in Figure 4-8. Again, the simulated and observed results are in reasonable agreement (except for the one-hour lag discussed above).

Summary

The SWMM Storage/Treatment Block is presented as a simulator capable of modeling a wide range of storage/treatment systems. The model can analyze non-detention as well as detention facilities. It can also characterize pollutants by their physical attributes in addition to their concentration in the waste stream. A simple procedure is used to compute capital and operation and maintenance costs.

Type: underground, rectangular, concrete tank

Service Area: 570 acres (231 ha)

Capacity: 3,800,000 gal (14,400 m^3)

Length: 420 ft (128 m)

Width: 75 ft (23 m)

Depth (weir height): 16 ft (4.9 m)

Depth + Freeboard: 21.5 ft (6.6 m)

Maximum allowable flow: 380 ft^3/sec (10.8 m^3/sec)

Inflow structure: sluice gates

Outflow structure: 75 ft (23 m) weir

Drawdown Mechanism: 3 combined sewage pumps, mixers

Chlorination: Pre-chlorination is provided for odor control
Post-chlorination is provided for tank overflows

Bar screen: Located in influent channel; 1.5 inch (3.8 cm) openings, mechanically cleaned.

Sectional Plan:

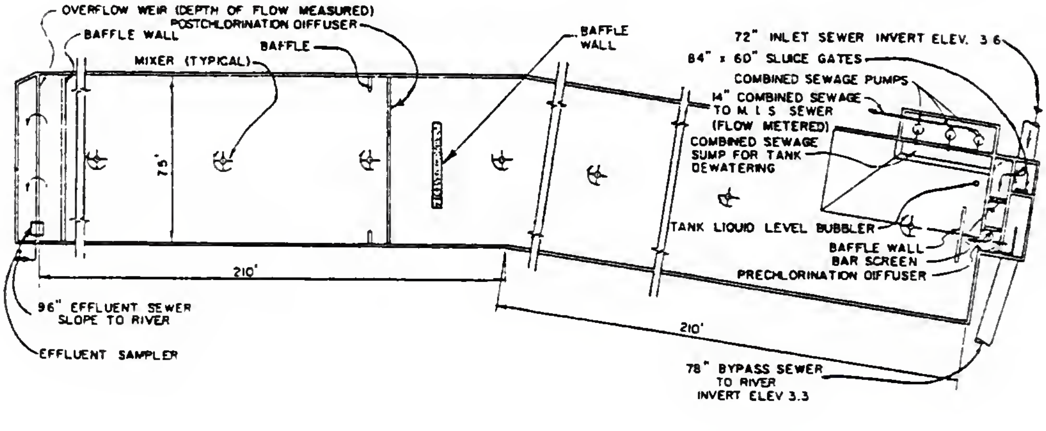

OVERFLOW WEIR (DEPTH OF FLOW MEASURED)
POSTCHLORINATION DIFFUSER
BAFFLE WALL
BAFFLE
MIXER (TYPICAL)
BAFFLE WALL
72" INLET SEWER INVERT ELEV. 3.6
84" x 60" SLUICE GATES
COMBINED SEWAGE PUMPS
14" COMBINED SEWAGE TO M.I.S. SEWER (FLOW METERED)
COMBINED SEWAGE SUMP FOR TANK DEWATERING
75'
TANK LIQUID LEVEL BUBBLER
BAFFLE WALL
BAR SCREEN
PRECHLORINATION DIFFUSER
210'
210'
96" EFFLUENT SEWER SLOPE TO RIVER
EFFLUENT SAMPLER
78" BYPASS SEWER TO RIVER INVERT ELEV 3.3

Figure 4-6. Design Details, Humboldt Avenue Detention Tank, Milwaukee, Wisconsin (City of Milwaukee et al., 1975).

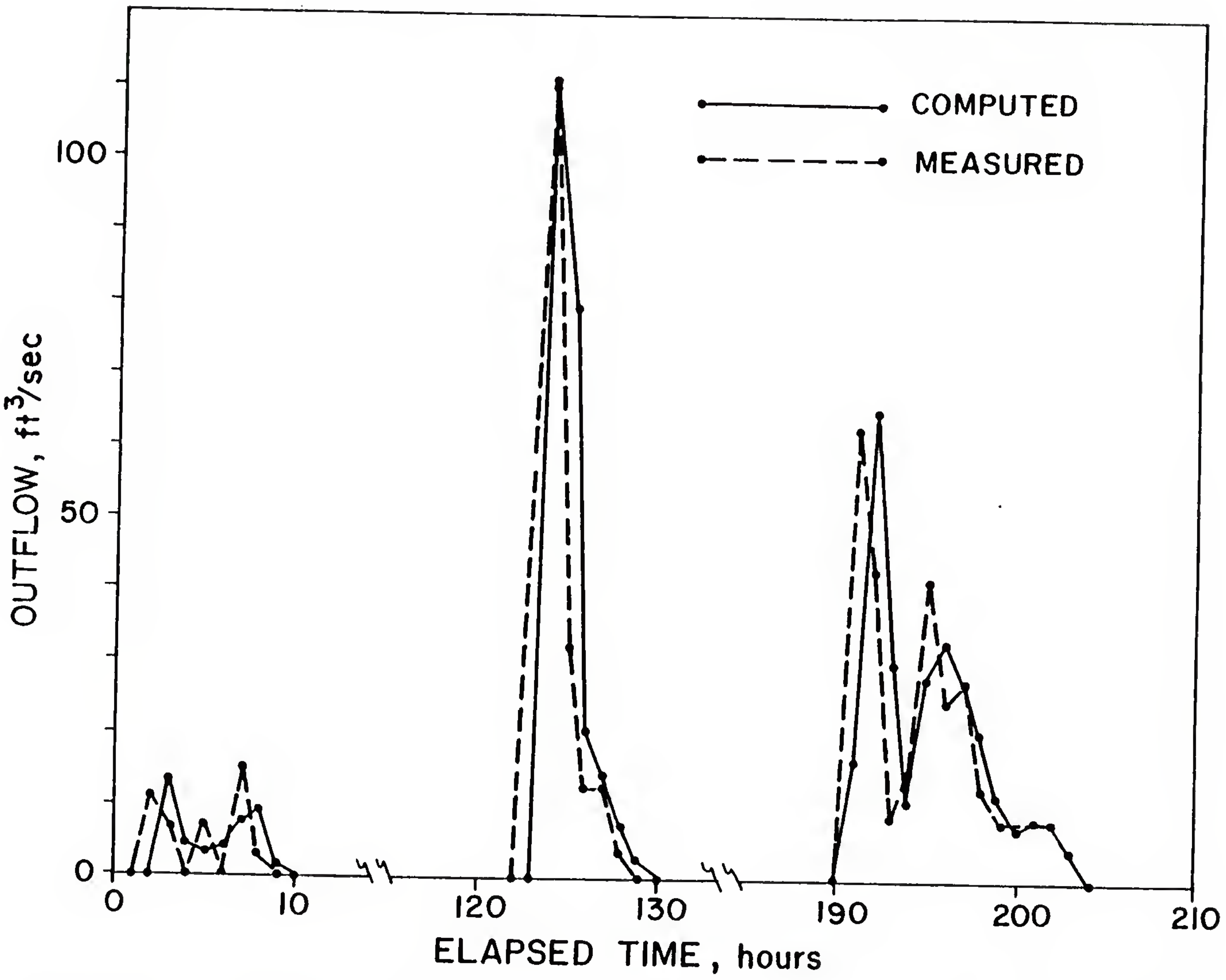


Figure 4-7. Effluent Flow Rate, September 12 (11:00 pm) to September 21 (10:00 am), 1972, Humboldt Avenue Detention Tank, Milwaukee, Wisconsin.

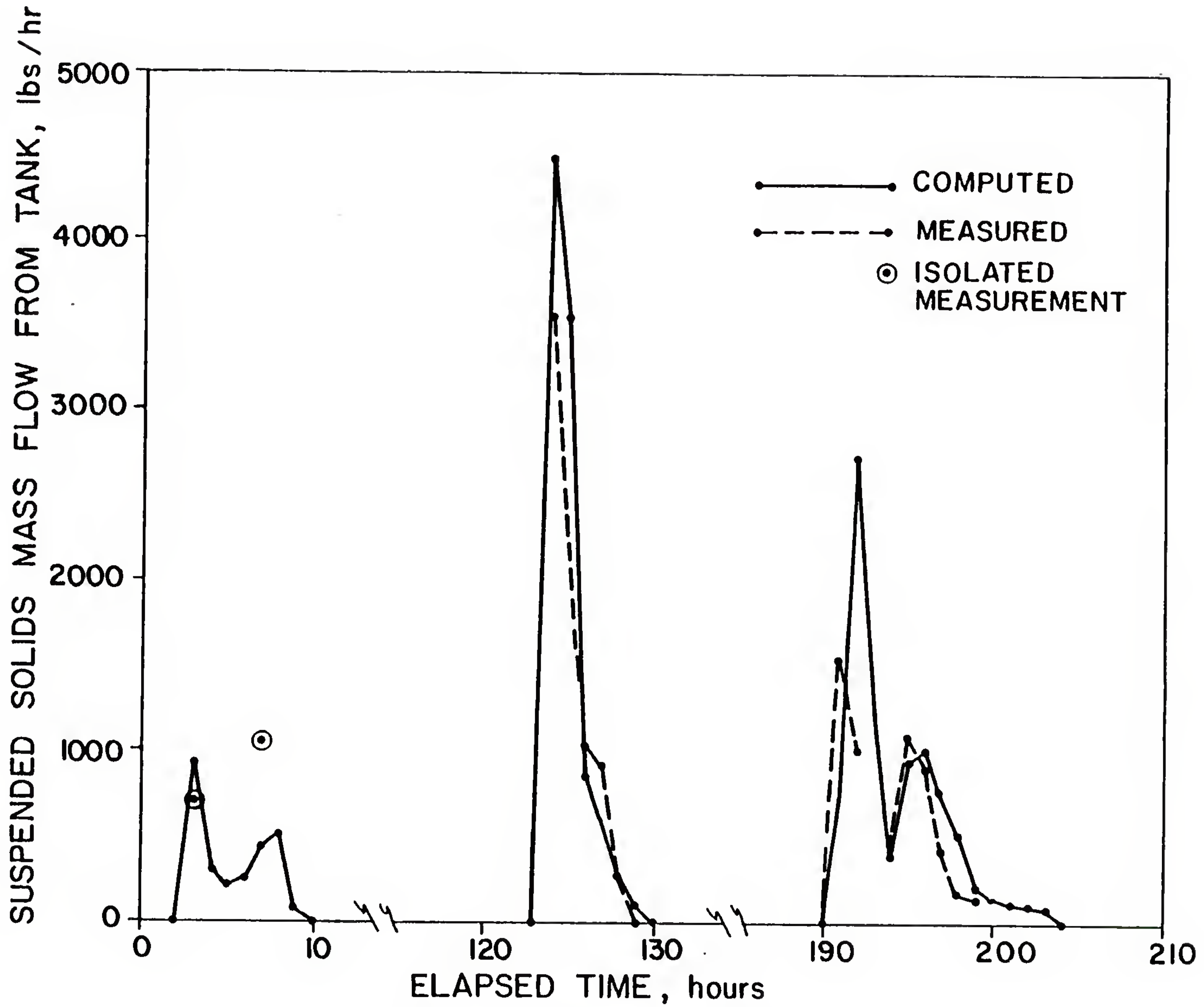


Figure 4-8. Effluent Suspended Solids Mass Rate, September 12 (11:00 pm) to September 21 (10:00 am), 1972, Humboldt Avenue Detention Tank, Milwaukee, Wisconsin.

More effort is needed to substantiate the model algorithms and explore their usefulness in the planning and design of stormwater storage/ treatment systems. Unfortunately, few data are presently available to validate the model procedures. Nevertheless, the model is capable of providing significant guidance in these endeavors.

The model's use as a storage/release system simulator is evident. It possesses capabilities and a flexibility unmatched by the statistical method of Chapter 3 and other simulators. This does not mean, however, that the S/T Block should always be used in lieu of other, less sophisticated methods. The statistical method and simpler simulators are valuable in the early stages of system planning.

CHAPTER 5

PRODUCTION THEORY AND STORAGE/RELEASE SYSTEMS

Introduction

A production function depicts the maximum output attainable through a production process by any set of inputs (Ferguson, 1975). Inputs are broadly defined as direct ingredients, resources, or provided services. Outputs are defined as any product increased in utility by the production process. The concept of a production function is a useful and convenient framework for the presentation of long-term storage/release system performance data, whether real or generated by a statistical method or simulator. More importantly, production functions often form the basis for useful economic analysis.

A stormwater storage/release system is a production process in which inputs or resources are used to generate an output, namely improved urban stormwater quality. The economics literature, and particularly the agricultural economics literature, is rich in its treatment of the production function. However, the use of the production function in water resources management, and more specifically in urban stormwater management, is relatively new.

This chapter will focus on the basic theory of the production function, possible mathematical representations, and some of the previous work in developing urban stormwater storage/release production functions. Much of this work has been undertaken by the author and others at the University of Florida and provides much of the impetus behind this study.

Production Theory and Production Functions

Introduction

A production process increases the utility of a commodity or commodities. However, certain technological relationships restrict the decision maker's options on input and output levels (Baumol, 1977). The production process is not limited to manufacturing; it can also describe processes such as transportation and storage. These are all productive acts in that they increase utility. The utility of a good can be enhanced by altering its form or its position in time or space. It requires resources to raise utility levels or create products. The relationship between resources (labor, capital and goods) and products is the production function.

Production functions are more precisely defined as the representation of the most technologically efficient use of inputs (resources) to produce outputs (products). Given a vector of inputs,

$$\underline{X} = (x_1, x_2, \ldots, x_i, \ldots, x_m) \tag{5-1}$$

where $\underline{X}$ = vector of inputs, and

x_i = input i,

and a vector of outputs,

$$\underline{Y} = (y_1, y_2, \ldots, y_j, \ldots, y_n) \tag{5-2}$$

where $\underline{Y}$ = vector of outputs, and

y_j = output j,

the technological relationship or production function is given as

$$g(\underline{Y}, \underline{X}) = 0 \qquad (5\text{-}3)$$

$$\underline{Y}, \underline{X} \geqq \underline{0}$$

The equality shown in equation 5-3 implies technical efficiency. The non-negativity requirement is intuitively obvious, i.e., there are no negative inputs or outputs. Consider the one-input, one-output function shown in Figure 5-1. The shaded region under the function represents the feasible production region. While it is possible to produce y' with x', it is also possible to produce y' with x". But x" > x' and, thus, the latter option represents an inefficient use of resource x and is rejected. The production function represents the production frontier. Obviously, the assumption of technological efficiency requires the production function to be single-valued.

Production Function Types

There are two broad classes of production functions: fixed- and variable-proportion. A production process is characterized by fixed proportions if, and only if, each level of output requires a unique set of inputs (Ferguson, 1975). Fixed proportion production functions are not useful in the context of storage/release production functions. A variable-proportion production function is one in which the same level of output may be produced by several combinations of inputs (Ferguson, 1975). These functions are assumed to be smooth and continuous and to have continuous first and second partial derivatives. Additionally, the inputs are assumed to be homogeneous, i.e., one unit of a specific input is identical to all other units of that input.

Production functions are developed only for variable inputs; fixed inputs are ignored because no control can be exercised over them. The

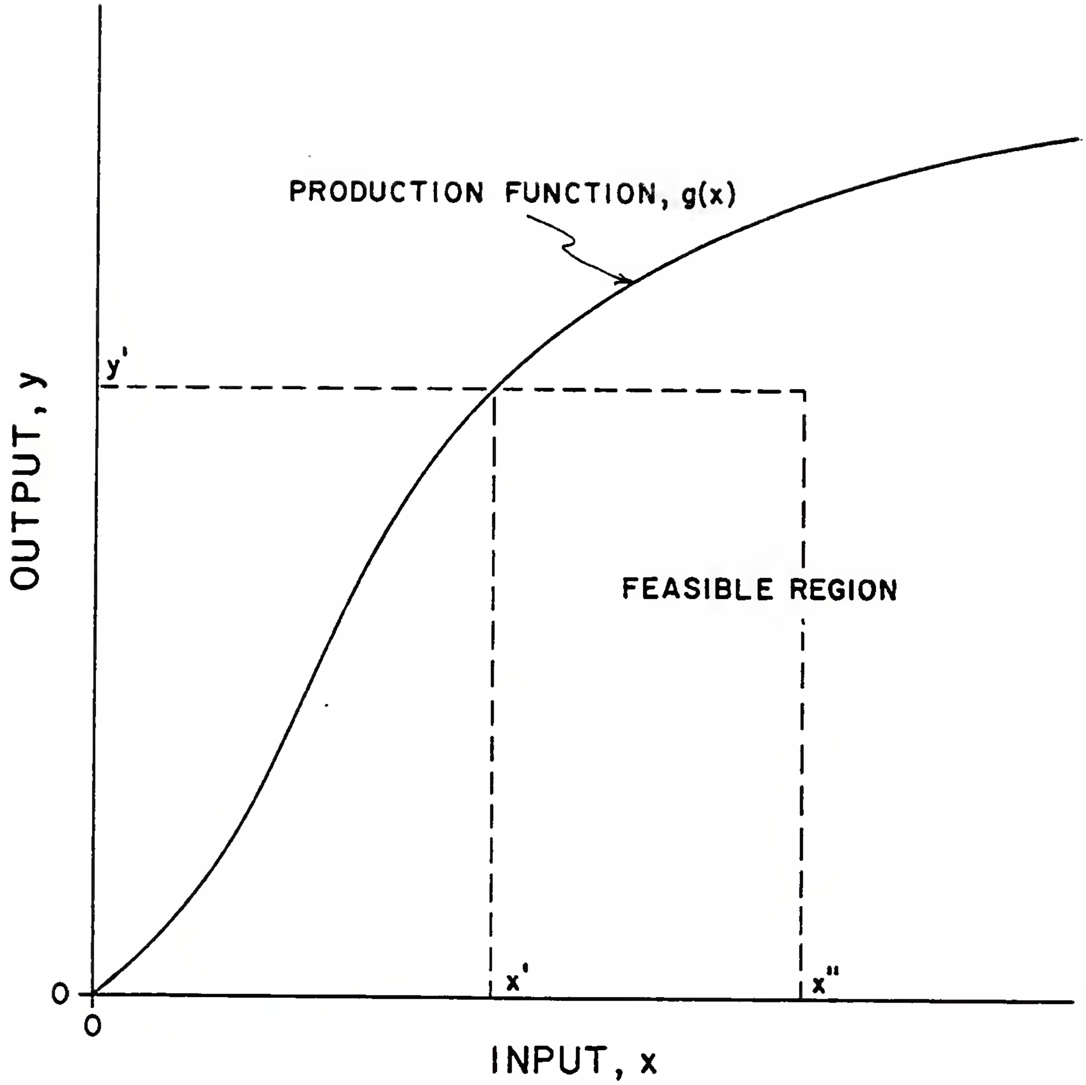


Figure 5-1. One-Input, One-Output Production Function.

terms "variable" and "fixed" used in this statement should not be confused with the previous mention of variable and fixed proportions.

Equation 5-3 implies that there is an infinite number of production function types with respect to the number of inputs and outputs. However, for the purposes herein, only the following types will be explored in any detail:

1) one input, one output,
2) two inputs, one output,
3) one input, two outputs, and
4) two inputs, two outputs.

The classical one-input, one-output production function, shown in Figure 5-1, is represented by

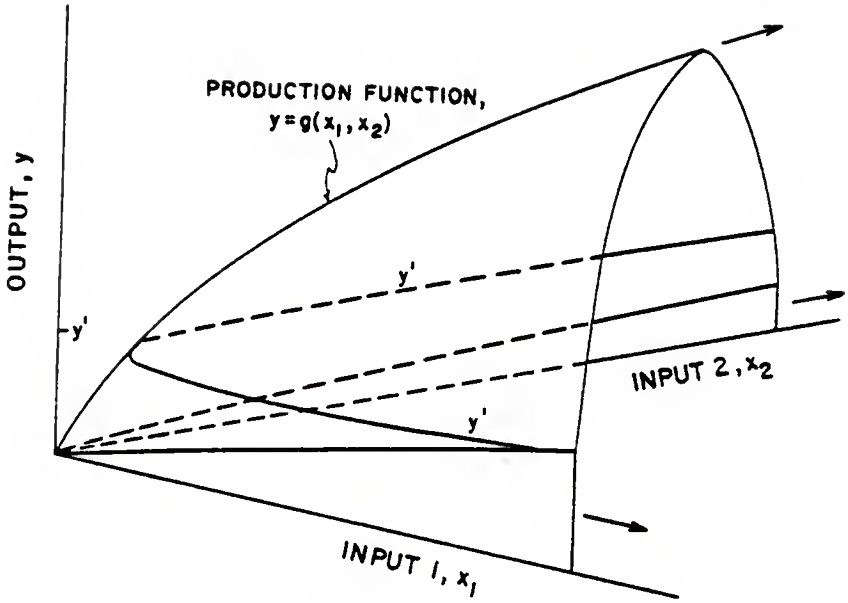
$$y = g(x) \tag{5-4}$$

where y = output, and

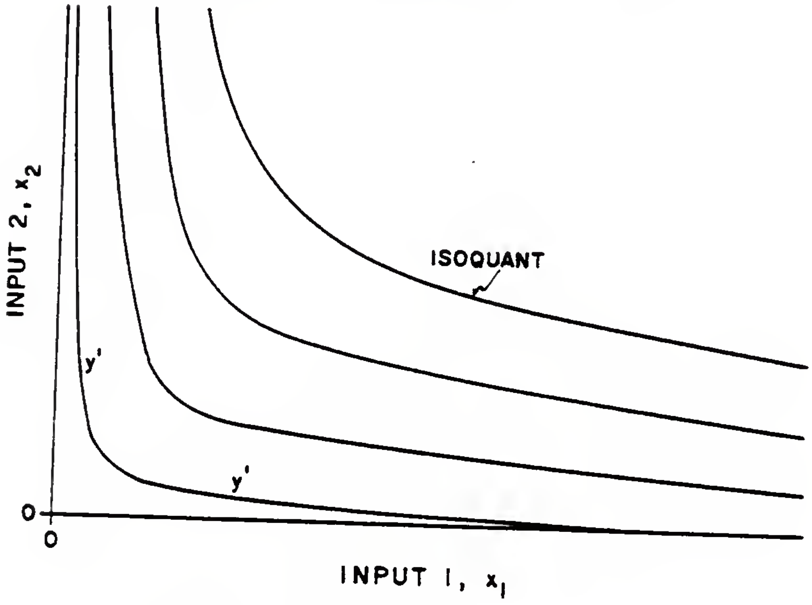
x = input.

Recall that the equality expressed in equation 5-4 expresses the assumption of technological efficiency.

The three-dimensional relationship for two inputs and one output, shown in Figure 5-2(a), is more conveniently drawn as the two-dimensional graph shown in Figure 5-2(b). The curves in Figure 5-2(b) are created by mapping the intersection of a plane, parallel to the input axes, and the production surface onto the plane formed by the input axes. Each curve represents a specific level of output and denotes the different combinations of input that can be used to achieve that level. These curves are commonly and aptly referred to as "isoquants." Isoquants never cross because the production function is single-valued. Mathematically, the two-input, one-output function can be written as



(a) Three-Dimensional Representation



(b) Two-Dimensional Representation

Figure 5-2. Two-Input, One-Output Production Function.

$$y = g(x_1, x_2) \tag{5-5}$$

where y = output, and

x_1, x_2 = imputs 1 and 2.

This type of function has played and will play a prominent role in the development of storage/release production functions and its properties are discussed in more detail in the next section.

The production function depicted in Figure 5-3(a) describes a one-input, two-output relationship that, like the two-input, one-output function, can be more conveniently shown as the two-dimensional map presented in Figure 5-3(b). In this case, the isoquants, more commonly known as product transformation curves, represent equal levels of input or resource used to produce two outputs. The curves are normally concave to the origin. This condition indicates the complementary nature of the two outputs. The function is described by

$$x = g(y_1, y_2) \tag{5-6}$$

where x = input, and

y_1, y_2 = outputs 1 and 2.

Two-input, two-output production functions are mathematically described by

$$g(y_1,y_2,x_1,x_2) = 0 \tag{5-7}$$

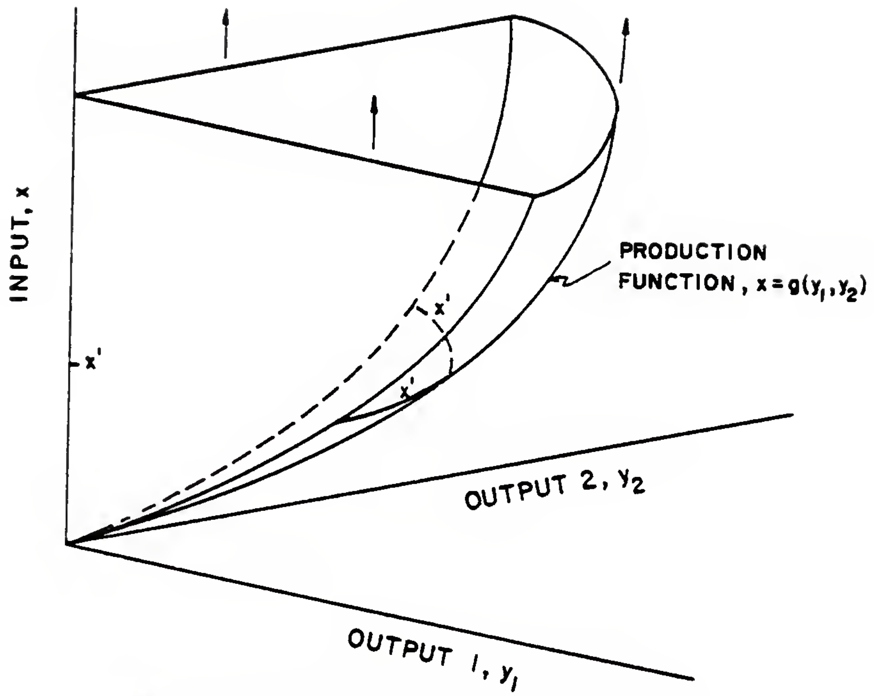
where y_1, y_2 = outputs 1 and 2, and

x_1, x_2 = inputs 1 and 2.

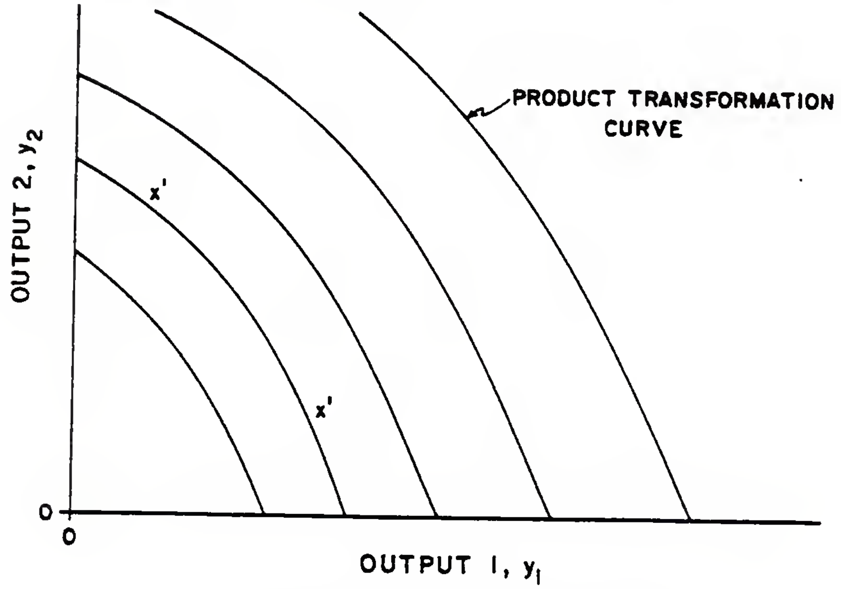
Equations 5-6 and 5-7 are useful when a single system serves two purposes (e.g., water quality improvement and flow control).

Properties of Production Functions

The one-input, one-output production function provides a convenient point to begin a look at the properties of production functions. Figure



(a) Three-Dimensional Representation



(b) Two-Dimensional Representation

Figure 5-3. One-Input, Two-Output Production Function.

5-4 shows the classical one-input, one-output function found in any production economics or microeconomics textbook.

An interesting empirical observation helps to understand the shape of the production function shown in Figure 5-4. Known as the "law" of diminishing marginal returns, it states that as more and more of some input i is used, and all other inputs are held constant, a point is eventually reached where additional quantities of input i will produce diminishing additional quantities of the product (Baumol, 1977). Intuition would tell us that as more of a particular input is used and all others are held constant, a point is reached where additional input becomes a disadvantage. For example, if a firm hires more labor but does not increase the work space, eventually the additional man will not lead to the kind of increase in productivity that the previously hired man did. This observation appears to be true in almost all production processes.

A more detailed discussion of production function properties requires the following definitions (Ferguson, 1966, 1975):

1) The marginal product, MP, of an input is the addition to the total output attributable to the addition of one unit of input to the production process, i.e.,

$$MP = \frac{dy}{dx} \tag{5-8}$$

2) The average product, AP, of an input is the total output divided by the amount of input used to produce that output, i.e.,

$$AP = \frac{y}{x} \tag{5-9}$$

The marginal and average product curves are plotted along with the production function in Figure 5-4. It is easy to see that the average

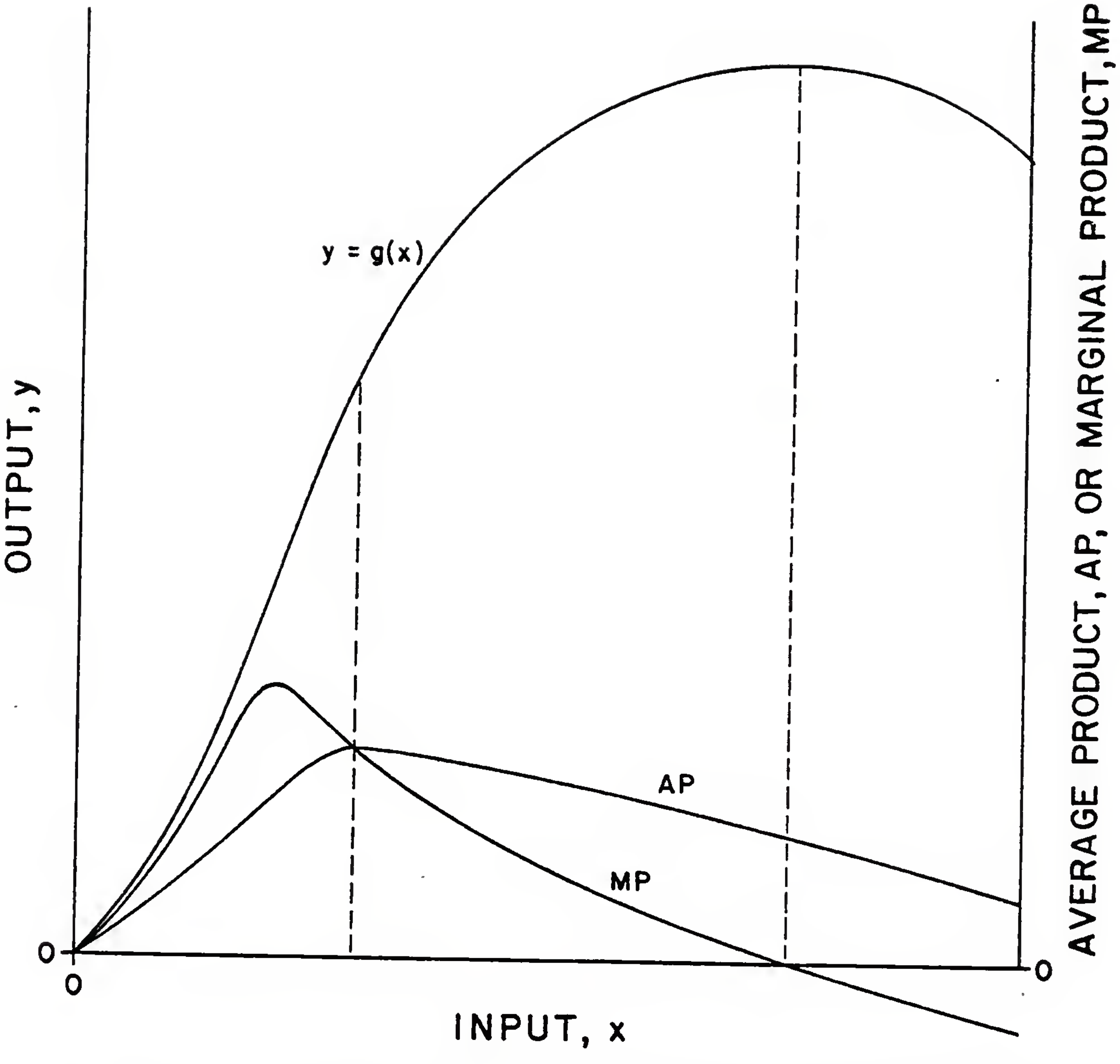


Figure 5-4. Classical One-Input, One-Output Production Function and the Average and Marginal Products.

product is at the maximum when a line from the origin to a point on the production function has its maximum slope (i.e., y/x). It is also easy to show that the average and marginal products are equal at this point. Given equations 5-8 and 5-9, the average product is at its maximum when

$$\frac{d(AP)}{dx} = \frac{d(y/x)}{dx} = \frac{1}{x}\left(\frac{dy}{dx} - \frac{y}{x}\right) = 0 \qquad (5\text{-}10)$$

or

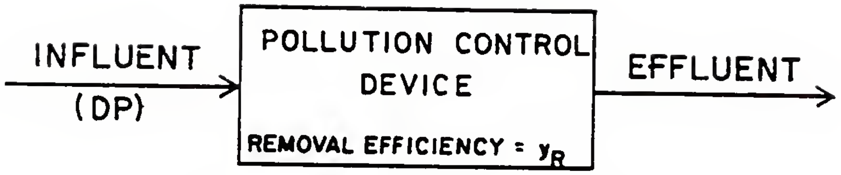
$$\frac{dy}{dx} = \frac{y}{x} \qquad \text{since } x > 0. \qquad (5\text{-}11)$$

The marginal product is greater than the average product to the left of this point and less than the average product to the right. At the point where the output begins to decline, the marginal product is zero; to the right of this point, it is negative.

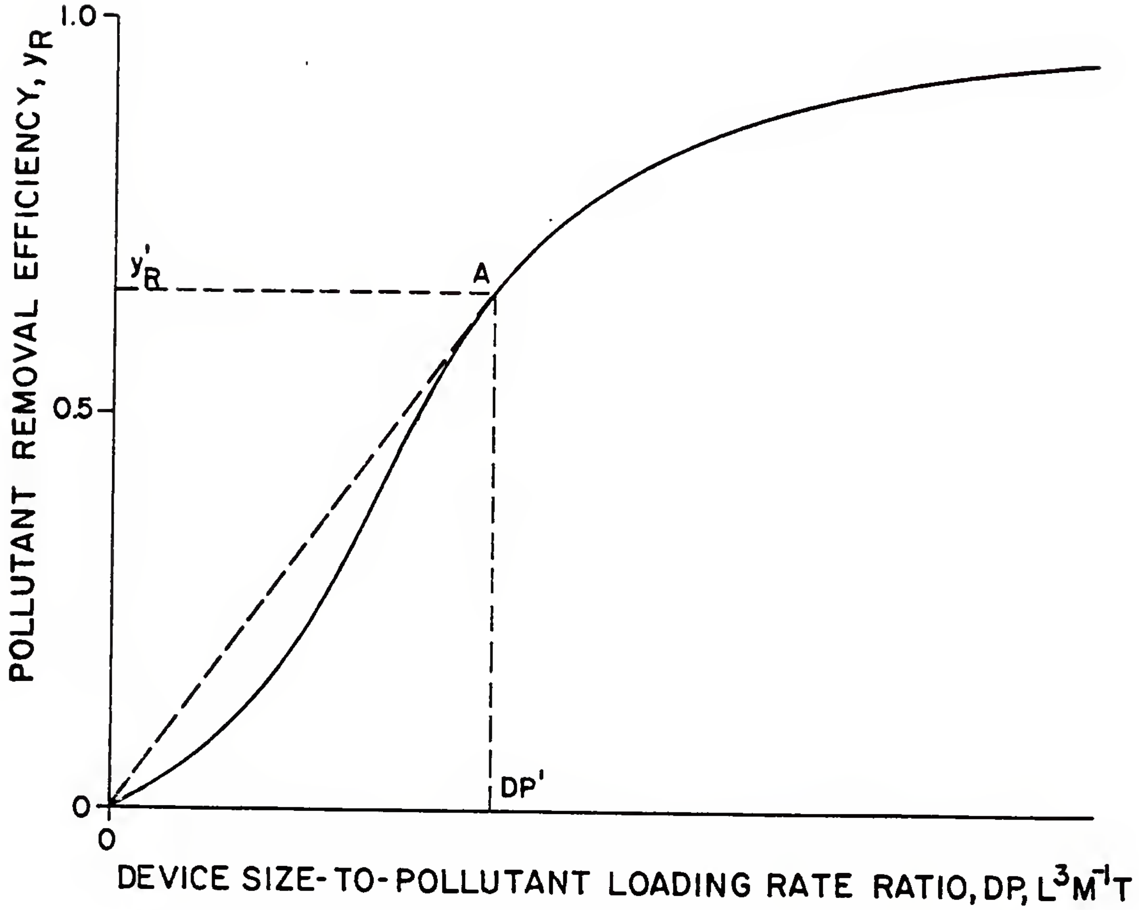
Environmental control processes will not usually display increasing average products (Heaney, 1979). Consider the production process schematized in Figure 5-5(a) and its production function shown in Figure 5-5(b). The production process represents a pollution control device accepting a constant flow of sanitary sewage. The output of the function is the pollutant removal efficiency, y_R, and the input is the ratio of the device size (L^3) to the pollutant loading rate (MT^{-1}), DP. As the ratio DP increases, the device performance improves. However, the production function demonstrates an increasing average product up to point A. Heaney (1979) argues that if the device was required to meet some objective between the origin and point A, the rational analyst would maintain the ratio at point A, DP', and only allow a fraction of the load, γ, to enter the device such that

$$y_R = \gamma y_R' \qquad (5\text{-}12)$$

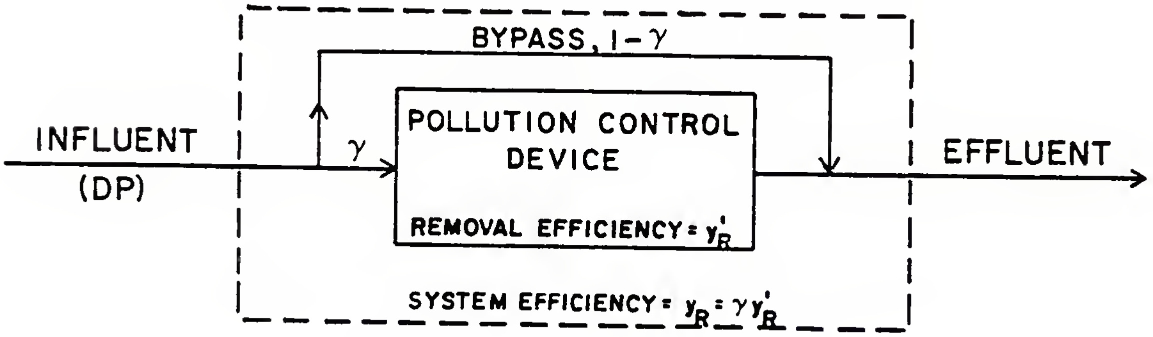
where y_R = overall removal efficiency, and



(a) Original Device Operation



(b) Pollution Control Device Production Function



(c) Revised Device Operation

Figure 5-5. Elimination of the Region of Increasing Average Product.

y_R' = removal efficiency associated with DP'.

This mode of operation is shown in Figure 5-5(c). The removal efficiency for all flows passing through the unit, while operating to the left of point A, is fixed at y_R' because the ratio of size to load entering the device remains fixed at DP'. The value of DP, the ratio for the entire system (including the circumvented load), is given as

$$DP = \gamma DP' \tag{5-13}$$

because the device size must be reduced to γ of its value at point A, but the load to the entire system is unaffected. Thus, in the region to the left of point A, the production function becomes the linear relationship

$$y_R = \frac{y_R'}{DP'} DP \tag{5-14}$$

This is shown by the dotted line drawn from the origin to point A in Figure 5-5(b).

Marginal and average products are also defined for multiple-input, multiple-output production functions:

$$MP_{ij} = \frac{\partial y_j}{\partial x_i} \tag{5-15}$$

and

$$AP_{ij} = \frac{y_j}{x_i} \tag{5-16}$$

where MP_{ij} = marginal product of input x_i with respect to output y_j, and

AP_{ij} = average product of input x_i with respect to output y_j.

Each marginal or average product is calculated with all other inputs or outputs held constant. An additional parameter, unique to multiple-input functions, is the marginal rate of substitution. For the two-input, one-output case, the marginal rate of substitution is defined as the number of units by which one input must be increased (reduced) to maintain a constant output when the other input is reduced (increased) by one unit, i.e.,

$$MRS_{21} = -\frac{\partial x_2}{\partial x_1} \tag{5-17}$$

where MRS_{21} = marginal rate of substitution of input 2 for input 1. A graphical representation of the marginal rate of substitution is shown in Figure 5-6. The value of MRS_{21} at any point is the negative of the slope of the isoquant. It can be shown that the marginal rate of substitution of x_2 for x_1 is equal to the ratio of the marginal products for x_1 and x_2. The total derivative of the production function is

$$dy = \frac{\partial g}{\partial x_1}\,\partial x_1 + \frac{\partial g}{\partial x_2}\,\partial x_2 \tag{5-18}$$

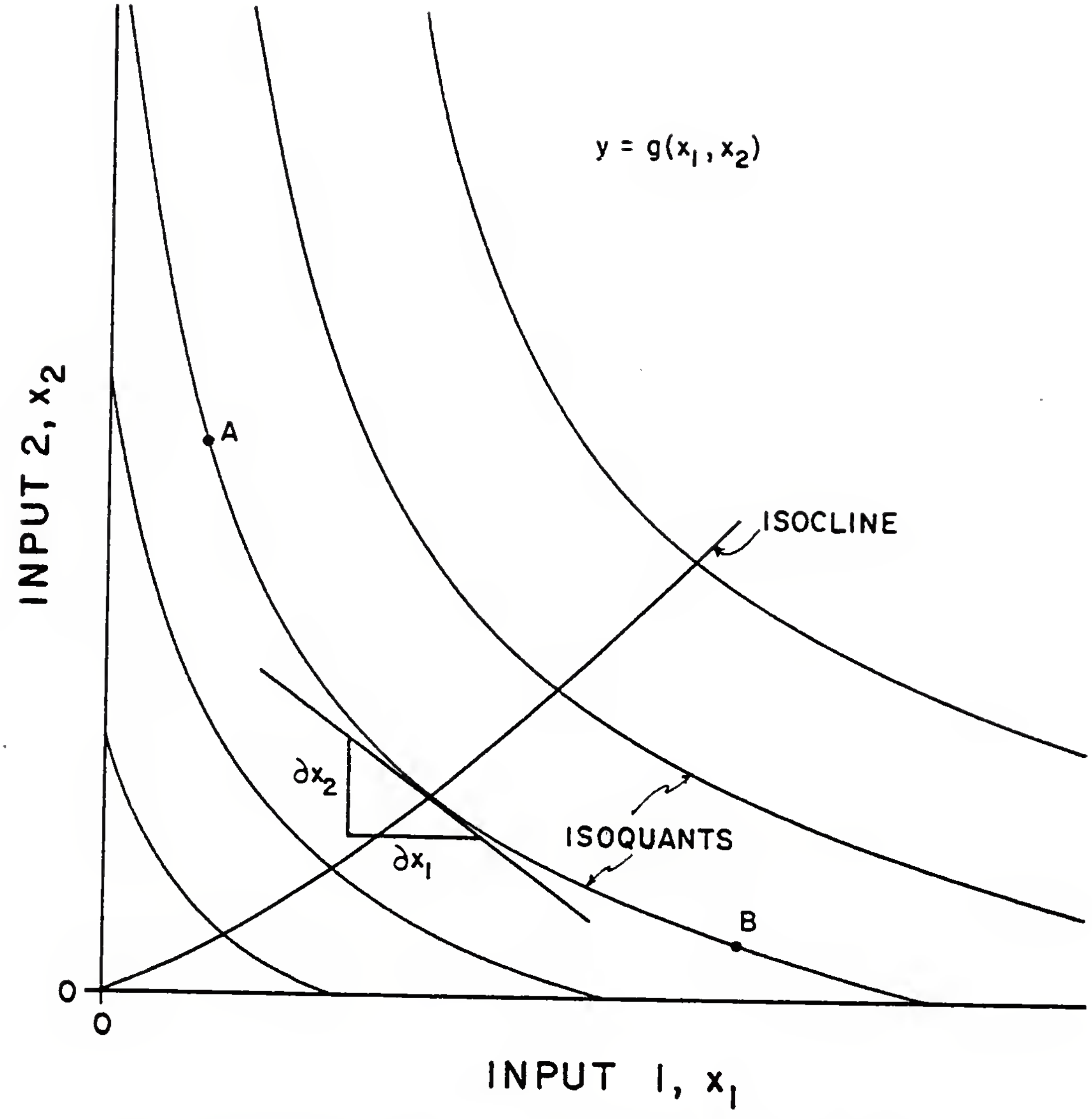
The output is constant along any isoquant, i.e., dy = 0. Thus,

$$\frac{\partial g}{\partial x_1}\partial x_1 + \frac{\partial g}{\partial x_2}\partial x_2 = 0 \tag{5-19}$$

or

$$-\frac{\partial x_2}{\partial x_1} = \frac{\dfrac{\partial g}{\partial x_1}}{\dfrac{\partial g}{\partial x_2}} = \frac{\dfrac{\partial y}{\partial x_1}}{\dfrac{\partial y}{\partial x_2}} = \frac{MP_{11}}{MP_{21}} \tag{5-20}$$

A line connecting a particular value of MRS_{21} on each isoquant is known as an isocline (see Figure 5-6). The isocline is useful in determining a region of the isoquant map known as the area of substitution. Consider the isoquant map shown in Figure 5-7. Line A represents the isocline for $MRS_{21} = 0$ and line B is the isocline for $MRS_{21} = \infty$. These two limiting isoclines are known as ridge lines. The two ridge lines form the area of substitution, i.e., the area where a constant output could be attained by substituting one input for another. Outside



NOTE: Isoclines do not necessarily emanate from the origin.

Figure 5-6. Graphical Representation of the Marginal Rate of Substitution and an Isocline.

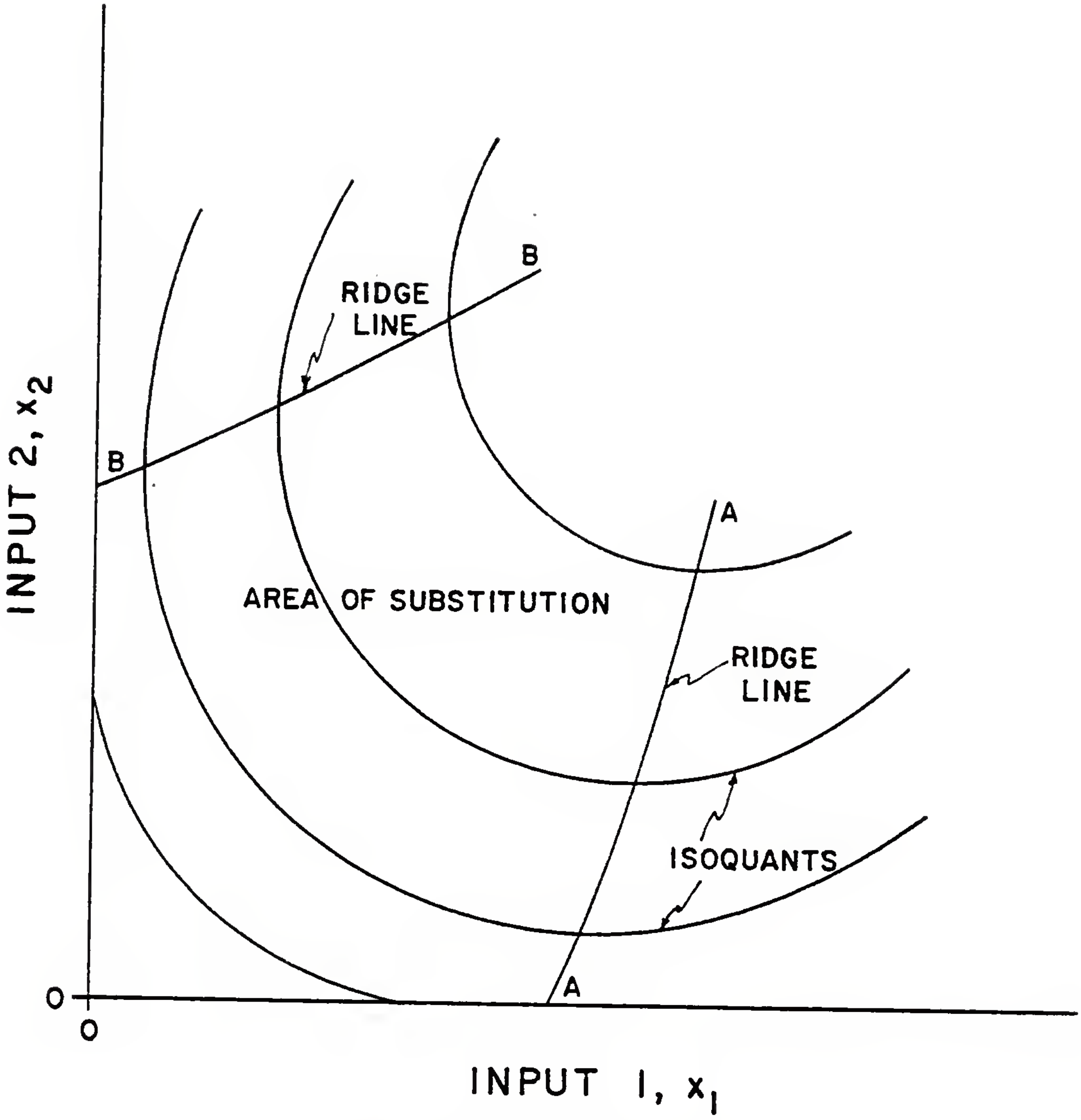


Figure 5-7. Determination of the Area of Substitution.

of this region both inputs would have to be increased to maintain a specific output. More precisely, line A represents, for a given value of x_2, the point at which the marginal product of x_1 becomes zero. To the right of line A the marginal product of x_1 is negative. Line B represents the same condition for input x_2 (i.e., the marginal product of x_2 is negative above line B). For the purposes of economic analysis, the region outside the area of substitution may be ignored because falling input costs will usually not be encountered (Ferguson, 1975).

The shape of the isoquants in the area of substitution is governed by the "principle" of diminishing marginal rate of substitution (Ferguson, 1966). This principle states that as one input is substituted for another, while maintaining a constant output (i.e., moving along an isoquant), it becomes more difficult for the substitution to maintain the output level. Consider the isoquant segment AB in Figure 5-6. When moving from point A to point B, it takes an increasing number of units of x_1 to replace each unit of x_2 and, thus, the value of dx_2/dx_1 is decreasing. The reverse situation (moving from B to A and substituting x_2 for x_1) also follows this "principle." Two factors contribute to this phenomenon:

1) the diminishing marginal returns of each input, and
2) the reduction in one input causes a downward shift in the marginal product curve of the other input because the production base is reduced.

A diminishing marginal rate of substitution insures that the isoquant set (over the area of substitution) will be convex to the origin.

In addition to the marginal and average product definitions given in equations 5-15 and 5-16, a parameter analogous to the marginal rate of substitution for the two-input, one-output production function can be developed for the one-input, two-output case. This parameter, known as the marginal rate of transformation, defines the ability of the production process to shift from one output to another with a given level of input (Ferguson, 1966). Mathematically,

$$MRT_{21} = - \frac{\partial y_2}{\partial y_1} \tag{5-21}$$

where MRT_{21} = marginal rate of transformation of output 2 for output 1.

The value of MRT_{21} corresponds to the negative of the slope of the product transformation curve. It can be demonstrated that MRT_{21} is equivalent to the ratio of the marginal products of the input with respect to outputs 2 and 1. Again, consider the total derivative of the production function:

$$dx = \frac{\partial g}{\partial y_1} \partial y_1 + \frac{\partial g}{\partial y_2} \partial y_2 \tag{5-22}$$

Along any product transformation curve the input level does not change, i.e., dx = 0. Thus,

$$\frac{\partial g}{\partial y_1} \partial y_1 + \frac{\partial g}{\partial y_2} \partial y_2 = 0 \tag{5-23}$$

or

$$- \frac{\partial y_2}{\partial y_1} = \frac{\frac{\partial g}{\partial y_1}}{\frac{\partial g}{\partial y_2}} = \frac{\frac{\partial x}{\partial y_1}}{\frac{\partial x}{\partial y_2}} = \frac{\frac{1}{MP_{11}}}{\frac{1}{MP_{12}}} = \frac{MP_{12}}{MP_{11}} \tag{5-24}$$

All of the parameters and properties discussed above are applicable for production functions with two or more inputs and two or more outputs. The presentation becomes unnecessarily difficult because of the dimensionality of these functions, and, thus, will no longer be pursued.

Mathematical Representations of Production Functions

Production functions can play an important role in the presentation of storage/release system performance data and in the application of optimization techniques to obtain economically efficient designs and operating policies (as will be shown in the next chapter). The manner in which the production data are represented has a distinct effect on any subsequent analysis, especially an economic analysis. If the production data can be adequately described by a mathematical function, a wide array of effective analytical techniques can be used to determine the optimal design. However, graphical techniques can perform the same analysis when a reasonable functional form is not available.

This section will describe the functional forms most commonly used by economists and some of their salient properties. These forms have a long history in describing production processes and, in general, possess qualities that make them amenable to economic decision making. The use of "form-free" representations is also discussed. These representations do not rely on a particular functional form to describe the production process.

Cobb-Douglas or Power Function

The well known Cobb-Douglas function for a two-input, one-output production process is given as follows (Hexem and Heady, 1978):

$$y = Ax_1^{\beta_1} x_2^{\beta_2} \tag{5-25}$$

where y = output,

x_1, x_2 = inputs 1 and 2,

A = coefficient, and

β_1, β_2 = coefficients, $0 < \beta_1 < 1$, $0 < \beta_2 < 1$.

The limits on coefficients β_1 and β_2 insure adherence to the "law" of diminishing marginal returns. This equation is readily generalized to denote n-input, n-output production functions but, for convenience, this simple form is analyzed here. The simplest form is, obviously, the one-input, one-output power function:

$$y = Ax^{\beta} \tag{5-26}$$

where β = coefficient, $0 < \beta < 1$.

The Cobb-Douglas function was originally developed for analyzing manufacturing industries in the first decade of this century but it has also become very popular in the field of agricultural economics to describe crop production (Hexem and Heady, 1978) and other processes. It has also been used to describe storage/release systems in the control of urban stormwater pollution (Nix et al., 1977). Its popularity stems primarily from its elegance as a mathematical tool rather than its ability to characterize production processes. A particularly troublesome feature is that any input is limitational, i.e., when any input is zero, production ceases ($y = 0$).

The single-input, single-output Cobb-Douglas production function appears as shown in Figure 5-8 along with the average and marginal products:

$$AP = Ax^{\beta-1} \tag{5-27}$$

and

$$MP = A\beta x^{\beta-1} \tag{5-28}$$

Recall that $\beta < 1$; thus, the average and marginal products decrease as

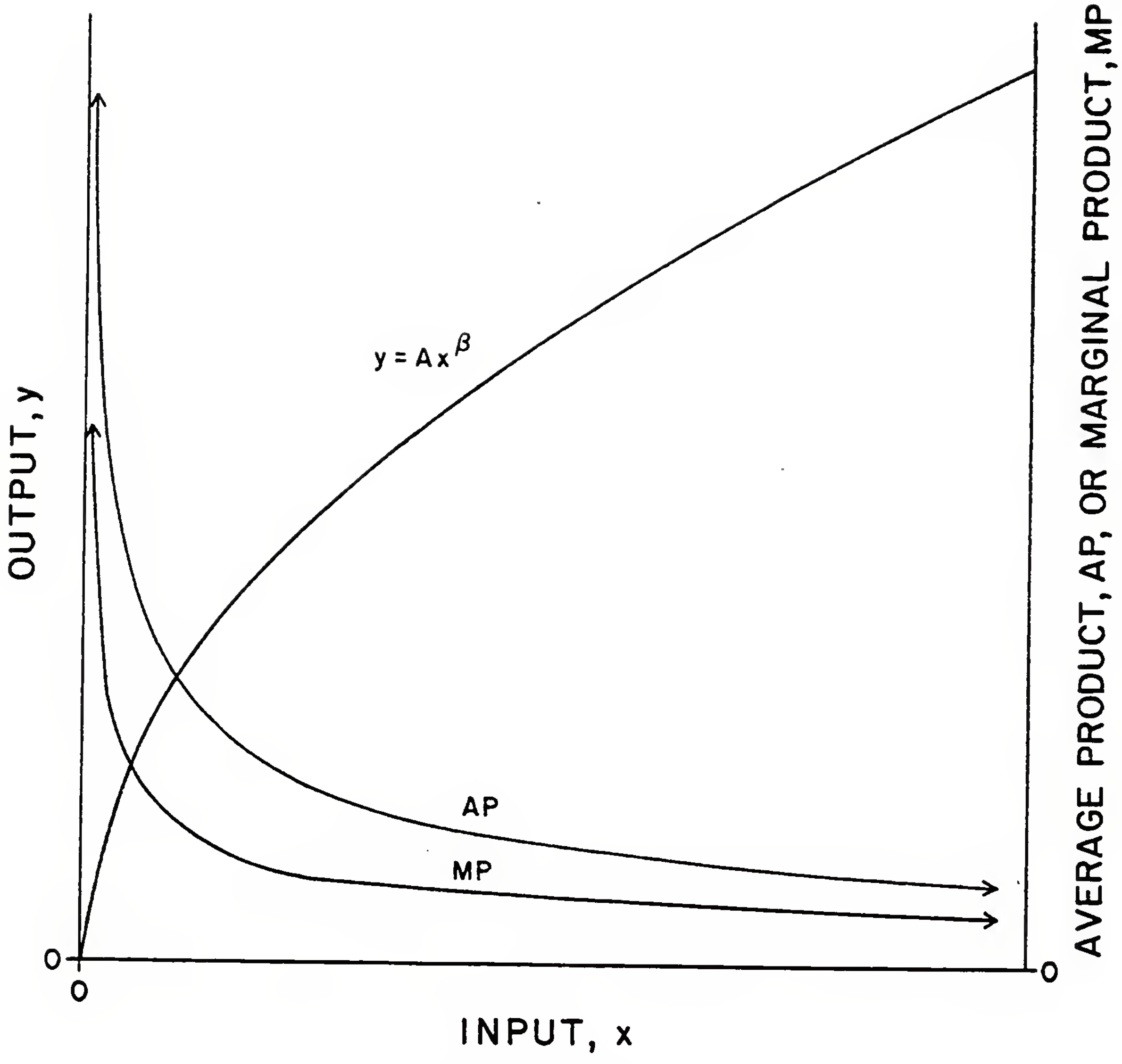


Figure 5-8. One-Input, One-Output Cobb-Douglas Production Function and the Average Marginal Products.

x increases and the average product is always greater than the marginal product. It is also evident that the marginal product is never negative, i.e., the function never reaches a maximum. In addition, the average and marginal products asymptotically approach zero as x approaches infinity and they approach infinity as x approaches zero.

The average products for the two-input, one-output Cobb-Douglas function are

$$AP_{11} = Ax_1^{\beta_1 - 1} x_2^{\beta_2} \tag{5-29}$$

and

$$AP_{21} = Ax_1^{\beta_1} x_2^{\beta_2 - 1} \tag{5-30}$$

The marginal products are

$$MP_{11} = A\beta_1 x_1^{\beta_1 - 1} x_2^{\beta_2} \tag{5-31}$$

and

$$MP_{21} = A\beta_2 x_1^{\beta_1} x_2^{\beta_2 - 1} \tag{5-32}$$

The same properties indicated for the average and marginal products in the one-input, one-output case are also evident in this case.

The marginal rate of substitution for a two-input, one-output Cobb-Douglas function is

$$MRS_{21} = \frac{MP_{11}}{MP_{21}} = \frac{A\beta_1 x_1^{\beta_1 - 1} x_2^{\beta_2}}{A\beta_2 x_1^{\beta_1} x_2^{\beta_2 - 1}} = \frac{\beta_1 x_2}{\beta_2 x_1} \tag{5-33}$$

For a specific value of MRS_{21}, equation 5-33 can be written as the equation of a linear isocline:

$$x_2 = MRS_{21} \frac{\beta_2}{\beta_1} x_1 \tag{5-34}$$

Thus, the marginal rate of substitution remains constant for a specific ratio of inputs. A few isoquants and isoclines for a two-input, one-output Cobb-Douglas function are shown in Figure 5-9. Note that the entire quadrant is in the area of substitution.

Mitscherlich-Spillman Functions

The generalized Mitscherlich function for m inputs and one output is written as (Hexem and Heady, 1978)

$$y = A[1-\exp(-\beta_1 x_1)][1-\exp(-\beta_2 x_2)] \ldots [1-\exp(-\beta_m x_m)] \qquad (5\text{-}35)$$

where y = output,

$x_1, x_2, \ldots, x_m$ = inputs 1 through m,

A = coefficient, and

$\beta_1, \beta_2, \ldots, \beta_m$ = coefficients, $\beta_1 > 0, \beta_2 > 0, \ldots, \beta_m > 0$.

Like the Cobb-Douglas function, all inputs are limitational.

The single-input form of equation 5-35 is

$$y = A[1-\exp(-\beta x)] \qquad (5\text{-}36)$$

and is displayed in Figure 5-10. The average and marginal products are

$$AP = \frac{A}{x}[1-\exp(-\beta x)] \qquad (5\text{-}37)$$

and

$$MP = A\beta\exp(-\beta x)$$

The marginal and average products are also displayed in Figure 5-10. As with the Cobb-Douglas equation, the average and marginal products are always decreasing, the average product is always greater than the marginal product, and the marginal product is never nonnegative. Additionally, the average and marginal products approach zero as x approaches infinity. When x is zero, the marginal product is Aβ and the average product is undefined. However, the average product approaches Aβ as x nears zero. The Mitscherlich function also has the potentially useful feature of asymptotically approaching the coefficient A as input usage increases. Thus, the coefficient A represents the maximum output.

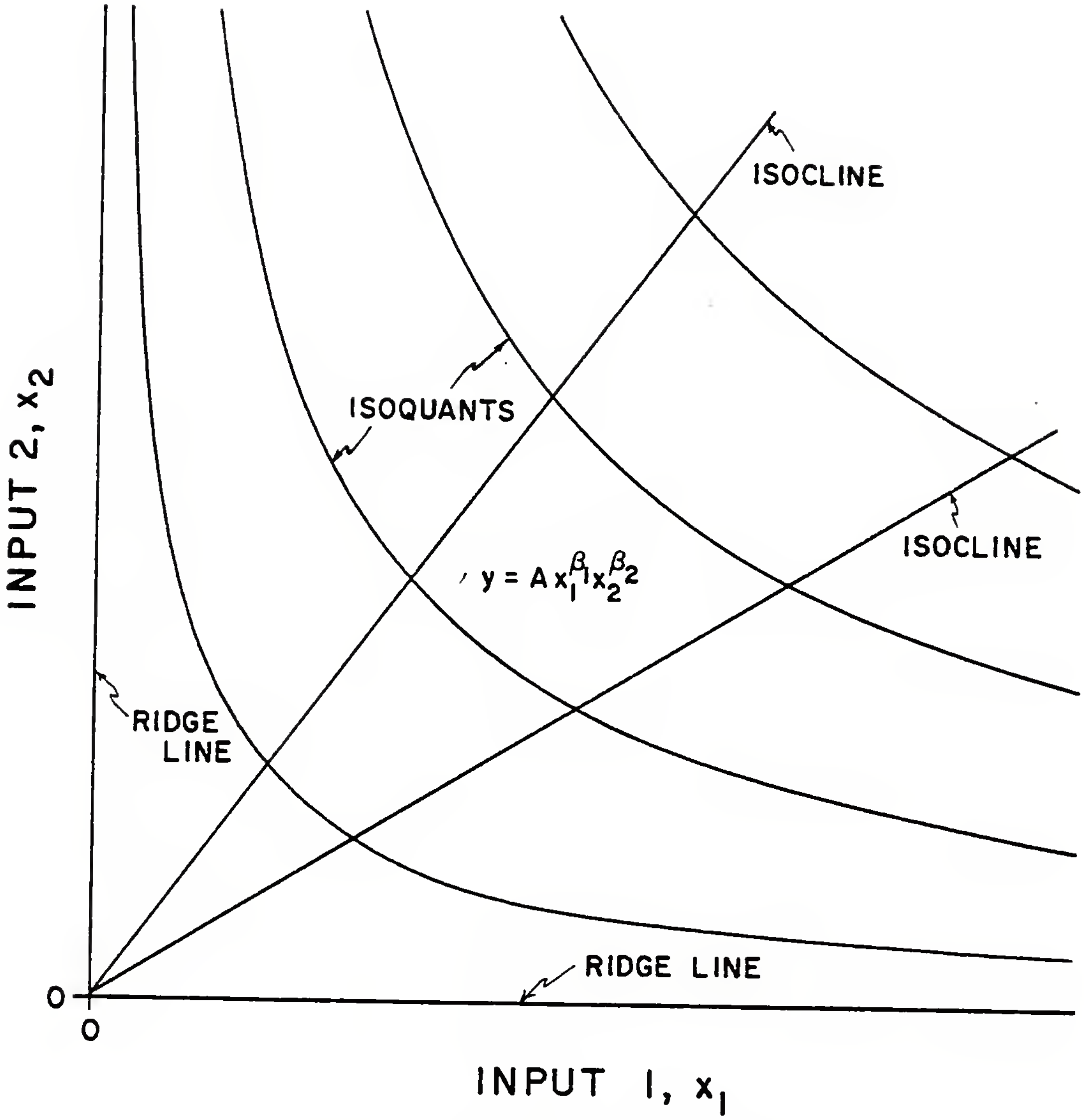


Figure 5-9. Two-Input, One-Output Cobb-Douglas Production Function with Isoclines and Ridge Lines.

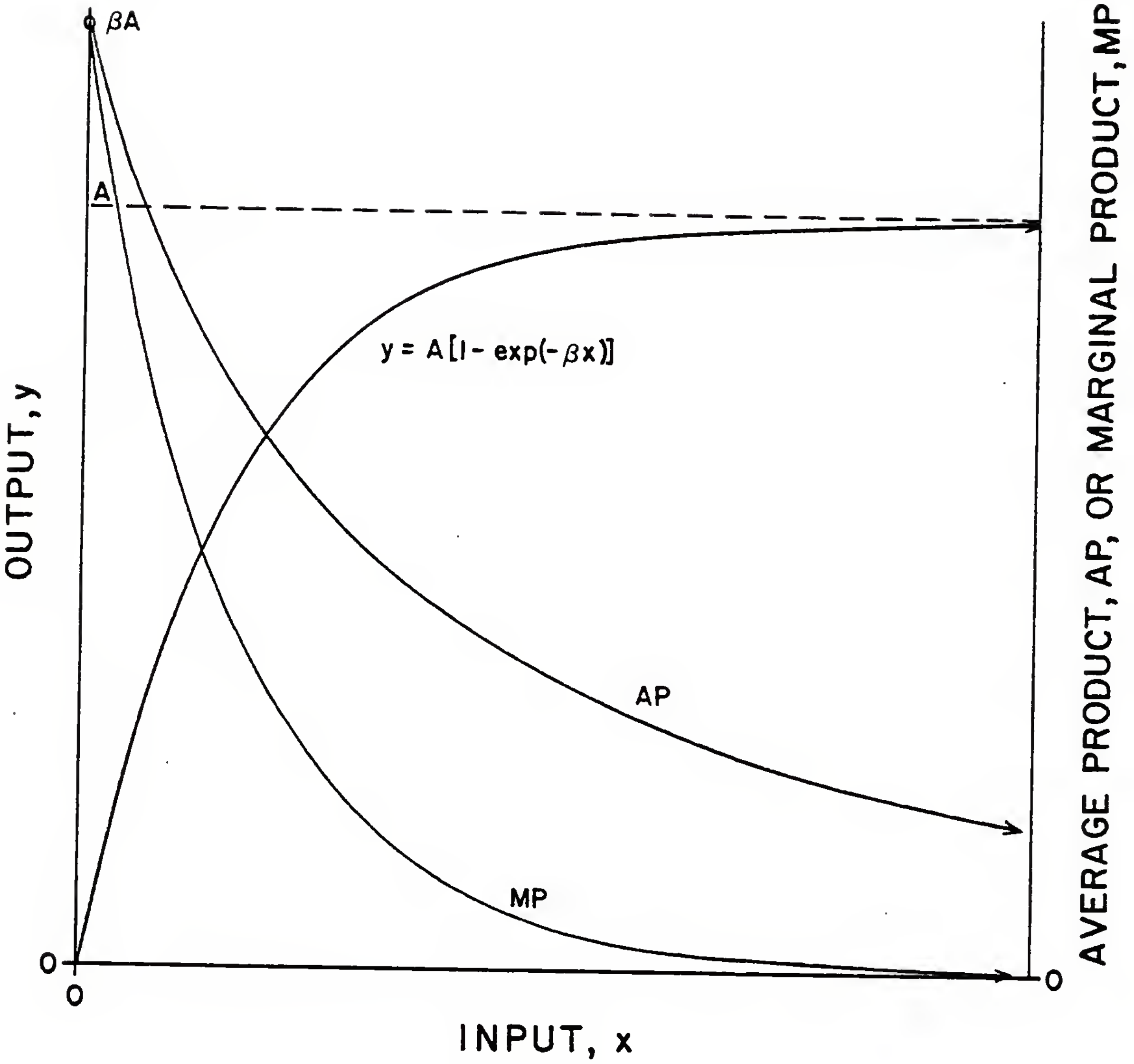


Figure 5-10. One-Input, One-Output Mitscherlich Production Function and the Average and Marginal Products.

The two-input, one-output Mitscherlich function is given as

$$y = A[1-\exp(-\beta_1 x_1)][1-\exp(-\beta_2 x_2)] \tag{5-38}$$

The average products are

$$AP_{11} = \frac{A}{x_1}\,[1-\exp(-\beta_1 x_1)][1-\exp(-\beta_2 x_2)] \tag{5-39}$$

and

$$AP_{21} = \frac{A}{x_2}\,[1-\exp(-\beta_1 x_1)][1-\exp(-\beta_2 x_2)] \tag{5-40}$$

The marginal products are

$$MP_{11} = A\beta_1[\exp(-\beta_1 x_1)][1-\exp(-\beta_2 x_2)] \tag{5-41}$$

and

$$MP_{21} = A\beta_2[1-\exp(-\beta_1 x_1)][\exp(-\beta_2 x_2)] \tag{5-42}$$

Equations 5-41 and 5-42 allow the marginal rate of substitution to be easily determined:

$$MRS_{21} = \frac{MP_{11}}{MP_{21}} = \frac{\beta_1[\exp(-\beta_1 x_1)][1-\exp(-\beta_2 x_2)]}{\beta_2[1-\exp(-\beta_1 x_1)][\exp(-\beta_2 x_2)]}$$

$$= \frac{\beta_1[1-\exp(\beta_2 x_2)]}{\beta_2[1-\exp(\beta_1 x_1)]} \tag{5-43}$$

The equation of an isocline becomes

$$x_2 = \frac{1}{\beta_2}\,\ln\{1-MRS_{21}\,\frac{\beta_2}{\beta_1}\,[1-\exp(\beta_1 x_1)]\} \tag{5-44}$$

A few isoclines and isoquants for a two-input, one-output Mitscherlich function are shown in Figure 5-11. These isoclines become nearly linear as they move away from the origin. Like the Cobb-Douglas function, the entire quadrant is in the area of substitution.

The Spillman production function has many of the same features as the Mitscherlich function (Hexem and Heady, 1978). The single-input, single-output form is

$$y = A(1-B^{x}) \tag{5-45}$$

where B = coefficient, $0 < B < 1$.

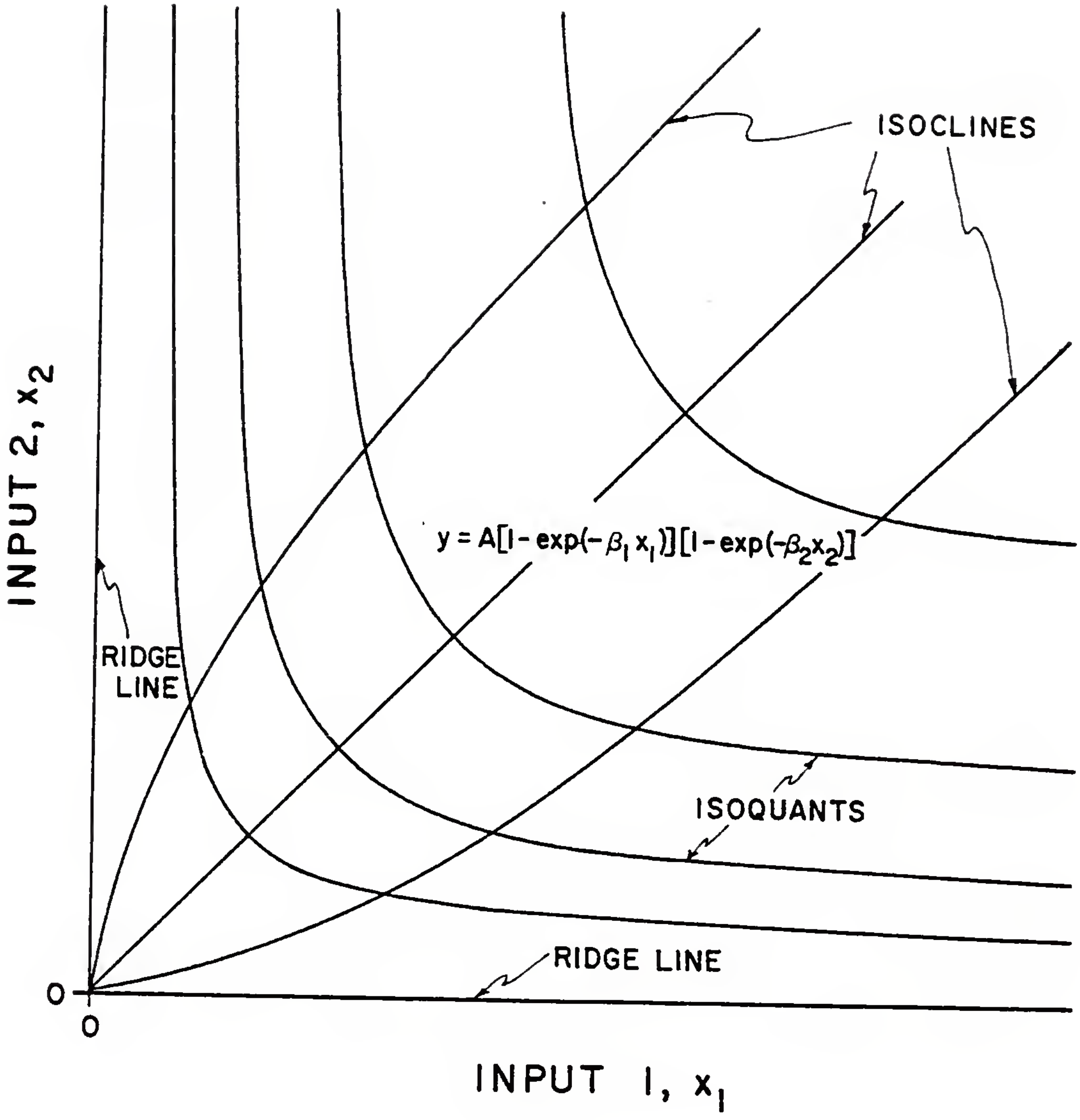


Figure 5-11. Two-Input, One-Output Mitscherlich Production Function with Isoclines and Ridge Lines.

The shape of this function is similar to the one depicted in Figure 5-10. The average and marginal products are

$$AP = \frac{A}{x}(1-B^{x}) \tag{5-46}$$

and

$$MP = -A[\ln(B)]\{\exp[x \ln(B)]\} \tag{5-47}$$

and each is similar to the average and marginal curves shown in Figure 5-10. The two-input, one-output Spillman function also performs much like the corresponding Mitscherlich function.

Polynomial (Quadratic) Equations

A polynomial equation is an expression involving a finite sum of terms, each of which has the form $a(x_1{}^{b}1)(x_2{}^{b}2) \ldots (x_m{}^{b}m)$ where a is a real coefficient and the exponents are positive integers. Polynomial equations are derived from a Taylor's series expansion about the point of interest. Given the simple function $y = g(x)$, a Taylor's series expansion in the vicinity of $x = x_o$ is

$$y = g(x_o) + g'(x_o)\frac{(x-x_o)}{1!} + g''(x_o)\frac{(x-x_o)^2}{2!}$$

$$+ g'''(x_o)\frac{(x-x_o)^3}{3!} + \ldots \tag{5-48}$$

An approximation of equation of 5-48 is created by ignoring the higher-ordered terms. If it is also recognized that the various differential terms are constants (i.e., $a_0 = g(x_o)$, $a_1 = g'(x_o)$, etc.) and that a minimum of a second-order polynomial (i.e., a quadratic equation) is needed for the possibility of decreasing marginal products, a useful approximation is given by

$$y \cong a_0 + a_1(x-x_o) + \frac{a_2}{2}(x-x_o)^2$$

$$= (a_0 - a_1a + a_2x_o{}^2) + (a_1 - a_2x_o)x + \frac{a_2}{2}x^2 \tag{5-49}$$

Grouping and renaming the constants yields the familiar quadratic form,

$$y = \beta_0 + \beta_1 x + \beta_2 x^2 \tag{5-50}$$

As a one-input, one-output production function, the quadratic form has the following average and marginal products:

$$AP = \frac{\beta_0}{x} + \beta_1 + \beta_2 x \tag{5-51}$$

and

$$MP = \beta_1 + 2\beta_2 x \tag{5-52}$$

If $\beta_0 = 0$, $\beta_1 > 0$ and $\beta_2 < 0$, then the quadratic equation will yield the production function shown in Figure 5-12. The average and marginal products are also displayed. The inspection of equations 5-50, 5-51 and 5-52 reveals that the quadratic form can represent production functions with other characteristics. The example in Figure 5-12 is only one possibility.

The two-input, one-output quadratic equation, displayed in Figure 5-13, is derived in a manner similar to equation 5-50 and is given by

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1^2 + \beta_4 x_2^2 + \beta_5 x_1 x_2 \tag{5-53}$$

The average and marginal products are given as

$$AP_{11} = \frac{\beta_0}{x_1} + \beta_1 + \beta_2\frac{x_2}{x_1} + \beta_3 x_1 + \beta_4\frac{x_2^2}{x_1} + \beta_5 x_2, \tag{5-54}$$

$$AP_{21} = \frac{\beta_0}{x_2} + \beta_1\frac{x_1}{x_2} + \beta_2 + \beta_3\frac{x_1^2}{x_2} + \beta_4 x_2 + \beta_5 \alpha_1, \tag{5-55}$$

$$MP_{11} = \beta_1 + 2\beta_3 x_1 + \beta_5 x_2, \tag{5-56}$$

and

$$MP_{21} = \beta_2 + 2\beta_4 x_2 + \beta_5 x_1 \tag{5-57}$$

Negative values for β_3 and β_4 are required if the law of diminishing marginal returns is to be displayed. In contrast to the previously discussed functional forms, the quadratic equation can represent

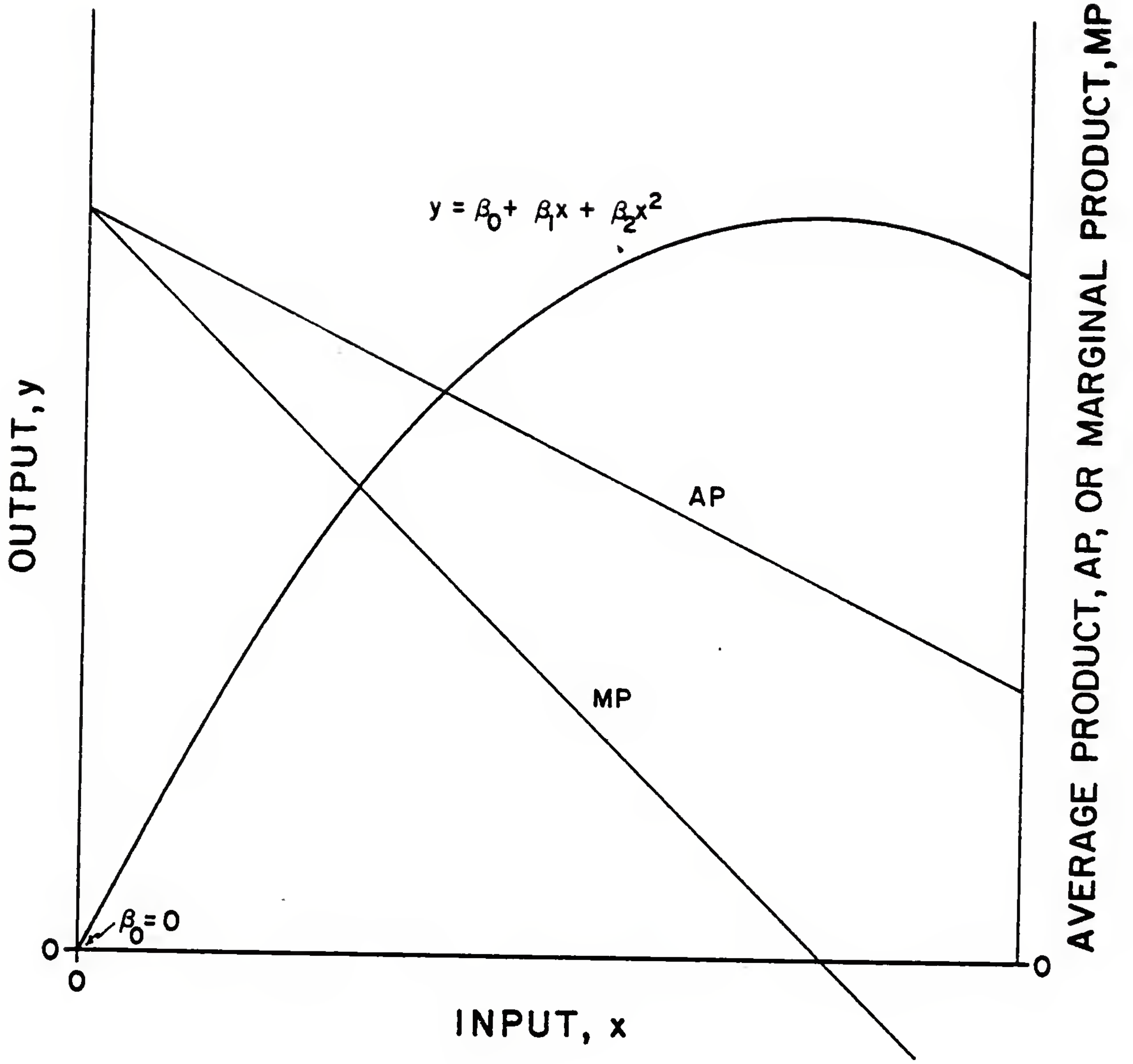


Figure 5-12. One-Input, One-Output Quadratic Production Function and the Average and Marginal Products.

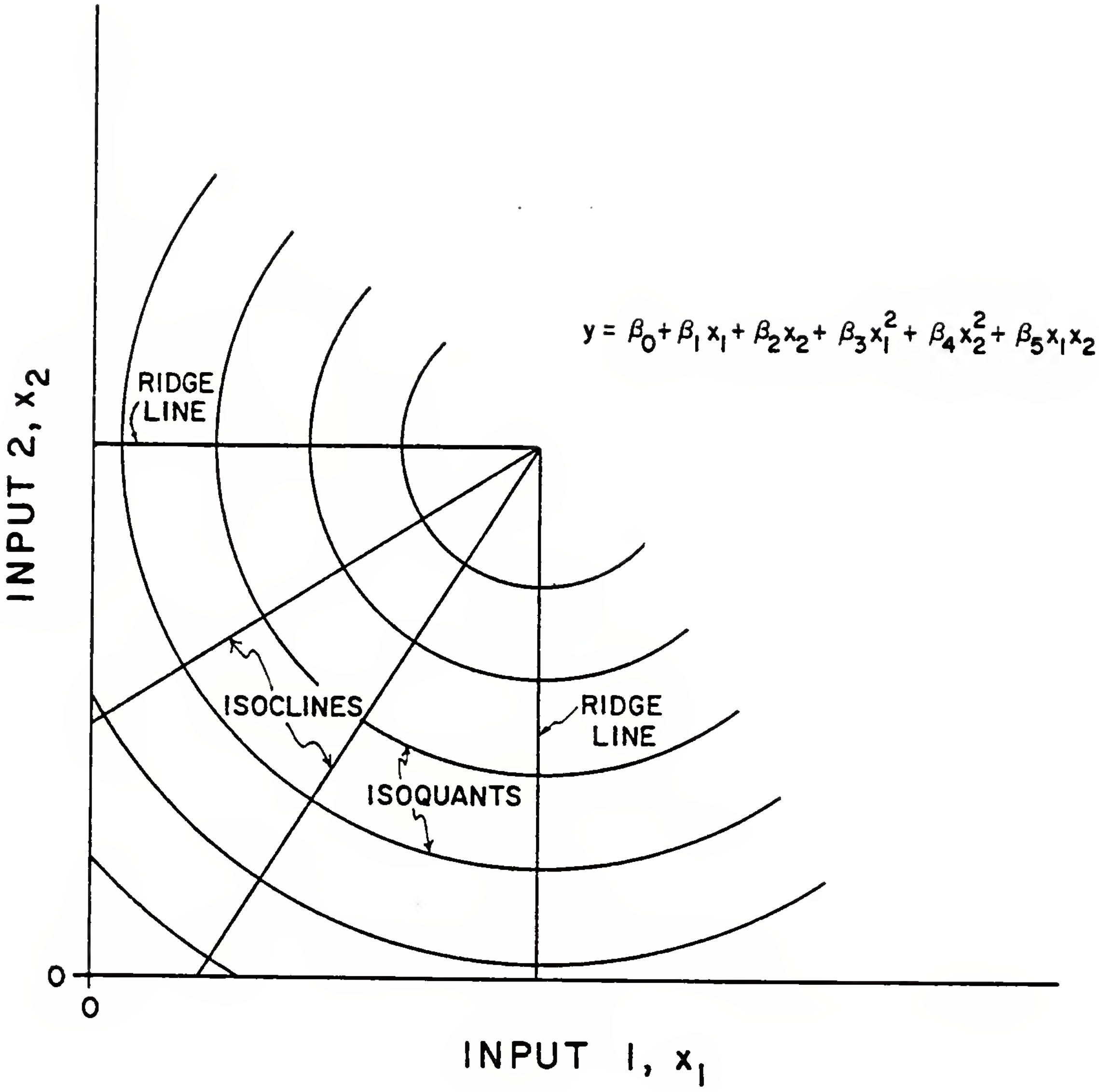

$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1^2 + \beta_4 x_2^2 + \beta_5 x_1 x_2$
RIDGE LINE
ISOCLINES
RIDGE LINE
ISOQUANTS
INPUT 2, x_2
INPUT 1, x_1
0
0

Figure 5-13. Two-Input, One-Output Quadratic Production Function with Isoclines and Ridge Lines.

isoquants demonstrating regions outside the area of substitution (see Figure 5-13). The marginal rate of substitution is calculated as

$$MRS_{21} = \frac{MP_{11}}{MP_{21}} = \frac{\beta_1 + 2\beta_3 x_1 + \beta_5 x_2}{\beta_2 + 2\beta_4 x_2 + \beta_5 x_1} \tag{5-58}$$

The isoclines are defined by

$$x_2 = \frac{\beta_2 MRS_{21} - \beta_1 + (\beta_5 MRS_{21} - 2\beta_3)x_1}{\beta_5 - 2\beta_4 MRS_{21}} \tag{5-59}$$

which is a linear function of x_1. However, unlike the previous functional forms, the isoclines do not emanate from the origin (except when $MRS_{21} = \frac{\beta_1}{\beta_2}$). When $MRS_{21} = 0$ or ∞, a ridge line is defined. Recall that ridge lines bound the area of substitution. One ridge line is formed when $MP_{11} = 0$, i.e.,

$$0 = \beta_1 + 2\beta_3 x_1 + \beta_5 x_2 \tag{5-60}$$

or

$$x_2 = -\frac{(\beta_1 + 2\beta_3 x_1)}{\beta_5} \tag{5-61}$$

The other is formed when $MP_{21} = 0$, i.e.,

$$0 = \beta_2 + 2\beta_4 x_2 + \beta_5 x_1 \tag{5-62}$$

or

$$x_2 = -\frac{(\beta_2 + \beta_5 x_1)}{2\beta_4} \tag{5-63}$$

A wide range of production surfaces can be represented by other polynomial equations. One-half and three-halves order "polynomial" functions are also commonly used (Hexem and Heady, 1978). However, polynomials of orders other than two can be more difficult to use in economic analysis.

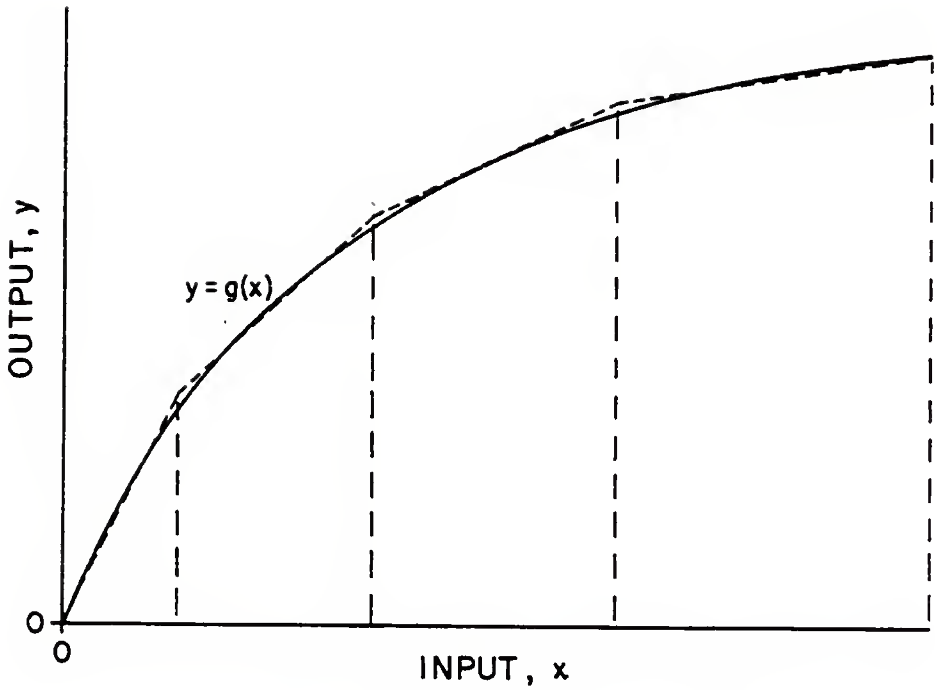
"Form-Free" Representation

Production processes are rarely slaves to the functional forms used to represent them. A function has a well defined set of properties;

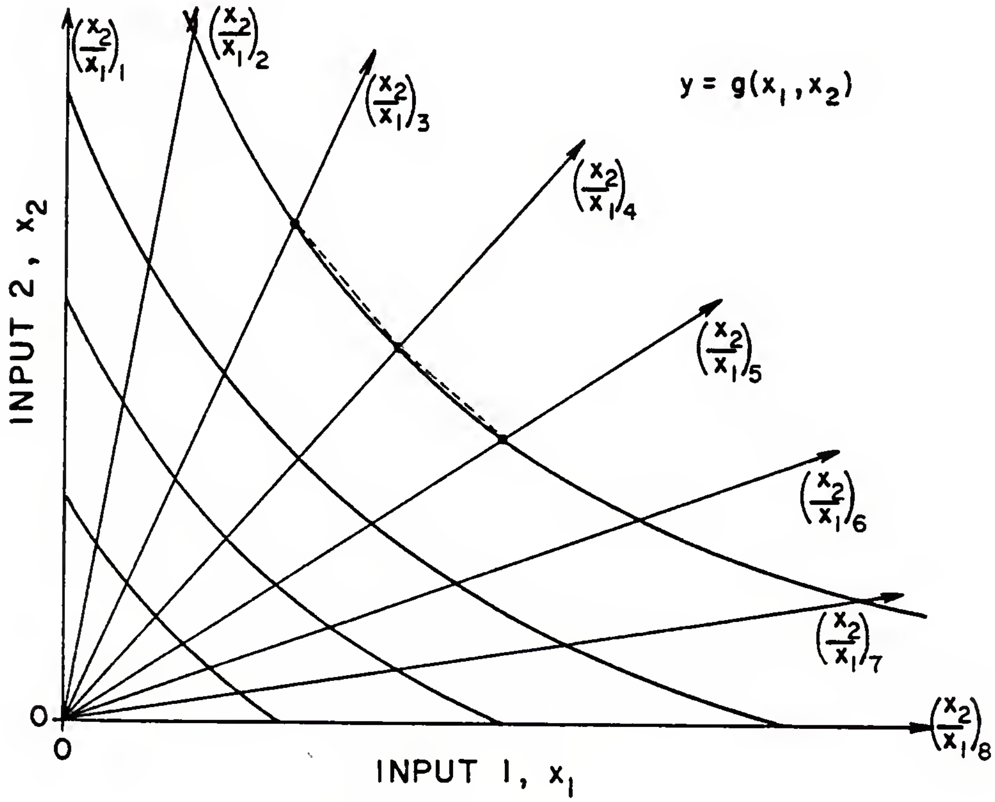
production processes rarely share the same set. However, a functional form can provide useful insights when it adequately describes a process. Fortunately, other approaches are available when a reasonable form is not available to fit the observed production surface.

The most obvious "form-free" representation is the graphical or visual fitting of curvilinear segments to the observed data. Naturally, this approach is limited to situations which can be visually displayed or, in other words, represented in one or two dimensions. Graphical representations can be difficult to construct, especially if the data base is small; but if the one constructing the curves is familiar with the expected production surface, this method can have a great advantage over "blind" functional forms.

A second approach is to approximate the output surface with series of linear segments (Hexem and Heady, 1978; Heaney, 1979) as shown in Figure 5-14(a) for the one-input, one-output case. A similar approach can be applied to the isoquants representing the two-input, one-output case. An alternative to linearizing each isoquant is to approximate the production function as a series of rays from the origin from which linear segments can be constructed between constant output values (Baumol, 1977; Heaney, 1979). Each ray represents a constant input usage ratio. The technique is shown in Figure 5-14(b). These methods are particularly amenable to economic analysis via linear programming. Obviously, the approximation improves as the discretization is improved. Production functions have also been approximated with grafted polynomials (Fuller, 1969).



(a) One-Input, One-Output Case


$\left(\frac{x_2}{x_1}\right)_1$
y
$\left(\frac{x_2}{x_1}\right)_2$
$\left(\frac{x_2}{x_1}\right)_3$
y = g(x₁, x₂)
$\left(\frac{x_2}{x_1}\right)_4$
$\left(\frac{x_2}{x_1}\right)_5$
$\left(\frac{x_2}{x_1}\right)_6$
$\left(\frac{x_2}{x_1}\right)_7$
$\left(\frac{x_2}{x_1}\right)_8$
INPUT 2, x₂
0
0
INPUT 1, x₁

(b) Two-Input, One-Output Case

Figure 5-14. Piecewise Linearization of a Production Function.

Estimating Production Functions

The most common technique used to fit production data to a linear function or "model" is the method of ordinary least squares. Essentially, the procedure estimates the parameters or coefficients of the linear model by minimizing the sum of squares of the deviations from the observed data by those predicted (Mendenhall, 1975). The procedure is well known and straightforward and, thus, will not be discussed in any detail.

Obviously, many production functions are not linear. However, if a function can be transformed to a linear form, the method of ordinary least squares can be used. The Cobb-Douglas and quadratic forms are transformable to linear equations. Taking the logarithms of both sides of the two-input, one-output Cobb-Douglas yields the following linear form:

$$\ln y = \ln A + \beta_1 \ln x_1 + \beta_2 \ln x_2 \qquad (5\text{-}64)$$

Equation 5-64 is linear in terms of the logarithms of y, x_1, and x_2. Thus, the logarithms of the output and input data are used in the least squares algorithm. The quadratic equation can be expressed in linear terms as

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3(x_1^2) + \beta_4(x_2^2) + \beta_5(x_1 x_2) \qquad (5\text{-}65)$$

where the terms in parentheses are treated as separate variables. Again, the raw data are transformed to these forms and the linear regression performed on the transformed data.

The basic concept of least squares can also be used to fit nonlinear equations (e.g., the Mitscherlich and Spillman functions) directly to a set of production or performance data. Several specific methods are available. Each method involves an interative technique requiring initial estimates of the model parameters (SAS Institute, Inc., 1979).

When comparing the merits of various models, it is important to note what indicates a "best" fit to the data. The coefficient of determination, r^2, is often used to determine how well a linear (or linearized) equation explains the variation of the dependent variable (e.g., y or ln y). It is calculated as (Freund and Littell, 1981)

$$r^2 = \frac{\sum_{k=1}^{N} (\hat{y}_k - \bar{y})^2}{\sum_{k=1}^{N} (y_k - \bar{y})^2}, \; 0 \leqq r^2 \leqq 1.0 \qquad (5\text{-}66)$$

where y_k = k^{th} observed value of y,

$\bar{y}$ = mean of observed values of y,

$\hat{y}_k$ = value generated by the regression line for the k^{th} observed value of the independent variable, x_k, and

N = total number of observed values.

The closer the value of r^2 is to 1.0, the better the regression line "explains" the variability of y. Unfortunately, the coefficient of determination is applicable only to linear (or linearized) functions and is sometimes difficult to interpret in comparisons of linear and linearized equations.

Several other methods are available to fit equations to production data (Arthanari and Dodge, 1981). Many of these methods are based on the analysis of the absolute deviations of the values of the dependent variable calculated by the model from the observed values. While these methods will not be used in this study to fit functional forms, the relative magnitude of the mean absolute deviation is a convenient tool for comparing the ability of the various forms to mimic the production data. This measure is especially useful because it is not limited to any particular class of equations and, thus, allows the direct comparison of linear (or linearized) and nonlinear equations. The mean absolute deviation and an observant eye are probably adequate indicators for the purposes of this study and those that might follow.

Production Functions for Storage/Release Systems

The purposes of this section are to establish the basic storage/release production process (i.e., inputs and outputs) and to review past efforts by the author, and others, in developing storage/release production functions for urban stormwater quality management. The past efforts provide a good base for the extensions and enhancements explored in Chapter 7.

Storage/Release Production Process

The major factors governing the performance of an urban stormwater storage/release system are shown in Figure 5-15. Most of these mechanisms or factors can be conceptualized or analyzed by the theories and methods outlined in earlier chapters. Obviously, these factors do not exhaust the myriad of complex actions and interactions that govern the behavior or storage/release systems and the topics discussed earlier cannot cover all contingencies. The purpose of this study is to develop a comprehensive, but usable set of analytical tools and, within this framework, little value is gained in pursuing more complex and detailed theoretical relationships. The theoretical scope presented in Chapter 2 and the analytical tools outlined in Chapters 2, 3, and 4 bound the level of technical sophistication used herein to develop storage/release production functions.

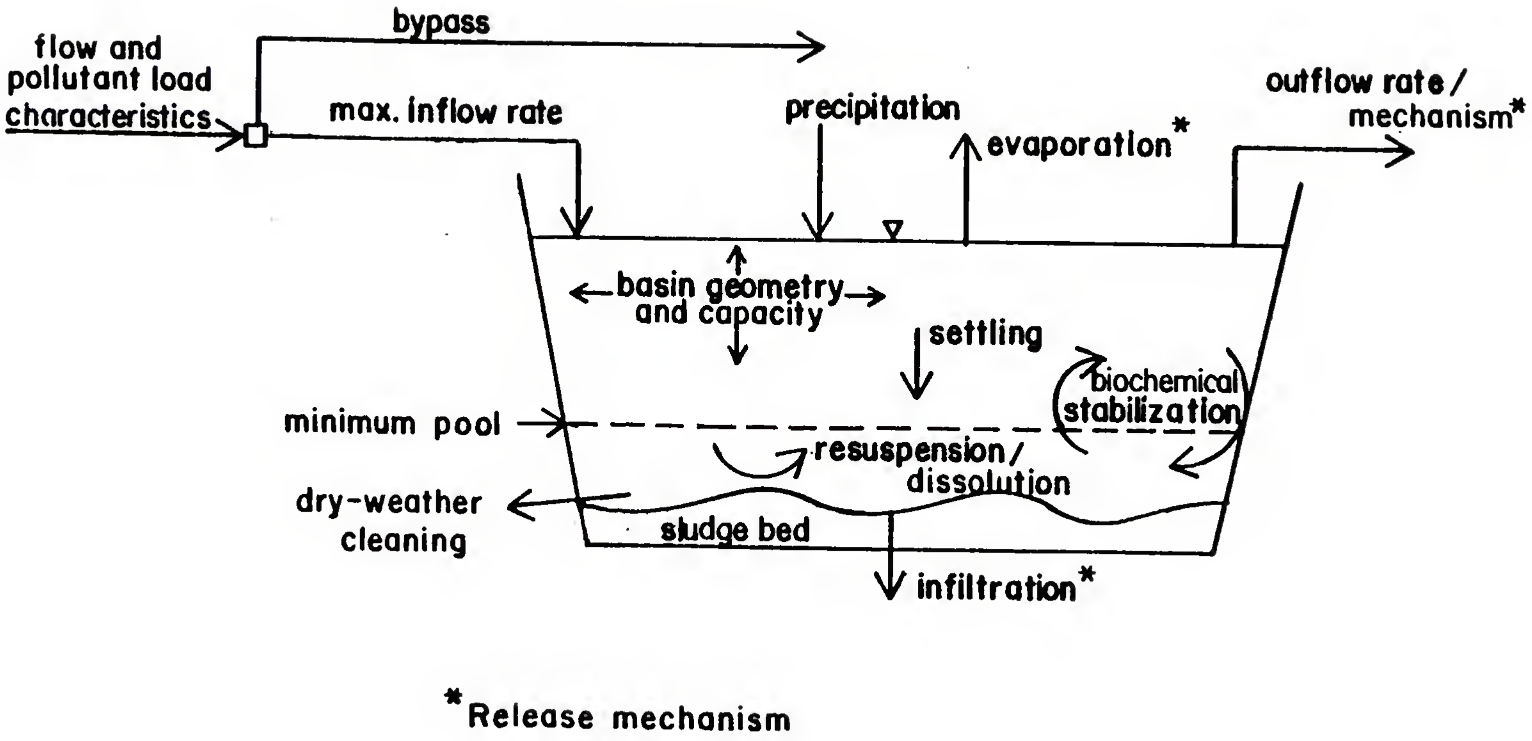

bypass
flow and pollutant load characteristics
max. inflow rate
precipitation
evaporation*
outflow rate / mechanism*
basin geometry and capacity
settling
biochemical stabilization
minimum pool
resuspension / dissolution
dry-weather cleaning
sludge bed
infiltration*
*Release mechanism

Figure 5-15. Factors Affecting the Quality Control Performance of Stormwater Storage/Release Systems (Nix et al., 1981).

Output. The output of a storage/release system used in urban stormwater quality management is some measure of its long-term ability to capture, redirect, remove, and equalize pollutant loads entering the system and/or improve conditions in receiving waters. This definition is imprecise and intended to be so. A precise definition is left to the individual application. However, most applications should fit the reliability classification scheme outlined in Chapter 2.

Inputs. The most important variable factors or inputs determining the performance of a storage/release system are the

1) release mechanism,
2) storage capacity,
3) mainstream capacity (in the off-line mode), and
4) the bypass mode.

Nearly all of the other factors are related to one or more of these primary factors or are fixed. The release mechanisms that can be defined as variable inputs include, among others, weirs, orifices, dry-weather drawdown, or fixed-rate pumping. Other release mechanisms such as evaporation and infiltration/percolation are normally, but not necessarily, fixed inputs. The release mechanism may also include, in the case of the off-line system, the ability of the mainstream capacity to accept released flows. The release mechanism can normally be expressed by the release rate (e.g., in ft^3/sec) at which the related facilities are designed. Basin geometry, especially depth, can be an important variable input but it is often fixed by sound engineering practices consistent with the intent of the system (e.g., long narrow basins for suspended solids settling.)

In storage/release systems where pollutant removal is a primary objective, the most important determinant of performance is detention time (Lager et al., 1977; Nix et al., 1981). Detention time determines the extent to which biological, chemical, and/or physical processes are able to remove or otherwise control pollutants. As demonstrated in Chapter 2, detention time is difficult to precisely analyze or calculate in storage/release systems receiving highly variable flows. However, the storage capacity and the release rate together determine the system's ability to capture and detain flows.

In some cases, pollutant removal is not the direct intent of the system. In these systems, the primary function is to divert the stored contents to an external treatment unit at a more equalized rate. The bypass mode is sometimes useful in these systems because it enables them to capture the early part of each event and, thereby, capture a disproportionately large portion of the load when the "first-flush" effect is prevalent.

For the purposes of specifying most production functions, the important input variables are the storage capacity and the design release rate. The other items listed above are of lesser importance. When developing production functions, it is the variable inputs that are of interest. The bypass mode is an "on" or "off" variable, and, thus, can be considered a fixed input in a particular application. If necessary, the bypass mode can be accounted for by producing two functions of the other variables. The mainstream capacity is only important in the off-line scheme and it can be combined with the release rate in certain cases (e.g., when all flows released from storage are sent back to the mainstream). If necessary, a third input can probably be most easily handled by creating a set of two input functions with each representing a different value of the third input.

Past Studies

Much effort has been expended in recent years to develop production functions for urban stormwater storage/release or storage/treatment systems. Most of the work was centered around constructing nomographic functions for preliminary planning. It is valuable to review these efforts for information regarding general forms, possible functional forms, and to provide a compendium of production functions for preliminary work.

Production functions based on STORM.

Through several reports and papers, personnel at the University of Florida, including the author, have developed and presented a set of urban stormwater storage/treatment production functions. These functions were developed primarily for use in a nationwide assessment of urban stormwater and combined sewer overflow control costs for each of the 248 urbanized areas in the United States (Heaney et al., 1977, 1979) and a similar study in Ontario, Canada (Sullivan et al., 1978). The estimates were based on storage/treatment production functions constructed for the developed areas of five major cities (Atlanta, Denver, Minneapolis, San Francisco, and Washington, D.C.) in the U.S. study and four cities (St. Catharines, Burlington, Kingston, and Sault Ste. Marie) in the Ontario study. These functions were extrapolated to other cities in the region represented by the reference city. This methodology was also generalized for a screening and planning procedure known as the Storm Water Management Model (SWMM): Level I (Heaney et al., 1976, 1978).

The Corps of Engineers STORM model (Hydrologic Engineering Center, 1977) was used to characterize the runoff from developed areas in each of the study cities and evaluate various storage/treatment combinations. The basic program operation was discussed in Chapter 2. The assumed storage/treatment system is shown in Figure 2-14. This system is identical to the off-line system described in Figure 2-1(b). The "treatment" unit provides no pollutant removal capability. The model simply tracks the portion of the runoff flow and pollutant load entering the "treatment" unit (either directly or by releases from storage). Recall that the off-line configuration directs flows to storage after the mainstream capacity, or in this case, the maximum "treatment" rate, is exceeded and releases the stored volume back to the mainstream if the capacity is available. In the generalized off-line configuration the rate at which the releases are made is not specified, but it must not cause the mainstream capacity to be exceeded. STORM assumes that the release rate is equal to the difference between the incoming stormwater flow and the maximum treatment rate. STORM also assumes that the bypass mode is in place; that is, flows directed to storage are bypassed out of the system once the storage unit is full.

STORM was run for a matrix of storage capacity and maximum treatment rate combinations for each reference city. A year of hourly rainfall data was used to drive the model. The specific year was selected on the basis of a comparison of each rainfall year against a twenty-five year record. The year that was best able to reproduce the long-term record, in terms of a frequency distribution, was selected to drive the model. This approach was taken to reduce computer costs.

The STORM runs generated a corresponding array of values for the percent runoff or volume control, y_V, and the percent pollutant (BOD) load control, y_L, for the various storage/treatment combinations. "Control" is defined as the percent of total runoff volume or pollutant load entering "treatment." From the plotted results, isoquants connecting the storage/treatment combinations yielding particular pollutant load or runoff volume control levels were constructed.

It was determined that each load and volume control isoquant was closely described by the following transcendental equation (Hasan, 1976; Heaney et al., 1977):

$$\Omega = \Omega_1 + (\Omega_2 - \Omega_1) \exp(-\beta_0 V_B) \qquad (5\text{-}67)$$

where Ω = treatment/release rate, inches/hour,

Ω_1 = value of Ω at which isoquant becomes parallel to the ordinate, inches/hour,

Ω_2 = value of Ω at which the isoquant intersects the abscissa, inches/hour,

V_B = storage basin capacity, inches, and

β_0 = coefficient, inches^{-1}.

Equation 5-67 is displayed in Figure 5-16. The values of Ω and V_B are normalized over the catchment area. The value of Ω_1 can be estimated by recognizing that it represents the maximum treatment or release rate at which no further storage capacity is needed to maintain a constant control level. In other words, at the point where the isoquant becomes parallel to the ordinate, the storage capacity is just large enough to route the controlled volume through the "treatment" unit at a uniform, continuous rate. Moving up the isoquant (i.e., increasing the storage capacity) provides no further control (as defined here) because the

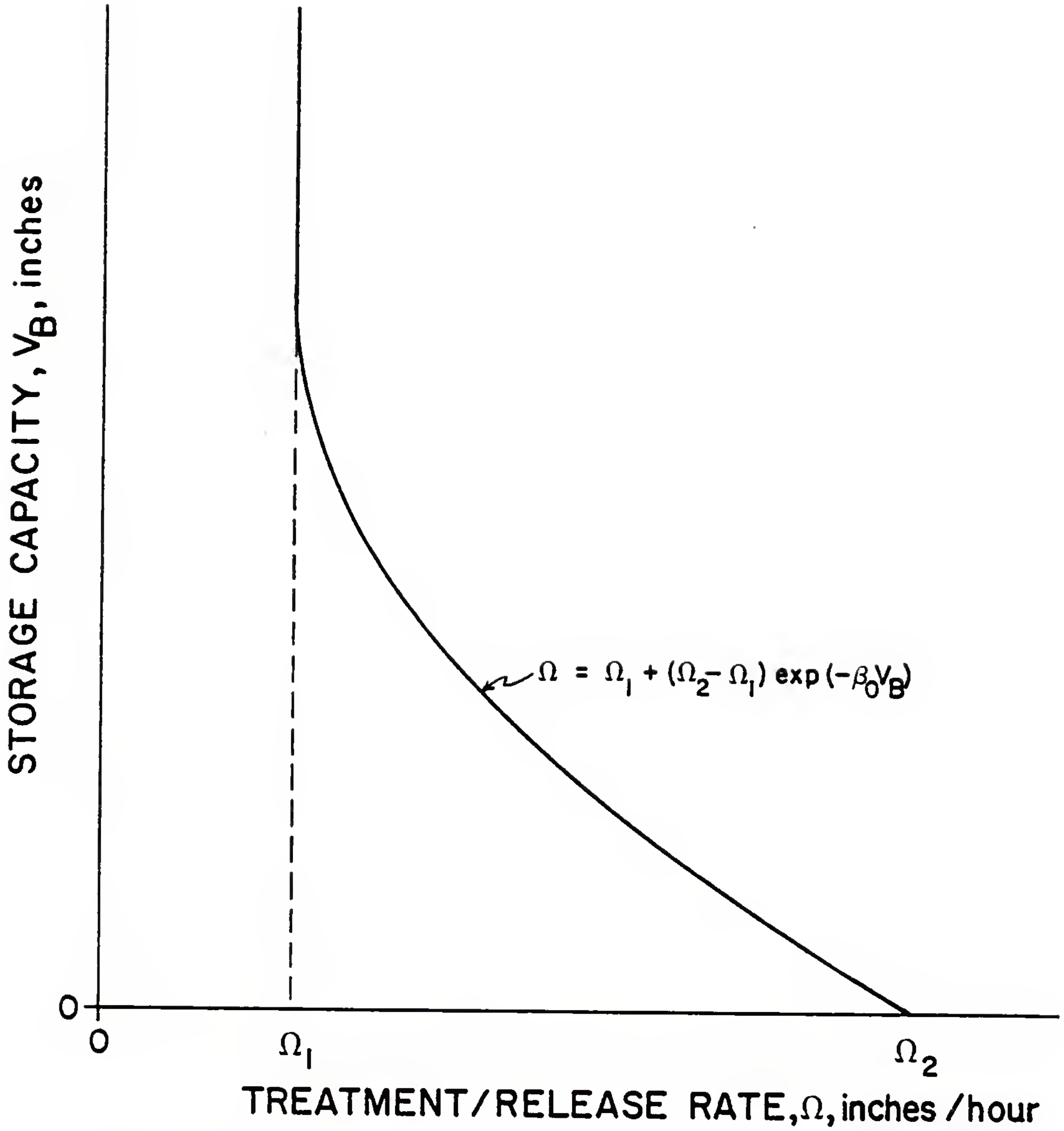


Figure 5-16. Storage/Treatment (or Release) Isoquant Produced by a Transcendental Production Function (Heaney et al., 1976).

treatment rate cannot be exceeded. The value of Ω_1 is calculated as

$$\Omega_1 = \frac{AR}{8760}\left(\frac{y_V}{100}\right) = \beta_1 y_V \tag{5-68}$$

where AR = annual runoff, inches/year,

y_V = percent runoff volume control,

8760 = hours in a year, and

β_1 = coefficient, inches/(hour•%).

If the pollutant concentrations in stormwater are not highly variable, Ω_1 for a pollutant load control isoquant is also approximated by equation 5-68 with the substitution of y_L for y_V.

The term $(\Omega_2 - \Omega_1)$ in equation 5-67 was found to be closely related to y_V (or y_L) by

$$\Omega_2 - \Omega_1 = \beta_2 \exp(\beta_3 y_V) \tag{5-69}$$

where β_2 = coefficient, inches/hour, and

β_3 = coefficient, $(\%)^{-1}$.

The coefficient β_0 was also found to be a function of the isoquant value, y_V (or y_L):

$$\beta_0 = \beta_4 \exp(-\beta_5 y_V) \tag{5-70}$$

where β_4 = coeffient, inches^{-1}, and

β_5 = coefficient, $(\%)^{-1}$.

Substituting equations 5-68, 5-69 and 5-70 into equation 5-67 yields the complete production function:

$$\Omega = \beta_1 y_V + \beta_2 \exp\{\beta_3 y_V - [\beta_4 \exp(-\beta_5 y_V) V_B]\} \tag{5-71}$$

where y_V may be replaced by y_L for the pollutant control production function. Equation 5-71 proved to be an excellent representation of the STORM results. However, it is an unwieldy function and it is certainly not one of the more commonly used functional forms. Fitting this relationship to the STORM results required independent ordinary least squares analyses to be performed on equations 5-69 and 5-70. The runoff volume control results for the U.S. reference cities are shown in Table 5-1. The pollutant load control results (in terms of BOD control) are shown in Table 5-2.

If pollutant concentrations are uniform, the runoff volume control production functions would also represent the pollutant load control production surface. However, the results of Table 5-2 were generated under the first-flush condition assumed by STORM, i.e., a larger portion of the pollutant load is flushed from the land surface during the first part of a storm. The STORM storage/treatment system operates in the bypass mode and, thus, is more effective in capturing the "first flush" of pollutants. Therefore, at a given storage/treatment combination, the first-flush isoquants will produce a greater control level. The first-flush BOD control isoquants for Minneapolis are reproduced in Figure 5-17. The set of graphically determined isoquants are shown with the isoquants produced by equation 5-71. In general, equation 5-71 faithfully reproduces the STORM results.

The STORM results for pollutant or BOD control were later fit to a Cobb-Douglas function of the following form (Nix et al., 1977):

$$y_L = A\Omega^{\beta_1} V_B^{\beta_2} \tag{5-72}$$

where A, β_1, β_2 = coefficients.

The transcendental function given by equation 5-71 was well suited to the STORM results but it is difficult to use in economic analyses. The

Table 5-1. Coefficients for the Transcendental Equation Representing the Percent Volume Control Production Function Produced by STORM (Heaney et al., 1977)

City	Annual Runoff, inches/year	Developed Area Population Density, persons/acre	β_1 inches/(hour·%)	β_2 inches/hour	β_3 $(\%)^{-1}$	β_4 inches^{-1}	β_5 $(\%)^{-1}$
San Francisco	9.37	9.96	0.0000107	0.0021466	0.0377090	108.5330	0.0335173
Denver	5.59	9.11	0.0000064	0.0012194	0.0397305	119.8106	0.0279204
Minneapolis	10.50	7.92	0.0000120	0.0012909	0.0487945	191.2782	0.0322136
Atlanta	16.18	8.24	0.0000185	0.0022832	0.0486532	112.2002	0.0348027
Washington, D.C.	17.22	20.02	0.0000197	0.0020464	0.0567454	117.5456	0.0398007

Table 5-2. Coefficients for the Transcendental Equation Representing the Percent Pollutant (BOD) Control Production Function Produced by STORM (Heaney et al., 1977)

City	Annual Runoff, inches/year	Developed Area Population Density, persons/acre	β_1 inches/(hour·%)	β_2 inches/hour	β_3 $(\%)^{-1}$	β_4 inches^{-1}	β_5 $(\%)^{-1}$
San Francisco	9.37	9.96	0.0000107	0.0021654	0.0388910	211.2763	0.0320226
Denver	5.59	9.11	0.0000064	0.0013631	0.0439822	184.9639	0.0279177
Minneapolis	10.50	7.92	0.0000120	0.0013656	0.0481981	241.6141	0.0301648
Atlanta	16.18	8.24	0.0000185	0.0025864	0.0468175	190.2240	0.0312484
Washington, D.C.	17.22	20.02	0.0000197	0.0018959	0.0487876	228.8434	0.0339322

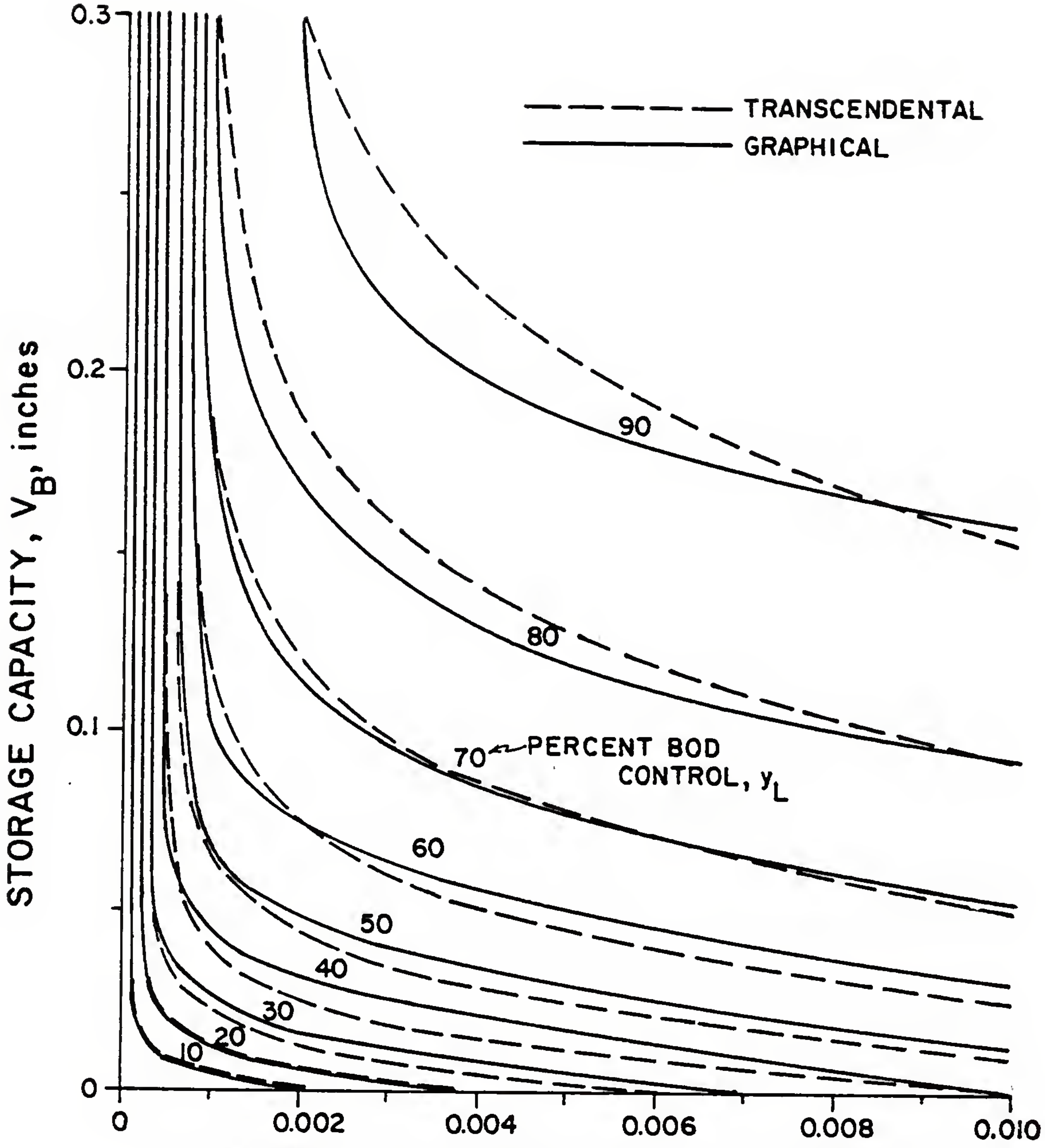


Figure 5-17. Storage/Treatment (or Release) Production Function for BOD Control Produced by STORM, Graphical and Transcendental Representations, Minneapolis, Minnesota (Heaney et al., 1976, 1977).

Cobb-Douglas function, as will be shown in Chapter 6, is remarkably easy to manipulate in economic optimization problems. A major limitation, however, is the function's inability to allow the isoquants to intersect axes. Nevertheless, an ordinary least squares analysis produced the results given in Table 5-3. The results for Minneapolis are plotted in Figure 5-18 along with the graphically derived isoquants. The Cobb-Douglas function does not appear to represent the STORM results as well as the transcendental equation. The importance of this discrepancy is a function of the level of analysis for which the production function is intended. The simplicity of the Cobb-Douglas function might be advantageous where a high level of accuracy is not required.

Statistically-derived production functions. DiToro and Small (1979) and Hydroscience, Inc. (1979) have shown that the methods used to analyze the storage/interceptor configuration, discussed in Chapter 3, can be used to develop a set of isoquants similar to those produced at the University of Florida. Their results for Denver, Colorado are shown in Figure 5-19 along with the superimposed STORM results (from Heaney et al., 1977). The two methods appear to produce reasonably similar results. The "interceptor" in the Hydroscience method is analogous to "treatment" in the STORM scenario. However, the release rate from storage, Ω, is assumed to be equal to the interceptor wet-weather capacity, Q_I, at all times in the Hydroscience method. STORM, on the other hand, assumes the release rate from storage varies between zero and the maximum "treatment" capacity depending on the flow entering the system. Both methods operate in the bypass mode.

Table 5-3. Coefficients for the Cobb-Douglas Equation Representing the Percent Pollutant (BOD) Control Production Function Produced by STORM (Nix et al., 1977)

City	Annual Runoff,[a] inches/year	Developed Area Population Density,[a] persons/acre	A	β_1	β_2
San Francisco	9.37	9.96	489	0.198	0.410
Denver	5.59	9.11	584	0.252	0.334
Minneapolis	10.50	7.92	464	0.213	0.333
Atlanta	16.18	8.24	660	0.246	0.490
Washington, D.C.	17.22	20.02	570	0.255	0.394

a. Source: Heaney et al., 1977.

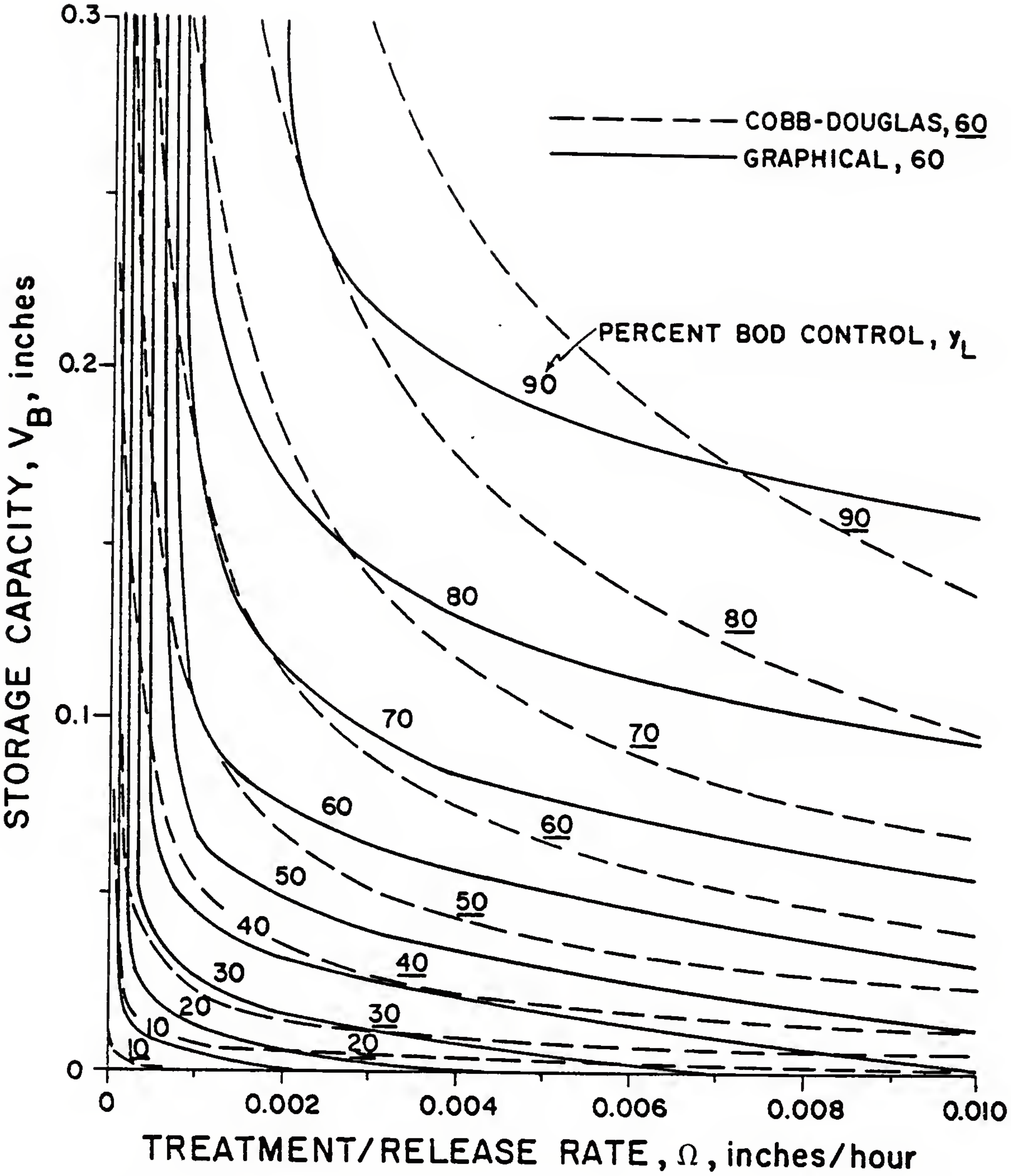


Figure 5-18. Storage/Treatment (or Release) Production Function for BOD Control Produced by STORM, Graphical and Cobb-Douglas Representations, Minneapolis, Minnesota (Nix et al., 1977).

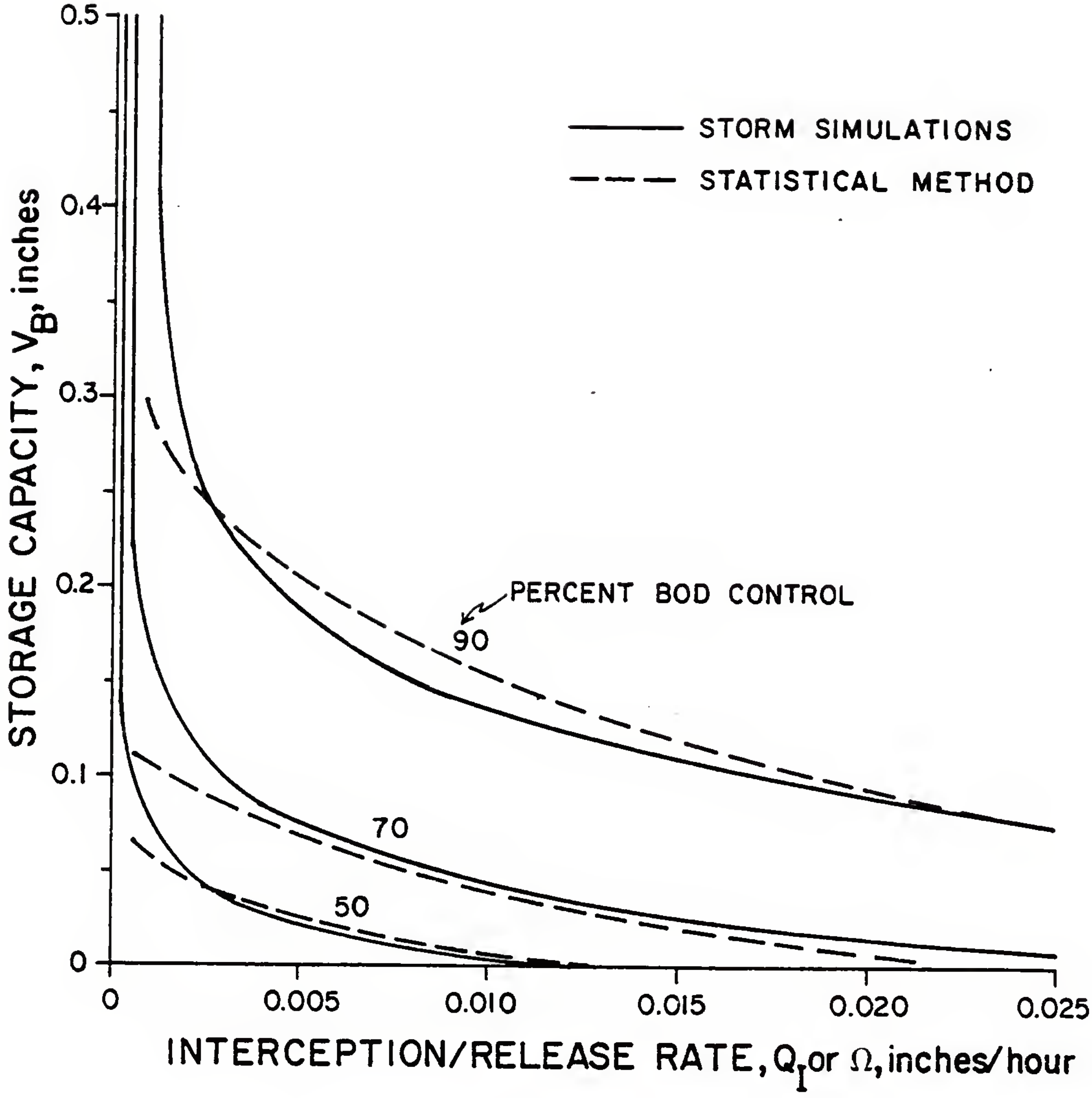


Figure 5-19. Off-Line Storage/Release Production Function for BOD Control Produced by Hydroscience Statistical Method, Compared with STORM-Generated Production Function, Denver, Colorado (DiToro and Small, 1979; Hydroscience, Inc., 1979).

The above application was for an off-line configuration. The method can also be used to develop production functions for in-line systems (E. D. Driscoll and Assoc., 1981; Goforth, 1981). The storage curves shown in Figures 3-7 and 3-8 can be more conveniently arranged as shown in Figure 5-20. The fraction of the total runoff volume or pollutant load (when concentrations are assumed to be independent of flows and durations) bypassing the storage unit, f_V, for any combination of storage capacity, V_B, and the release rate, Ω, is determined by the following steps:

1) Enter the abscissa of the right-hand set of curves at the desired value of V_B (point A) and proceed vertically to the curve representing the desired value of Ω (point B). (Each axis is normalized by the mean runoff event volume, V_R, to enhance their applicability. The parameter Δ_R is the mean time between runoff events.)
2) Move horizontally from point B through the common axis (representing the mean effective storage capacity, V_E) to the curve in the left-hand figure representing the appropriate coefficient of variation for runoff event volumes, ν_{vR} (point C).
3) From point C, move down to the corresponding value of f_V (point D).

Repeating these steps for several combinations of V_B and Ω, with ν_{vR} = 1.73, produced the production function shown in Figure 5-21. Each isoquant represents a different value of $1-f_V$ (the control level).

An analysis performed by E. D. Driscoll and Assoc. (1981) took isoquants of the type shown in Figure 5-21 one step further. By linking

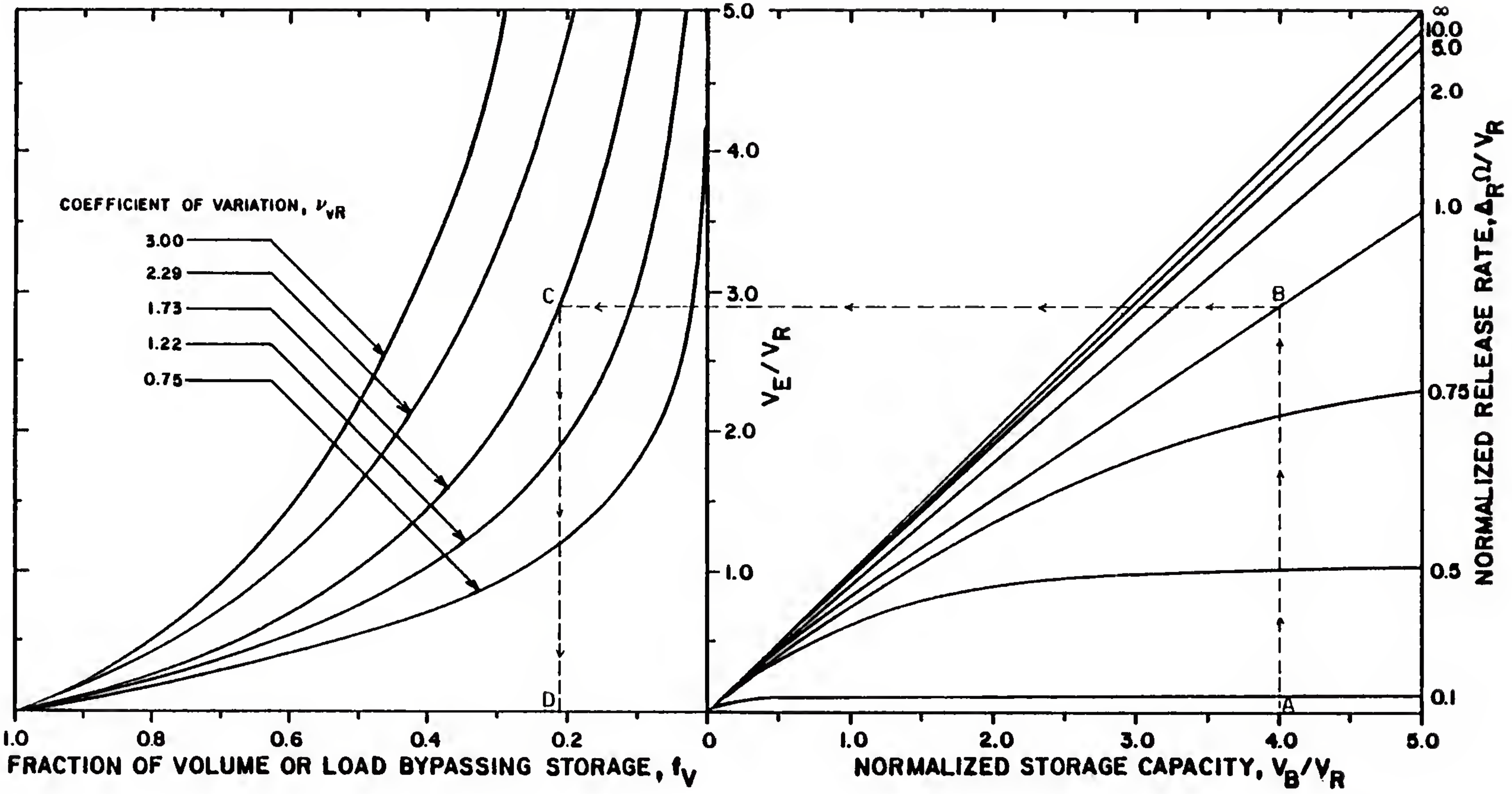


Figure 5-20. Derivation of Production Functions for In-Line Storage/Release Systems from Hydroscience Statistical Method (Goforth, 1981).

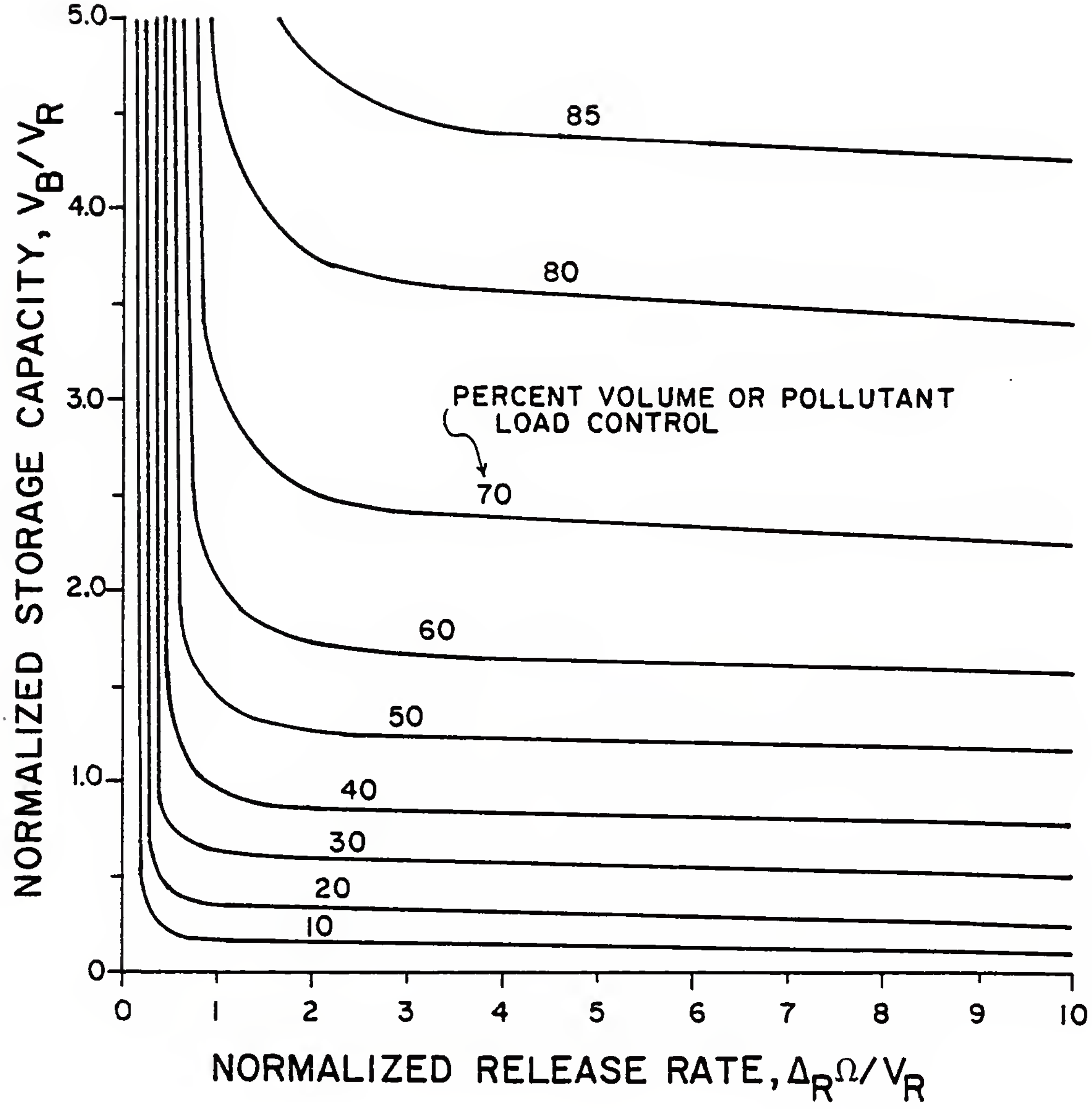


Figure 5-21. In-Line Storage/Release Production Function for Pollutant Load or Volume Control Produced by Hydroscience Statistical Method, $\nu_{vR} = 1.73$.

the system performance to the long-term receiving water response (in terms of the mean and variance of pollutant concentration), a connection between the resource usage and meaningful physical benefits could be forged. The analysis was performed on the control of combined sewer discharges from Kingston, New York. These flows contain high fecal coliform concentrations and when they are discharged to the nearby Hudson River, a possible health hazard is created at two recreational beaches. The effect of the storage/release system on the fecal coliform concentration at these beaches was translated to the number of additional days the beaches could be kept open during the summer season because fecal coliform standards were not violated. The results are shown in Figure 5-22. This type of production function provides more useful information than Figure 5-21 because it expresses performance in terms of obvious physical benefits. Figure 5-22 is much more useful to the decision-making process.

Howard et al. (1979) also used their simplified statistical method (described in Chapter 2) in an attempt to duplicate the storage/treatment isoquants developed by the University of Florida with STORM (Heaney et al., 1977). The results of Howard's method were compared in tabular form to the STORM results. The purpose of the comparison was to demonstrate that a simple statistical method could be used to reasonably mirror the results of a computer simulator. Howard et al. (1979) also compared results obtained by the Hydroscience method (Hydroscience, Inc., 1979) to those produced by STORM and Howard's method. Overall, both statistical methods reproduced the STORM results for the five U.S. reference cities relatively well. However, the Hydroscience method was able to provide more "accurate" results.

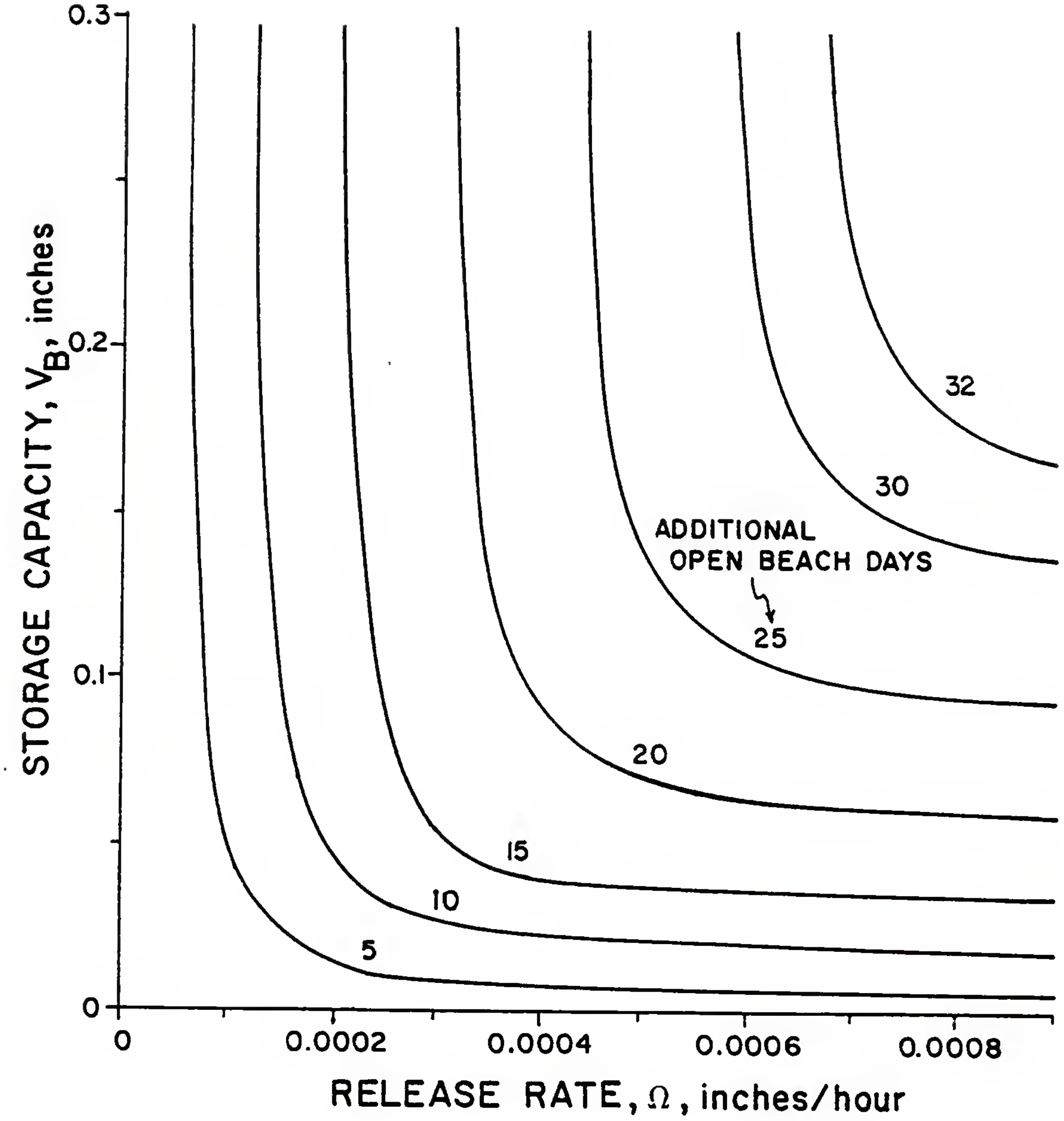


Figure 5-22. In-Line Storage/Release Production Function for Additional Open Beach Days Produced by Hydroscience Statistical Method, Kingston, New York (E.D. Driscoll and Assoc., 1981).

The more interesting result produced by Howard's method (Charles Howard and Assoc., 1979; Howard et al., 1979) was a pollutant removal production function generated with the modification for pollutant removal in storage discussed in Chapter 2 (equation 2-71). The production function for pollutant removal from the developed areas of Minneapolis, Minnesota is shown in Figure 5-23. These isoquants show a markedly different feature from the pollutant control isoquants produced by the other studies; i.e., they show a large range outside of the area of substitution. This occurs because as the release rate or "treatment" capacity is increased, a point is reached where the loss of detention ability overwhelms the benefit of providing a larger effective storage capacity at the beginning of each runoff event. In other words, the marginal product of the release rate is negative. This effect is not evident in the other studies because the effect of detention on pollutant removal is not addressed.

Production functions from the SWMM Storage/Treatment Block. The SWMM Storage/Treatment Block is a relatively sophisticated model capable of simulating the two basic storage/release configurations (among other storage/treatment groups). The model is fully described in Chapter 4.

Early in its development, the S/T Block was tested by modeling a hypothetical storage basin operating in Atlanta, Georgia. The basin received a year's worth of hourly runoff flows and pollutant loads generated by STORM. The STORM model was driven with the local hourly rainfall record for 1969. The two models were linked through a simple interfacing program. Releases from the basin were acomplished by fixed-

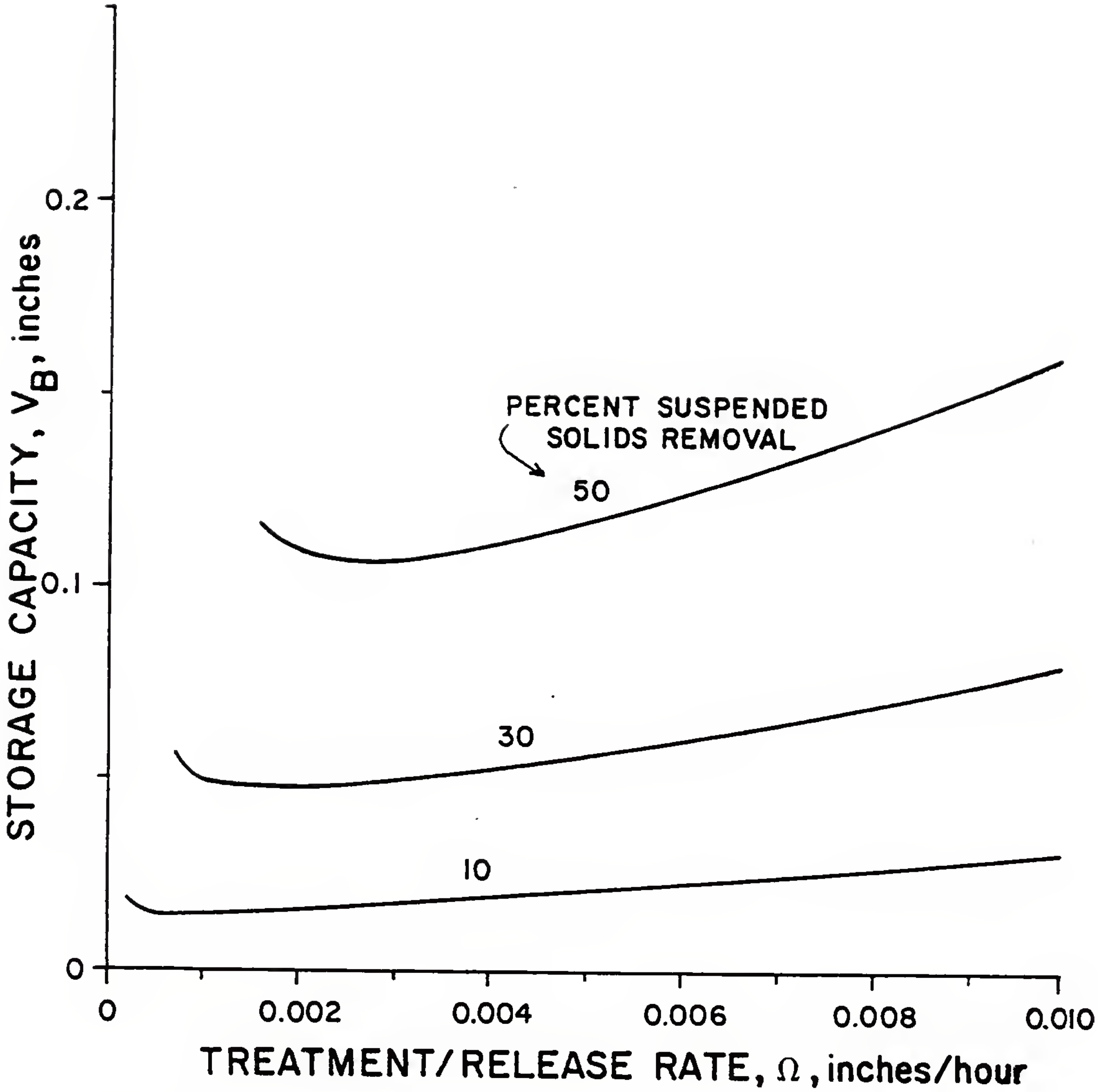


Figure 5-23. In-Line Storage/Treatment (or Release) Production Function for Suspended Solids Control Produced by Howard's Statistical Method, Minneapolis, Minnesota (Charles Howard and Assoc., 1979; Howard et al., 1979).

rate pumping. The basin was set up as a plug-flow reactor and a simple pollutant removal equation similar to equation 4-13 was used to simulate BOD removal. Running the model for several basin capacity/pumping rate combinations produced the production function shown in Figure 5-24. As with Howard's method, these isoquants reveal a region outside the area of substitution. Goforth (1981) produced a similar set of isoquants for a hypothetical basin using a "draft" version of the Storage/Treatment Block.

Summary

Production functions are useful tools in summarizing the technological relationships evident in urban stormwater storage/release systems. These functions can usually be expressed as two-input, one-output relationships in which the inputs are storage capacity and the design release rate. The output is not precisely defined but is related to the specific application. Possible examples include the long-term average pollution control level, increased recreational opportunities, or the ability of the system to capture events of a particular size.

Production functions can be mathematically expressed in terms of specific functional forms. However, a reasonable form which adequately "fits" the data may not be available. "Form-free" representations (by graphical or visual curve fitting) can be applied in these cases. A review of past efforts provides examples of both approaches and serves as a good base from which to explore extensions and enhancements of the production function in the context of urban stormwater quality management. With the production data expressed in either fashion, the technological relationships are summarized and the economic analysis can proceed.

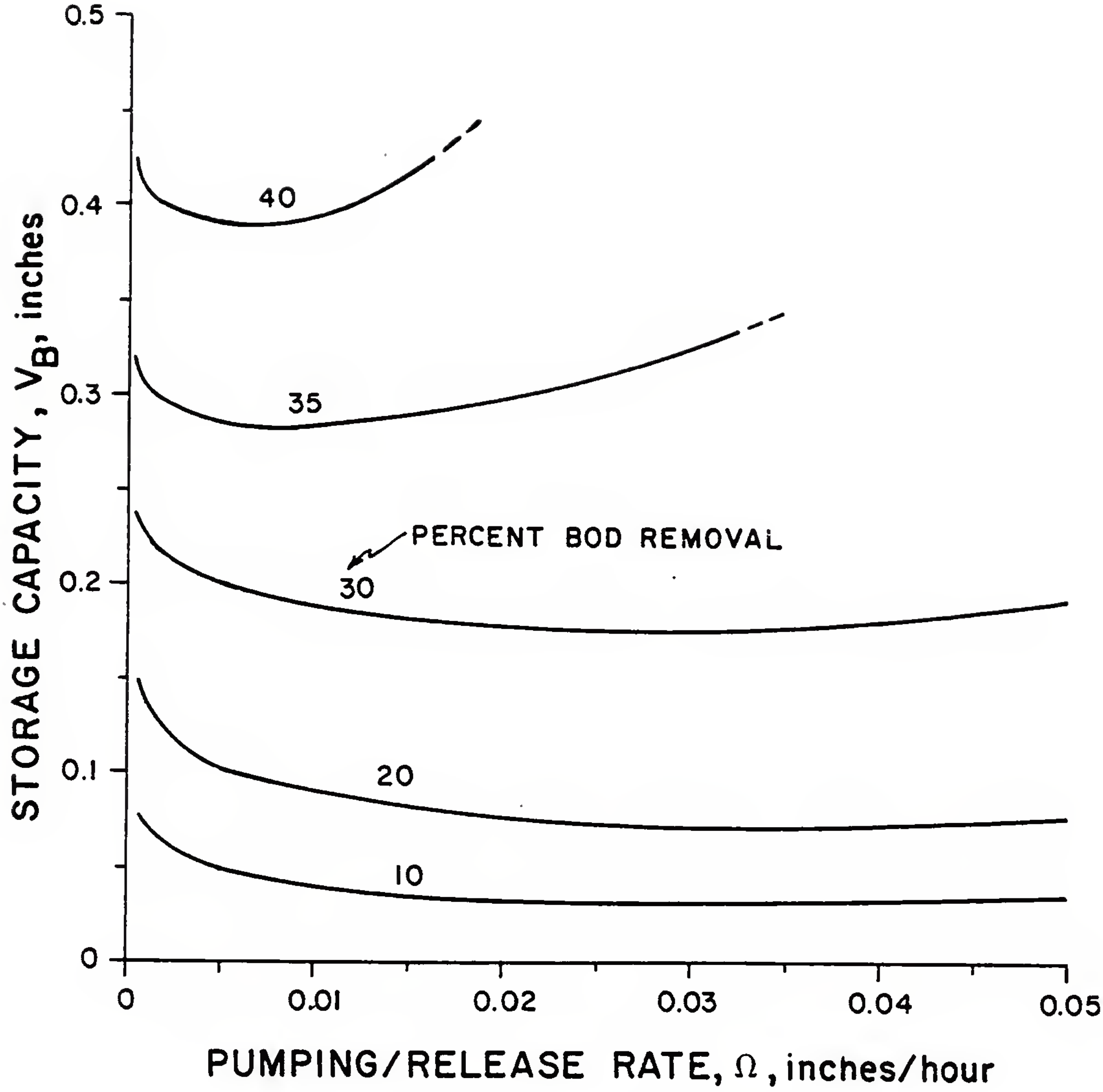


Figure 5-24. In-Line Storage/Release Production Function for BOD Control Produced by SWMM Storage/Treatment Block, Atlanta, Georgia.

CHAPTER 6

STORAGE/RELEASE SYSTEM OPTIMIZATION

Introduction

The previous chapters dealt with the analysis of storage/release systems in the context of urban stormwater quality management and methods to characterize their performance (defined in terms of the specific application) through production functions. The inevitable question facing the analyst or engineer is how to size and design the system in the manner most beneficial to society. In other words, what is the optimal design and operating strategy?

The production function describes the efficient technological possibilities, i.e., the most efficient use of resource or input combinations to obtain various levels of output or outputs. With the storage/release system production function and the costs associated with each input, the optimization problem becomes an exercise in microeconomic theory and welfare economics (James and Lee, 1971). The solution to the problem is a function of the objective to be achieved and the technical (production) and budgetary constraints. Three common forms of this problem are

1) to maximize the net benefits (total benefits less total costs) to society, constrained by the performance of the storage/release system;

2) to minimize the costs to society, constrained by pre-established performance levels; or

3) to maximize the performance level constrained by a budget ceiling.

The first formulation is the all-encompassing problem statement and is, in one form or another, the objective of any environmental management project. However, because of the difficulty in quantifying benefits in economic terms, one of the other two formulations is used. In the second problem statement, the bane of benefit assessment is externalized and placed in other hands (e.g., a regulatory agency). For example, a required pollution control level is often linked to meeting a specific receiving water standard. The problem of benefit assessment does not disappear--it should manifest itself in the establishment of the standard. The fact that receiving water standards are often criticized for their vagueness, lack of scientific substantiation, and poor accountability to social and economic well-being points to the difficulty (and/or lack of bureaucratic sensitivity). The third problem statement completely divorces itself from benefit assessment. This is an unwise and generally unacceptable approach as any economic decision should address potential benefits in some manner.

Design Optimization Problem

Assume a situation in which a storage/release system is being used to provide stormwater quality enhancement. Also assume that the design release rate and the storage basin capacity are the predominant inputs to the production process (as discussed in Chapter 5). If costs are to be minimized for a specified performance or production level, the optimization problem can be stated as follows:

$$\text{minimize } Z = h(\Omega, V_B) \tag{6-1}$$

$$\text{such that } y = g(\Omega, V_B) \tag{6-2}$$

$$y \geqq y^o \tag{6-3}$$

$$\Omega, V_B, y \geqq 0 \tag{6-4}$$

where z = total cost, dollars,

y = system performance,

Ω = release rate, L^3T^{-1},

V_B = storage basin capacity, L^3,

$h(\Omega, V_B)$ = cost function for inputs Ω and V_B,

$g(\Omega, V_B)$ = production function for inputs Ω and V_B, and

y^o = specified performance level.

Equation 6-1 is commonly known as the objective function and equations 6-2 through 6-4 are referred to as the constraint set. The entire set is called a constrained optimization problem.

The problem outlined above states that the cost is to be minimized (equation 6-1) subject to the storage/release system technology (equation 6-2). Equation 6-3 requires the system performance to meet or exceed the specified level. The specified level is often set by a regulatory agency and represents a social choice mechanism through which the desired level of performance is selected. This circumvents the need for the assessment of benefits at the "on-site" problem level. Ideally, the regulatory agency or other "decision maker" will have weighed the benefits against the costs when selecting the required performance level. However, most are based on other considerations. Much of the criticism of receiving water (and air) quality standards has pointed to the lack of evidence proving that the standards have increased social welfare at all much less maximized it. This controversy will remain. Benefit assessment is a risky, subjective, and presumptuous venture. Any assessment method is imprecise and the results are vulnerable--each individual has his own measure of value. Although several methods have been used to assess the monetary value of benefits, the valuation of public safety and health or environmental quality is primarily a socio-political process in which the engineer or other analyst can only estimate physical effects, evaluate direct benefits and, perhaps, suggest the desired performance level. The final selection of a performance level is not solely within the analyst's domain nor should it be. For these reasons, the methodology presented in this study only looks at the cost side of the design problem and avoids the exercise of benefit assessment. More precisely, the goal is to present the minimum expenditure needed to achieve any performance levels by solving equations 6-1 through 6-4 for a wide range of values for y^o. This result is particularly useful in aiding the selection of a desired performance level within the socio-political arena.

An example of how a range of minimum cost information might be used to select a performance level is shown in Figure 6-1. This figure delineates the cost of providing additional swimming days at a beach on the Hudson River near Kingston, New York (E. D. Driscoll and Assoc., 1981). A reasonable decision could be made from this curve as to how much extra recreation the citizenry would be willing to "purchase." Obviously, this would be a subjective selection process given the fact that costs and benefits are not compared on the same scale (i.e., dollars). However, if information concerning the monetary value of benefits was available, a set of curves similar to Figure 6-2 could be use to locate the performance level where the net benefits are maximized.

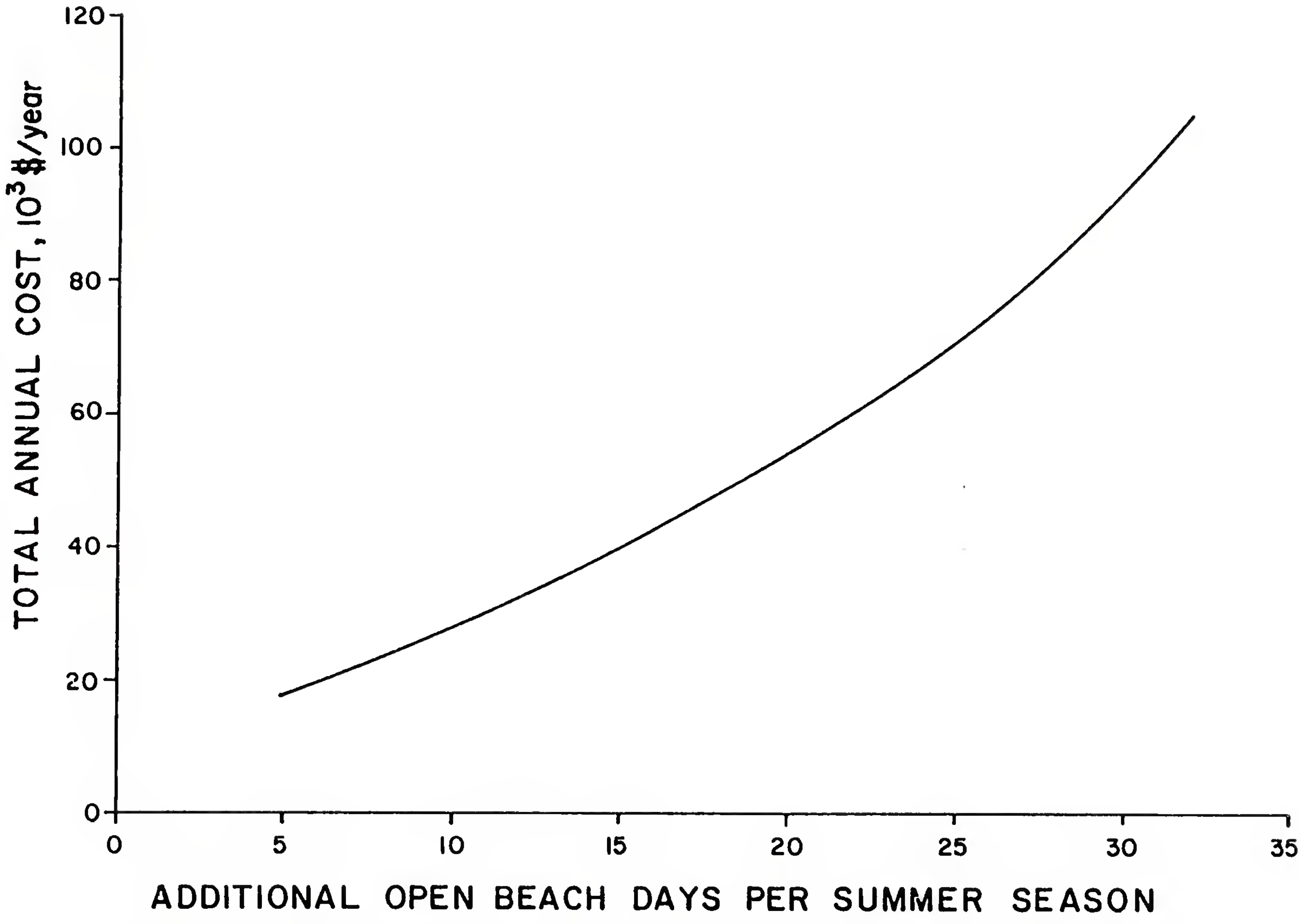


Figure 6-1. Annual Cost Versus Additional Open Beach Days as a Result of Fecal Coliform Control With a Storage/Release System, Kingston, New York (E.D. Driscoll and Assoc, 1981).

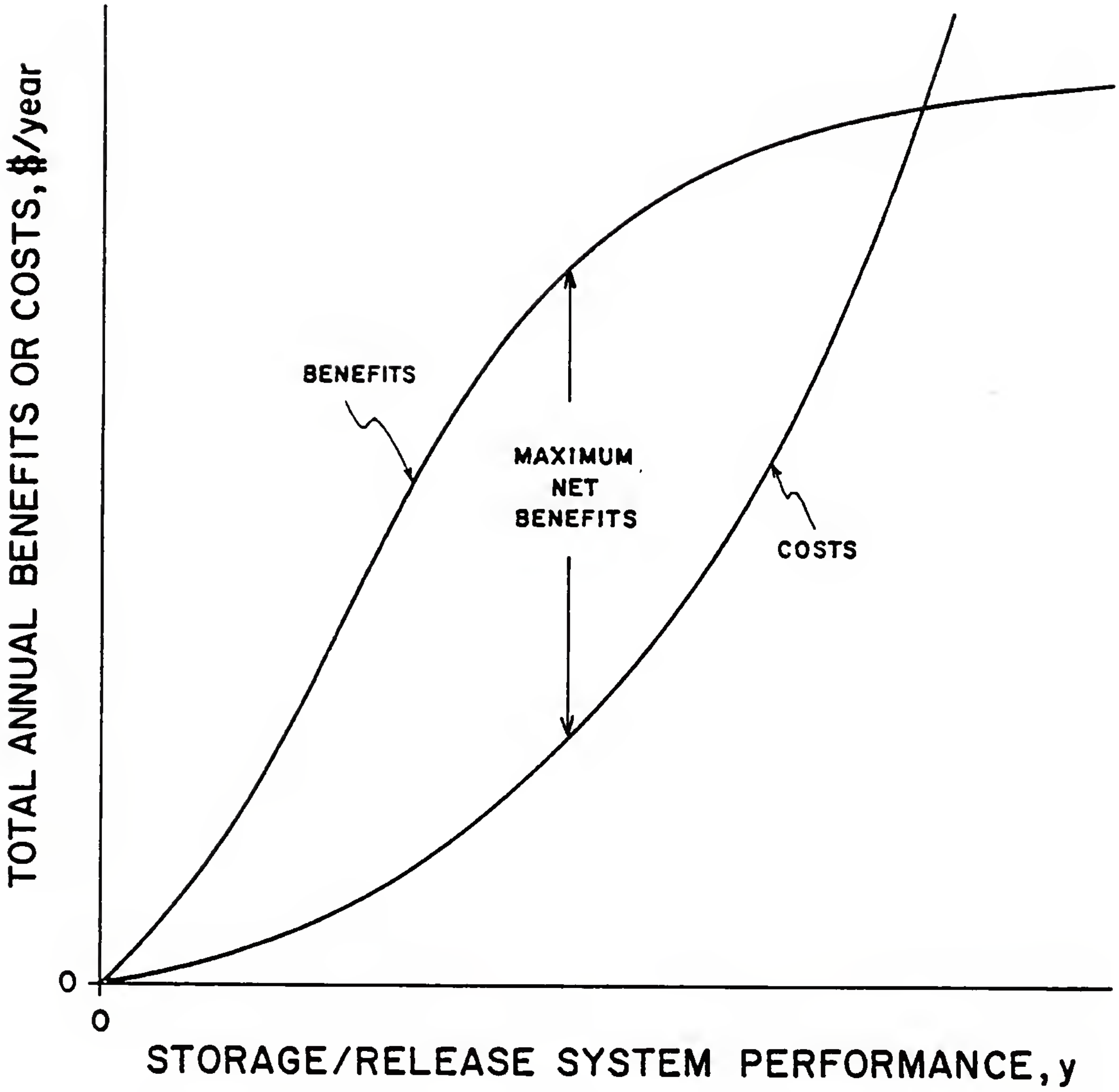


Figure 6-2. Determination of Maximum Net Benefits.

Figures 6-1 and 6-2 represent two levels of benefit assessment. In one case, the physical benefits are known and a subjective decision must be made to maximize social welfare. In the second case, the physical benefits are evaluated with regard to their monetary value and, if this "yardstick" is acceptable, an objective selection can be made. However, it is important to note that the socio-political facet is still very evident in the selection of the monetary "yardstick." When a curve such as Figure 6-1 is presented to the public, its representatives, regulatory agencies and other interested parties, the decision process indirectly places a value on the benefits received. If benefits can be quantified in monetary terms, then the decision process is that much more informed. To reiterate, the goal of this study is to demonstrate a methodology by which the necessary cost information can be produced.

Cost Functions

The optimization of a storage/release system requires information about the cost of the system components. The costs of the system are expressed as

$$Z = h(\Omega, V_B) \tag{6-5}$$

where Ω = design release rate, L^3T^{-1}, and

V_B = storage basin capacity, L^3.

These costs can be divided into two parts, i.e.,

$$Z = \text{variable costs} + \text{fixed costs} \tag{6-6}$$

Variable costs are those that change with input usage. Fixed costs are implemented at any input level and are constant over all levels. However, the temporal domain of the problem must be considered in order to distinguish between fixed and variable costs. For decision purposes, fixed costs are unimportant except when one input can be used without the other. When this occurs, the presence of an input becomes part of the decision process.

Costs can also be divided into capital costs and operation and maintenance costs. Capital costs are incurred by construction, engineering, legal matters, administration, land, and interest during construction (Benjes, 1976). Operation and maintenance costs include post-construction labor, power, chemicals, administration, laboratory/sampling, site maintenance, replacement parts, and other miscellaneous costs. Operation and maintenance costs are normally reported on an annual basis. Capital costs should be amortized over the service life of the facility so that all dollars spent are equivalent in kind. The interest rate at which to amortize capital costs is often set at the rate suggested by the Water Resources Council and used by the Environmental Protection Agency in its construction grants program (E.D. Driscoll and Assoc., 1981). In this study, all costs are expressed as annual costs.

There are essentially two ways to obtain the necessary cost data. One method is to collect a cross section of data from storage/release systems constructed around the nation or within the inclusive region or local area. This route is only usable for attaining "first-cut" estimates of costs over a range of storage capacities or release rates. Costs vary widely from location to location and, thus, the more the cross-sectional data can be localized, the better the estimates will be. Great care should be taken when using cross-sectional data because designs are rarely identical.

The second, and preferred, method of collecting relevant cost data is to obtain estimates for basic materials, land, labor, and engineering. Benjes (1976) has compiled a summary of cost relationships for combined sewer overflow storage/treatment facilities using this method. An EPA manual (Municipal Environmental Research Laboratory, 1976) has also presented similarly developed cost information (in addition to some cross-sectional data). These reports use data that are averages for the U.S. and thus, care is necessary when applying them locally. Ideally, local cost data can be collected to develop a site-specific cost function in lieu of using these general cost functions. Nevertheless, the approach taken by Benjes (1976) is probably superior to using data taken from a cross-section of facilities.

The costs given by each source should be expressed in terms of equivalent dollars; in other words, they should be indexed in some manner so that a facility built or materials used at one point in time can be properly compared with units built and materials used at some other time. The most widely used index is the Engineering News Record's (ENR) Construction Cost Index and Building Cost Index. However, the ENR index is based on materials and labor needed for projects much different than urban stormwater storage/release systems or other water utility projects and, thus, is generally inappropriate (Benjes, 1976). The EPA has developed a cost index reflective of the types of materials used in sewage treatment plants. The index is reported periodically in the Journal of the Water Pollution Control Federation. This index is probably more suitable to urban stormwater storage/release system but can only serve as an approximation.

All cost indices are used in the following manner:

$$\mathrm{Cost}_b = \left(\frac{\mathrm{Index}_b}{\mathrm{Index}_a}\right) \mathrm{Cost}_a \qquad (6\text{-}7)$$

where a and b represent two different points in time. Such an index should also be applied when accounting for cost changes in the period between planning and construction.

Variable input costs are most commonly expressed as power functions. Thus, equation 6-5 can be expressed as

$$Z = c_1\Omega^{\alpha_1} + c_2 V_B^{\alpha_2} + c_1' + c_2' \qquad (6\text{-}8)$$

where Z = total annual cost, dollars/year,

c_1, c_2 = coefficients, $c_1 > 0$, $c_2 > 0$,

α_1, α_2 = coefficients, $\alpha_1 > 0$, $\alpha_2 > 0$, and

c_1', c_2' = fixed annual costs associated with the release rate, Ω, and the storage basin capacity, V_B, respectively, dollars/year.

The shape of either variable cost term in equation 6-8 is determined by the value of its exponent (i.e., α_1 or α_2). Not considered here is the possibility that input costs will decrease with increased usage (i.e., $\alpha_1 < 0$, $\alpha_2 < 0$). This is a very rare situation (Ferguson, 1975) and warrants little concern. The plausible shapes are presented in Figure 6-3. For storage/release systems and wastewater systems, these exponents are normally less than one (Benjes, 1976; Municipal Environmental Research Laboratory, 1976; Heaney et al., 1976, 1977; Heaney, 1979). This fact tends to favor the construction of a single, large system in lieu of several smaller ones. If a power function does not adequately fit the variable cost data, a graphical representation can be used. It will be shown that functional forms are not required to complete the analysis.

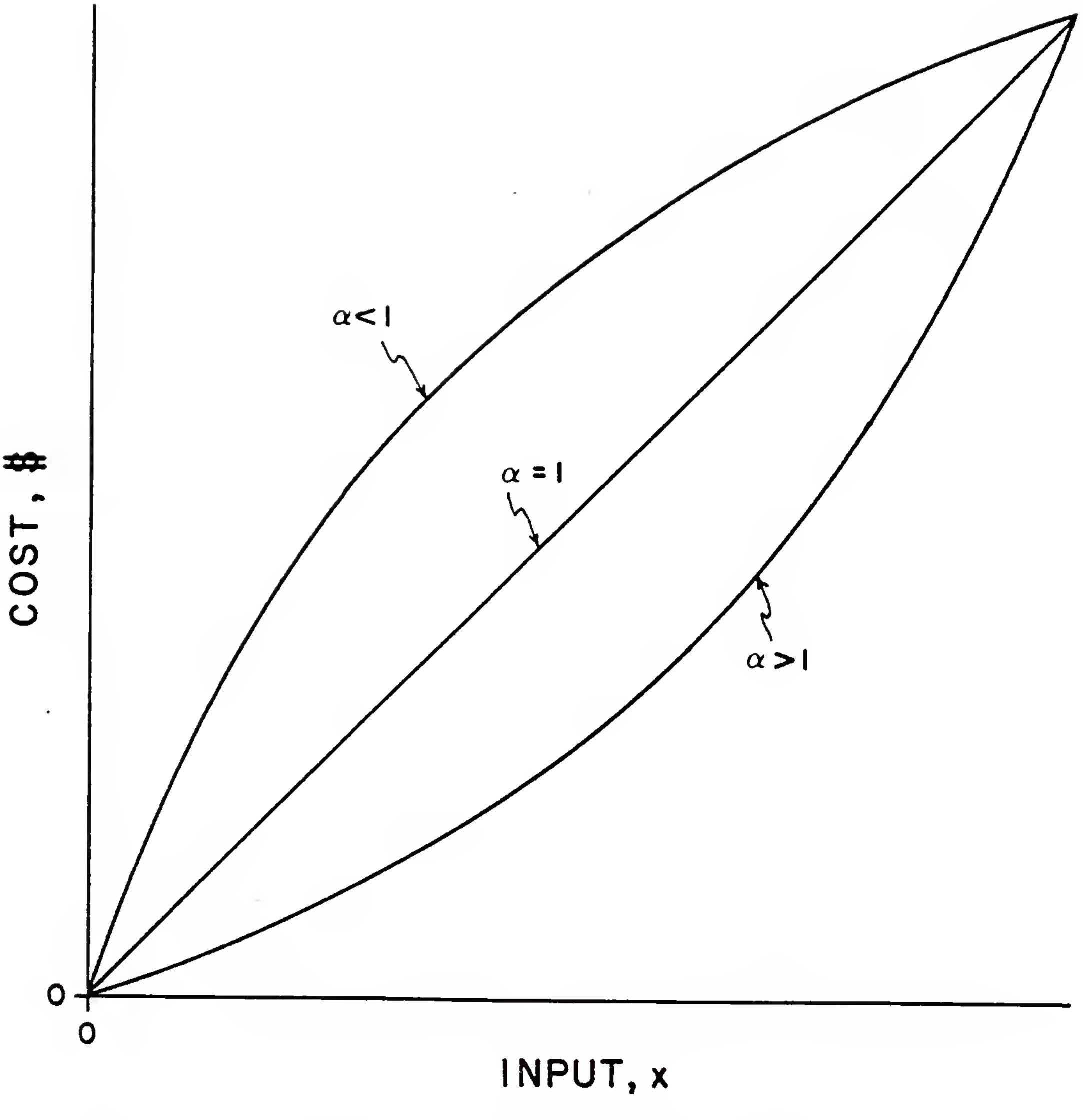


Figure 6-3. General Input Cost Function Types.

Typical cost curves or functions for earthen, and covered and uncovered concrete storage basins are shown in Figures 6-4, 6-5, and 6-6. Each is a function of storage capacity. Cost functions for the release rate are more difficult to present because of the many ways in which a storage basin can be emptied. In all cases, the cost function for the release mechanism must include all costs associated with the released stormwater. These items might include conveyance systems, treatment facility costs (if routed to a treatment facility), pumping facilities and any other cost associated with moving and possibly treating the releases. The reports by Benjes (1976) and the Municipal Environmental Research Laboratory (1976) contain cost functions which might be useful for this purpose.

Optimization Techniques

Overview

The storage/release optimization problem will probably have nonlinear components, either in the objective function or the constraints. Thus, the solution relies upon a class of techniques known as nonlinear optimization. Unfortunately, many of these methods become unwieldy when the nonlinear components become mildly complex. The purpose here is not to explore the gamut of techniques but to simplify the analysis as much as possible. The optimization problem presented in equations 6-1 through 6-4 is not very complex in structure and while it is possible to subject it to a myriad of complex optimization techniques, it is not necessary or desirable.

Most nonlinear optimization techniques can be classified into one of the following groups (Stark and Nicholls, 1972):

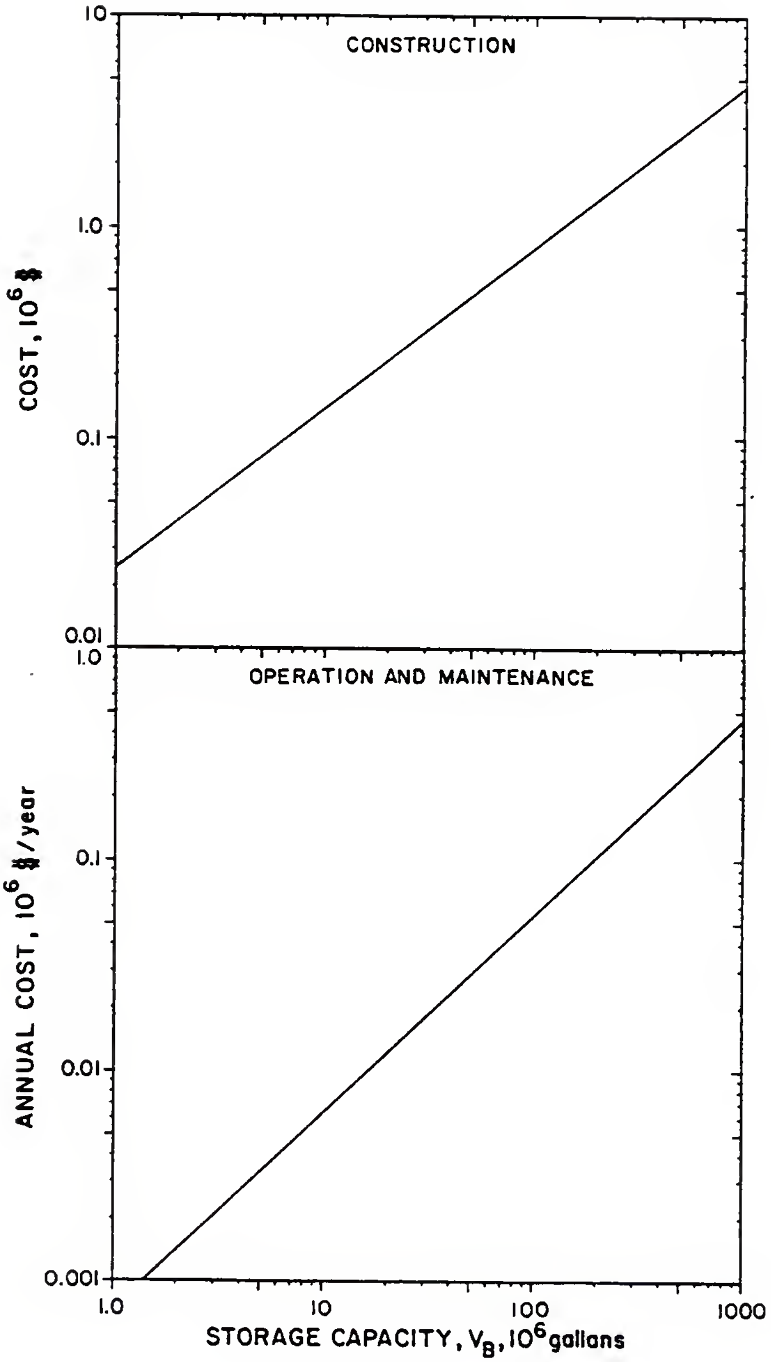


Figure 6-4. Costs for Earthen Storage Basins, September, 1976 (Municipal Environmental Research Laboratory, 1976).

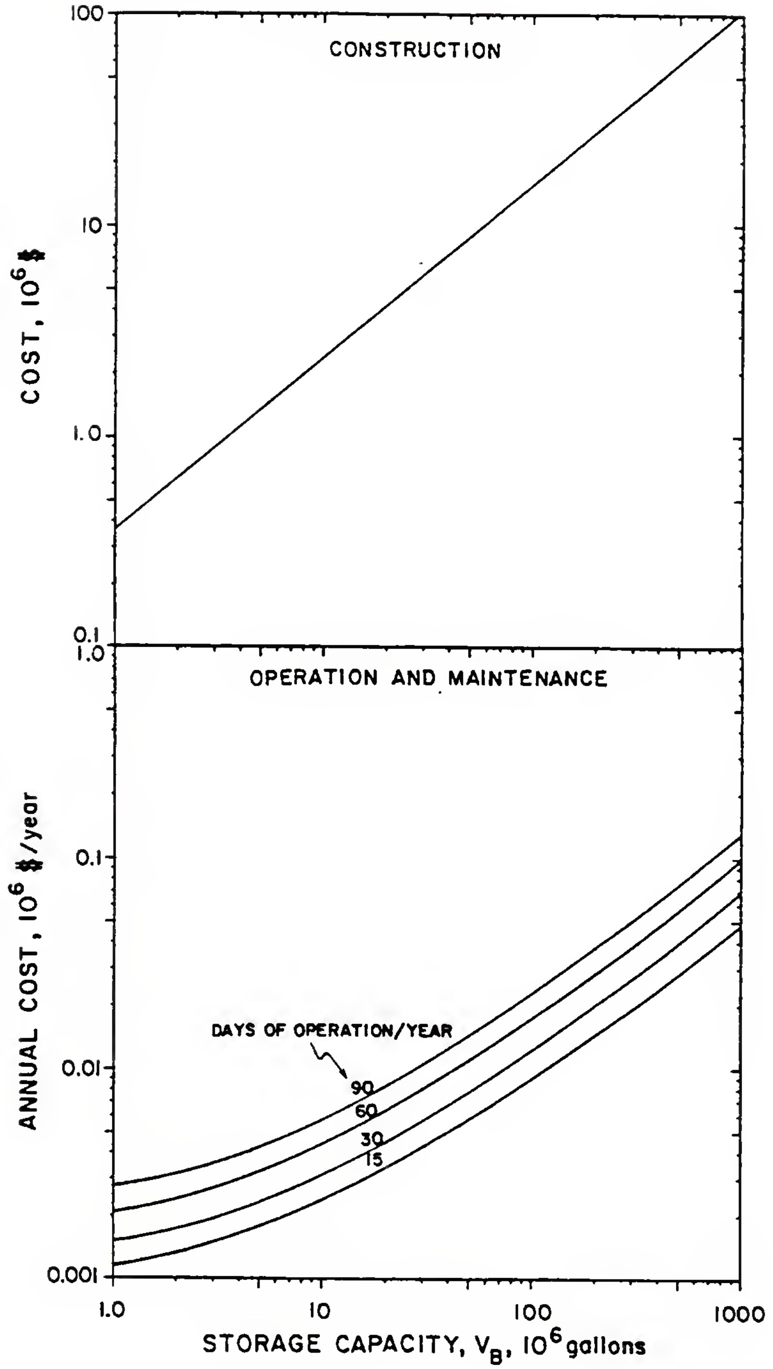


Figure 6-5. Costs for Covered Concrete Storage Basins, September, 1976 (Municipal Environmental Research Laboratory, 1976).

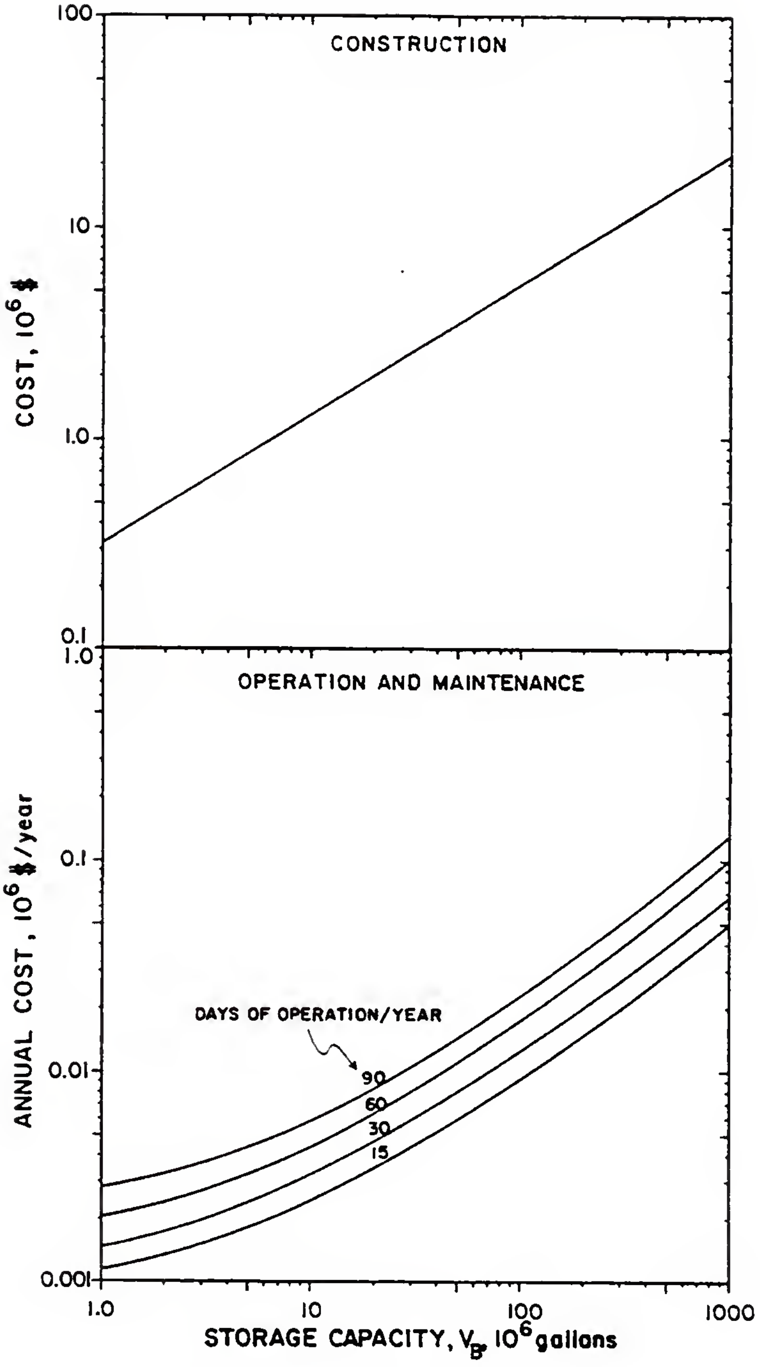


Figure 6-6. Costs for Uncovered Concrete Storage Basins, September, 1976 (Municipal Environmental Research Laboratory, 1976).

1) classical optimization,
2) search techniques,
3) linear transformation/approximation, and
4) graphical techniques.

Classical optimization techniques usually involve solving a set of equations, and, thus, are generally useful as long as the set is solvable. Search techniques use iterative algorithms to improve an initial guess at the solution and are useful in situations where classical techniques are impractical. The third technique approximates a nonlinear component by piecewise linearization or by directly transforming the objective and/or constraints to linear forms. Such a modification allows the analyst to take advantage of the well-understood, efficient linear programming algorithms. Graphical optimization techniques involve the plotting of production and cost data in such a manner that the optimal solution can be found by inspection.

After discussing two useful mathematical properties, two classical optimization techniques will be reviewed and a simple, flexible graphical technique introduced. A common thread running throughout this chapter is the assertion that complicated analytical techniques are unnecessary for the problem at hand. The techniques emphasized in this section, while not a panacea, will handle most storage/release optimization problems in a rigorous fashion. The graphical technique will prove to be particularly useful. The presentation is generalized in order to maintain continuity with the prefatory discussion of production theory in Chapter 5.

Useful Properties

Monotonicity. An optimization problem can be simplified if it can be written in terms of monotonically increasing or decreasing functions (Wilde, 1978). A function, $f(\underline{X})$, of a vector of variables, $\underline{X} = (x_1, x_2, \ldots, x_i, \ldots, x_m)$, is monotonic if and only if it is either increasing or decreasing with respect to a single independent variable, x_i, for $\underline{X} > \underline{0}$. More specifically, $f(\underline{X})$ is

1) monotonically increasing with respect to x_i, if and only if $\partial f/\partial x_i > 0$ for $\underline{X} > \underline{0}$,
2) monotonically decreasing with respect to x_i, if and only if $\partial f/\partial x_i < 0$ for $\underline{X} > \underline{0}$, or
3) independent of x_i, if and only if $\partial f/\partial x_i = 0$ for $\underline{X} > \underline{0}$.

A set of functions is collectively termed "monotonic" with respect to x_i if and only if each function is either monotonically increasing or decreasing with respect to x_i or independent of x_i (with at least one dependent function). If all functions within the set are monotonically increasing (decreasing), the set is said to be monotonically increasing (decreasing). An increasing function and a decreasing function are said to have "opposite" monotonicities with respect to x_i. If two or more functions are all increasing or all decreasing, they are said to have the same monotonicity.

The sum of two monotonic functions, $f_1 + f_2$, is also monotonic if both functions have the same monotonicity. This is easily shown by the sum of the differentials, $\partial f_1/\partial x_i + \partial f_2/\partial x_i$. If both terms are greater (less) than zero, then the sum is greater (less) than zero. If f_1 and f_2 have the same monotonicity, the product $f_1 f_2$ will also be monotonic.

This can be shown by

$$\frac{\partial(f_1 f_2)}{\partial x_i} = (\frac{\partial f_1}{\partial x_i})f_2 + (\frac{\partial f_2}{\partial x_i})f_1 \qquad (6\text{-}9)$$

in which $\partial f_1/\partial x_i$ and $\partial f_2/\partial x_i$ are both greater (less) than zero.

If a monotonic function is raised to the power α, the result is also monotonic, in the same sense, whenever $\alpha > 0$. This is demonstrated by the following:

$$\frac{\partial(f^\alpha)}{\partial x_i} = \alpha f^{\alpha-1}\frac{\partial f}{\partial x_i} \qquad (6\text{-}10)$$

Thus, when $\partial f/\partial x_i$ is greater (less) than zero, $\partial(f^\alpha)/\partial x_i$ is also greater (less) than zero. If $\alpha < 0$, f^α will have the opposite monotonicity of f. When two monotonic functions are used to form a composite function $f_1(f_2)$, differentiation by the chain rule yields

$$\frac{\partial f_1(f_2)}{\partial x_i} = (\frac{\partial f_1}{\partial f_2})(\frac{\partial f_2}{\partial x_i}) \qquad (6\text{-}11)$$

When f_1 and f_2 have the same monotonicity, the value of $\partial f_1(f_2)/\partial x_i$ is positive and thus, $f_1(f_2)$ is monotonically increasing. If f_1 and f_2 have opposite monotonicities, $f_1(f_2)$ is monotonically decreasing.

Convexity. A convex function has a unique global minimum. A necessary condition for locating a minimum point is that the first derivative must disappear. However, when the first derivative is zero and nothing about the convexity of the function is known, it can only be said that a local minimum or maximum has been found. However, a convex function has a unique global minimum and when the function of interest is convex the first derivative locates that minimum. A function, $f(\underline{X})$, of a vector of variables, $\underline{X}$, is convex if for any pair of points, $\underline{X}'$ and $\underline{X}''$, a line segment joining them is everywhere above the function (when referenced from the origin). A single-variable convex function is shown in Figure 6-7(a). A concave function is shown in Figure 6-7(b). More generally, f($\underline{X}$) is convex if the pure second-order derivatives are nonnegative and the determinant of the matrix of second-order derivatives is nonnegative over the range of interest (Stark and Nicholls, 1972):

$$f_{ij} = \frac{\partial^2 f}{\partial x_i \partial x_j} \geqq 0 \text{ for } i = j, \tag{6-12}$$

and

$$\begin{vmatrix} f_{11} & \cdots & f_{ij} & \cdots & f_{1m} \\ \vdots & & \vdots & & \vdots \\ f_{i1} & \cdots & f_{ij} & \cdots & f_{im} \\ \vdots & & \vdots & & \vdots \\ f_{m1} & \cdots & f_{mj} & \cdots & f_{mm} \end{vmatrix} \geqq 0 \tag{6-13}$$

If the pure second-order derivatives are nonpositive, f($\underline{X}$) is concave. For example, a function of two variables, $f(x_1, x_2)$ is convex if

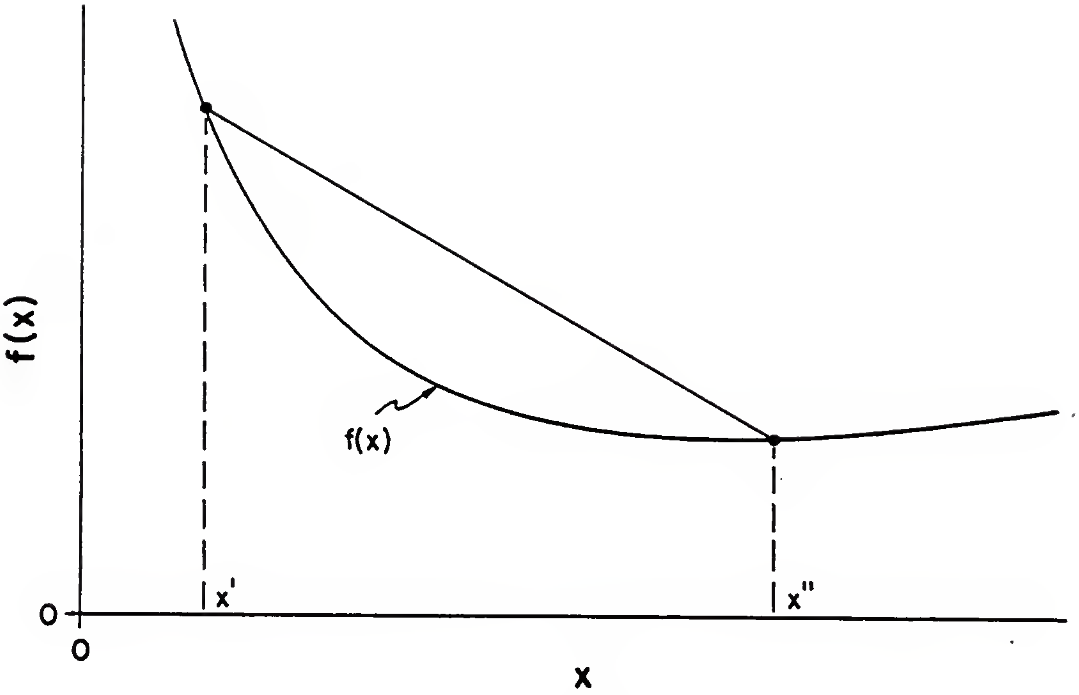
$$\begin{vmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{vmatrix} = f_{11}f_{22} - f_{21}f_{12} \geqq 0 \tag{6-14}$$

and $f_{11} \geqq 0$ and $f_{22} \geqq 0$. If $f_{11} \leqq 0$ and $f_{22} \leqq 0$ and $f_{11}f_{22} - f_{21}f_{12} \geqq 0$, then f_1 is concave. For a single-variable function, f(x), convexity is guaranteed if $\partial^2 f(x)/\partial x^2 \geqq 0$.

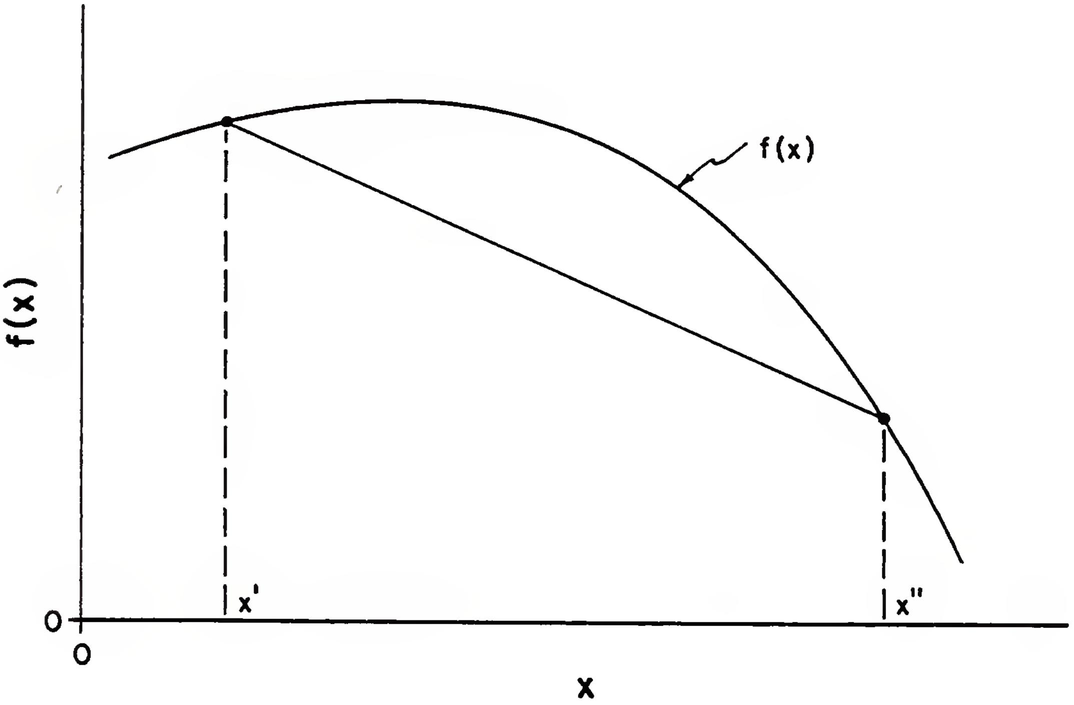
Simple Classical Optimization

For illustrative purposes, consider the following optimization problem:

$$\text{minimize } Z = c_1 x_1^{\alpha_1} + c_2 x_2^{\alpha_2} \tag{6-15}$$



(a) Convex Function



(b) Concave Function

Figure 6-7. Convex and Concave Functions.

such that $y = Ax_1^{\beta_1} x_2^{\beta_2}$ (6-16)

$y \geqq y^o$ (6-17)

$x_1, x_2, y \geqq 0$ (6-18)

where z = total annual cost, dollars/year,

x_1, x_2 = inputs 1 and 2,

y = output,

y^o = specified output level,

c_1, c_2 = coefficients, $c_1 > 0$, $c_2 > 0$,

α_1, α_2 = coefficients, $\alpha_1 > 0$, $\alpha_2 > 0$,

β_1, β_2 = coefficients, $0 < \beta_1 < 1$, $0 < \beta_2 < 1$.

In this problem, the production process is represented by a Cobb-Douglas function and the input costs are represented by power funtions. For convenience, fixed costs are assumed to be nonexistent. Recall that all of the coefficients are nonnegative (see also Chapter 5). This problem can be simplified by noting that the monotonicity of the Cobb-Douglas equation is increasing for each input and the monotonicity of the objective or cost function is also increasing for each input. Because of this property, the inequality constraint, $y \geqq y^o$, must be active (i.e., $y = y^o$) at the optimum solution. Thus, the problem becomes to

minimize $Z = c_1 x_1^{\alpha_1} + c_2 x_2^{\alpha_2}$ (6-19)

such that $y^o = Ax_1^{\beta_1} x_2^{\beta_2}$ (6-20)

$x_1, x_2 \geqq 0$ (6-21)

Substituting the production constraint into the objective function yields

$$\text{minimize } Z = c_1\left(\frac{y^o}{A}\right)^{\alpha_1/\beta_1} x_2^{-\alpha_1\beta_2/\beta_1} + c_2 x_2^{\alpha_2} \qquad (6\text{-}22)$$

The problem becomes a single-variable, unconstrained (except for the nonnegativity requirements) optimization problem which is easily solved by differential calculus. In other words, a minimum can be found by taking the first derivative with respect to x_2, setting the result equal to zero and solving for the optimal value of x_2, x_2*, i.e.,

$$\frac{\partial Z}{\partial x_2} = c_1\left(\frac{y^o}{A}\right)^{\alpha_1/\beta_2}\left(-\frac{\alpha_1\beta_2}{\beta_1}\right)x_2^{-\alpha_1\beta_2/\beta_1-1} + c_2\alpha_2 x_2^{\alpha_2-1} = 0 \qquad (6\text{-}23)$$

or

$$x_2* = \left(\frac{c_1\alpha_1\beta_2}{c_2\alpha_2\beta_1}\right)^{\frac{\beta_1}{\alpha_1\beta_2+\alpha_2\beta_1}} \left(\frac{y^o}{A}\right)^{\frac{\alpha_1}{\alpha_1\beta_2+\alpha_2\beta_1}} \qquad (6\text{-}24)$$

The value of x_1* is easily calculated from the production function or in a manner similar to x_2*:

$$x_1* = \left(\frac{c_2\alpha_2\beta_1}{c_1\alpha_1\beta_2}\right)^{\frac{\beta_2}{\alpha_1\beta_2+\alpha_2\beta_1}} \left(\frac{y^o}{A}\right)^{\frac{\alpha_2}{\alpha_1\beta_2+\alpha_2\beta_1}}. \qquad (6\text{-}25)$$

The minimum cost is given by

$$Z* = \left[c_1\left(\frac{c_2\alpha_2\beta_1}{c_1\alpha_1\beta_2}\right)^{\frac{\alpha_1\beta_2}{\alpha_1\beta_2+\alpha_2\beta_1}} + c_2\left(\frac{c_1\alpha_1\beta_2}{c_2\alpha_2\beta_1}\right)^{\frac{\alpha_2\beta_1}{\alpha_1\beta_2+\alpha_2\beta_1}}\right] \left(\frac{y^o}{A}\right)^{\frac{\alpha_1\alpha_2}{\alpha_1\beta_2+\alpha_2\beta_1}} \qquad (6\text{-}26)$$

Once the array of coefficients is condensed, equation 6-26 becomes a simple power function of the specified performance level, y^o. Recall that y^o can be treated as variable although it is treated as a constant in the problem statement.

A quick perusal of equations 6-24 and 6-25 shows that the non-negativity requirements are not challenged. However, the convexity of equation 6-22 is not guaranteed. This can be shown by the second derivative:

$$\frac{\partial^2 Z}{\partial x_2^{\,2}} = c_1 \left(\frac{y^o}{A}\right)^{\alpha_1/\beta_1} \left(\frac{\alpha_1\beta_2}{\beta_1}\right)\left(\frac{\alpha_1\beta_2}{\beta_1} + 1\right) x_2^{-\alpha_1\beta_2/\beta_1-2}$$

$$+ c_2\alpha_2(\alpha_2-1)x_2^{\alpha_2-2} \qquad (6\text{-}27)$$

The first term is always positive but the second term will be negative when $\alpha_2 < 1$, and thus, it is possible that $\partial^2 Z/\partial x_2^{\,2} < 0$. However, it n be shown by the monoticity of the terms in equation 6-22 that the solution derived above is a global minimum. The two variable terms of equation 6-22 are individually plotted in Figure 6-8. Curve A represents the first term; with all positive coefficients, it is decreasing at a diminishing rate. As x_2 approaches 0, this term appropaches infinity. Curves B and C represent the two possible shapes of the second term (a linear possibility is also implied). Both are monotonically increasing and as x_2 approaches infinity, so does $c_2x_2^{\alpha_2}$. Equation 6-22 is continuous and the sum of the terms must be less than infinity within $x_2 = 0$ and $x_2 = \infty$. Therefore, a lower bound or minimum exists. The first derivative yields a unique solution and thus, it must be the global minimum.

The Cobb-Douglas function proved to relatively easy to manipulate. This partially explains its attractiveness. Other functions are often

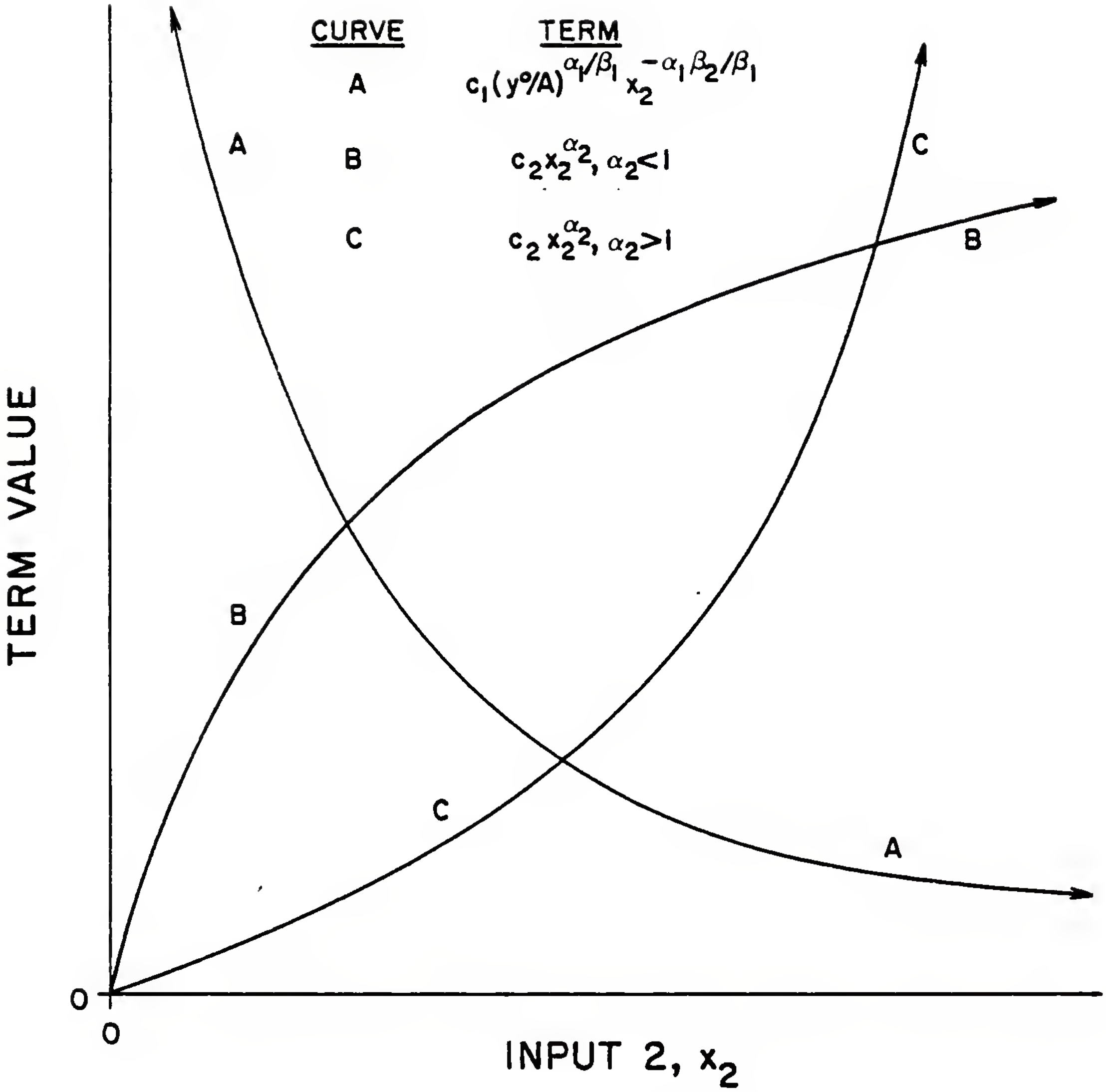


Figure 6-8. Terms of Equation 6-22.

not as readily handled. Consider the above optimization problem formulated with a quadratic production function:

$$\text{minimize } Z = c_1 x_1^{\alpha_1} + c_2 x_2^{\alpha_2} \tag{6-28}$$

$$\text{such that } y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1^2 + \beta_4 x_2^2 + \beta_5 x_1 x_2 \tag{6-29}$$

$$y \geqq y^o \tag{6-30}$$

$$x_1,\ x_2,\ y \geqq 0 \tag{6-31}$$

Although the quadratic function is not necessarily monotonically increasing, it is in the area of substitution and the following simplification is acceptable:

$$\text{minimize } Z = c_1 x_1^{\alpha_1} + c_2 x_2^{\alpha_2} \tag{6-32}$$

$$\text{such that } y^o = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1^2 + \beta_4 x_2^2 + \beta_5 x_1 x_2 \tag{6-33}$$

$$x_1,\ x_2 \geqq 0 \tag{6-34}$$

The problem can be further simplified by substituting $x_1 = -(\beta_0 - y^o + \beta_2 x_2 + \beta_3 x_1^2 + \beta_4 x_2^2 + \beta_5 x_1 x_2)/\beta_1$ into equation 6-32:

$$\text{minimize } Z = -\frac{c_1}{\beta_1^{\alpha_1}} (\beta_0 - y^o + \beta_2 x_2 + \beta_3 x_1^2 + \beta_4 x_2^2 + \beta_5 x_1 x_2)^{\alpha_1} + c_2 x_2^{\alpha_2} \tag{6-35}$$

In order to find the minimum of equation 6-35, the first partial derivatives of Z with respect to each variable must vanish. The resulting equations are solved for the optimal solution; x_1^*, x_2^*. In effect, a set of two nonlinear simultaneous equations must be solved. A numerical, iterative, solution technique such as the Newton-Raphson approximation method is required (Stark and Nicholls, 1972). Iterative techniques are not always successful because of convergence problems, but they can often produce a solution. Numerical techniques such as these will not be presented here.

The problem is simplified by assuming a linear cost equation, i.e., $\alpha_1 = \alpha_2 = 1$:

$$\text{minimize } Z = c_1x_1 + c_2x_2 \tag{6-36}$$

$$\text{such that } y^o = \beta_0 + \beta_1x_1 + \beta_2x_2 + \beta_3x_1^2 + \beta_4x_2^2 + \beta_5x_1x_2 \tag{6-37}$$

$$x_1, x_2 \geqq 0 \tag{6-38}$$

Unfortunately, simple substitution does not work in this case because y^o disappears in the differentiation (see equation 6-35 with $\alpha_1 = \alpha_2 = 1$). Therefore, in order to insure the inclusion of y^o, the use of Lagrange multipliers is required (Stark and Nicholls, 1972), i.e.,

$$\text{minimize } Z = c_1x_1 + c_2x_2 + \lambda(\beta_0 - y^o + \beta_1x_1 + \beta_2x_2 + \beta_3x_1^2 + \beta_4x_2^2 + \beta_5x_1x_2) \tag{6-39}$$

where λ = Lagrange multiplier.

The intricacies of this technique will not be discussed here. In this case, the Lagrange multiplier is an artificial variable introduced to insure the proper role of the constraint by including $\partial Z/\partial\lambda = 0$ in the set of first derivatives. This set yields the following results for x_1^* and x_2^*:

$$(\beta_3 + \beta_4H^2 + \beta_5H)(x_1^*)^2 + (\beta_1 + \beta_2H + 2\beta_4GH + \beta_5G)x_1^* + (\beta_0 - y^o + \beta_2G + \beta_4G^2) = 0 \tag{6-40}$$

and

$$x_2^* = G + Hx_1^* \tag{6-41}$$

where $G = -\dfrac{\beta_1 - c_1\beta_4/c_2}{\beta_5 - 2c_1\beta_4/c_2}$, and

$$H = \frac{\beta_5 - 2\beta_2}{\beta_5 - 2c_1\beta_4/c_2} .$$

Equation 6-40 is readily solved by the quadratic formula. Convexity (and the assurance of locating a global minimum) is checked in a somewhat more complicated method than that discussed earlier (Ferguson, 1975).

Negative values for x_1* and x_2* are possible when using these simple optimization techniques and a quadratic production function. If this occurs, the true solution is located at one of the axes of the nonnegative quadrant. This problem can be handled by the visual inspection of a graphical representation (see later discussion) or by use of the Kuhn-Tucker conditions to establish optimal solutions where nonnegativity of the variables is possible (Stark and Nicholls, 1972). A graphical technique would probably be more than adequate and a discussion of the Kuhn-Tucker conditions is not necessary.

Constrained problems formulated with the Mitscherlich function can also be simplified by direct substitution. However, the solution of the first derivative requires a numerical procedure. The same is true for the Spillman function.

Geometric Programming

Geometric programming is a recent development in nonlinear optimization (Zener, 1961; Duffin et al., 1967; Beightler and Phillips, 1976). It differs from other classical optimization techniques in that it emphasizes the relative magnitude of the terms in the objective function (cost function, in this case) rather than on the variables (inputs or resources). In other words, geometric programming first allocates costs among the components of the objective function; the optimal input levels are found indirectly. For certain types of problems, the geometric programming technique arrives at a solution with much less effort than other classical techniques. Unfortunately, there are limitations to its usefulness.

Geometric programming is restricted to a class of relationships known as posynomial functions. Posynomials are similar to polynomials in that both are finite sums of terms, each containing a product of power terms. However, they are different from polynomials in that each additive term must be positive and all exponents are unrestricted, real numbers. Consider the following unconstrained (except for the nonnegativity requirements) problem written as a posynomial:

$$\text{minimize } Z = \sum_{j=1}^{n} a_j \, p_j(\underline{X}) \qquad (6\text{-}42)$$

where a_j = coefficient, $a_j > 0$,

$\underline{X}$ = vector of variables, $(x_1, x_2, \ldots, x_i, \ldots, x_m)$,

$p_j(\underline{X}) = \prod_{i=1}^{m} x_i^{b_{ij}}$,

b_{ij} = coefficient,

n = number of additive terms, and

m = number of variables.

With the ordinary classical techniques, the minimum value of Z, Z*, is found by causing the first-order derivatives to disappear:

$$\frac{\partial Z}{\partial x_k} = \sum_{j=1}^{n} a_j b_{kj} x_k^{b_{kj}-1} \prod_{\substack{i=1 \\ i \neq k}}^{m} x_i^{b_{ij}} = 0 \text{ for } k = 1, \ldots, m \qquad (6\text{-}43)$$

or

$$\frac{\partial Z}{\partial x_k} = \sum_{j=1}^{n} a_j b_{kj} x_k^{-1} \prod_{i=1}^{m} x_i^{b_{ij}} = 0 \text{ for } k = 1, \ldots, m. \qquad (6\text{-}44)$$

where x_k = k^{th} variable, and

B_{fj} = coefficient.

These equations are then solved in order to find the optimal solution $\underline{X}*$ and the resulting value of Z*. The results are subsequently checked to insure a global minimum. Geometric programming takes an alternative approach by optimizing the allocation of Z among each term $a_j p_j(\underline{X})$. At the optimal solution, this distribution can be defined by a series of weights, i.e.,

$$w_j = \frac{a_j p_j(\underline{X}*)}{Z*} = \frac{a_j \prod_{i=1}^{m} (x_i*)^{b_{ij}}}{Z*} \quad \text{for } j = 1, \ldots, n. \tag{6-45}$$

Obviously, these weights must sum to one:

$$\sum_{j=1}^{n} w_j = 1 \tag{6-46}$$

Equation 6-46 is referred to as the normality condition. As the optimal solution equation 6-44 becomes

$$\frac{\partial Z}{\partial x_k} = \sum_{j=1}^{n} a_j b_{kj} (x_k*)^{-1} \prod_{i=1}^{n} (x_i*)^{b_{ij}} = 0 \text{ for } k = 1, \ldots, m. \tag{6-47}$$

With reference to euation 6-45, equation 6-47 can be written as

$$\sum_{j=1}^{n} b_{kj} (x_k*)^{-1} w_j Z* = 0 \text{ for } k = 1, \ldots, m \tag{6-48}$$

or, since $\underline{X}* > \underline{0}$ and $Z* > 0$ for a meaningful problem,

$$\sum_{j=1}^{n} b_{ij} w_j = 0 \text{ for } i = 1, \ldots, m. \tag{6-49}$$

Equation 6-49 is called the orthogonality condition.

Recalling equations 6-45 and 6-46, the value of Z* can be found from knowing the values of w_j, i.e.,

$$Z^* = \prod_{j=1}^{n} (Z^*)^{w_j} = \prod_{j=1}^{n} \left[\frac{a_j p_j(\underline{X}^*)}{w_j}\right]^{w_j} = \prod_{j=1}^{n} \left[\frac{a_j}{w_j}\right]^{w_j} \prod_{j=1}^{n} \left[p_j(\underline{X}^*)\right]^{w_j} \qquad (6\text{-}50)$$

However, from equation 6-49 the following statement can be made:

$$\prod_{j=1}^{n} [p_j(\underline{X}^*)]^{w_j} = \prod_{i=1}^{m} \prod_{j=1}^{n} (x_i^*)^{b_{ij}w_j} = \prod_{i=1}^{m} (x_i^*)^{\sum_{j=1}^{n} b_{ij}w_j}$$

$$= \prod_{i=1}^{m} (x_i^*)^0 = 1 \qquad (6\text{-}51)$$

Thus, equation 6-50 becomes

$$Z^* = \prod_{j=i}^{n} \left[\frac{a_j}{w_j}\right]^{w_j} \qquad (6\text{-}52)$$

The original unconstrained optimization problem can now be written as follows:

$$\text{minimize } Z = \prod_{j=1}^{n} \left[\frac{a_j}{w_j}\right]^{w_j} \qquad (6\text{-}53)$$

$$\text{such that } = \sum_{j=1}^{n} b_{ij}w_j = 0 \text{ for } i = 1, \ldots, m \qquad (6\text{-}54)$$

$$\sum_{j=1}^{n} w_j = 1 \qquad (6\text{-}55)$$

$$w_j > 0 \text{ for } j = 1, \ldots, n \qquad (6\text{-}56)$$

Geometric programming is also capable of solving constrained optimization problems. However, the procedure is somewhat more difficult and will not be presented here. Beightler and Phillips (1976) present an excellent discussion of this application.

Consider the simple optimization problem outlined by equations 6-15 through 6-18 and more specifically, the unconstrained form given by equation 6-22. This optimization problem, formulated with the Cobb-Douglas production function, is easily handled within the framework of unconstrained geometric programming. In this case, $a_1 = c_1\left(\frac{y^o}{A}\right)^{\alpha_1/\beta_2}$, $a_2 = c_2$, $b_{11} = -\alpha_1\beta_2/\beta_1$ and $b_{12} = \alpha_2$. The orthogonality condition requires

$$\frac{-\alpha_1\beta_2}{\beta_1} w_1 + \alpha_2 w_2 = 0 \qquad (6\text{-}57)$$

for the single variable x_2. The normality condition requires

$$w_1 + w_2 = 1 \qquad (6\text{-}58)$$

From these two equations the weights w_1 and w_2 are determined to be

$$w_1 = \frac{\alpha_2\beta_1}{\alpha_2\beta_1 + \alpha_1\beta_2} \qquad (6\text{-}59)$$

and

$$w_2 = \frac{\alpha_1\beta_2}{\alpha_2\beta_1 + \alpha_1\beta_2} \qquad (6\text{-}60)$$

Thus, the relative costs attributable to each additive term have been determined. The minimum cost is calculated from equation 6-52:

$$Z^* = \left[\frac{c_1\left(\frac{y^o}{A}\right)^{\alpha_1/\beta_1}}{\left(\frac{\alpha_2\beta_1}{\alpha_2\beta_1 + \alpha_1\beta_2}\right)}\right]^{\frac{\alpha_2\beta_1}{\alpha_2\beta_1 + \alpha_1\beta_2}} \left[\frac{c_2}{\left(\frac{\alpha_1\beta_2}{\alpha_2\beta_1 + \alpha_1\beta_2}\right)}\right]^{\frac{\alpha_1\beta_2}{\alpha_2\beta_1 + \alpha_1\beta_2}} \qquad (6\text{-}61)$$

Once again, a cost equation relating Z^* and y^o is developed. Equation 6-61 is mathematically identical to the result derived by more conventional means (i.e., equation 6-26). The optimal input values are calculated from the original cost equation:

$$x_1^* = \left(\frac{w_1 Z^*}{c_1}\right)^{1/\alpha_1} \qquad (6\text{-}62)$$

and

$$x_2^* = \left(\frac{w_2 Z^*}{c_2}\right)^{1/\alpha_2} \qquad (6\text{-}63)$$

The optimization problem given by equation 6-22 is well suited to the geometric programming algorithm because there is one more term than there are variables. This allows the number of equations (the orthogonality and normality conditions) to equal the number of weights. This problem is said to possess zero degrees of difficulty. The degree of difficulty is calculated as n-(m+1) where n is the number of additive terms and m is the number of variables in the unconstrained problem. A problem with a zero degree of difficulty has a unique global solution that is easily found.

When the problem possesses one or more degrees of difficulty, this simple geometric programming technique does not yield a unique solution. However, in the case of storage/release optimization, a more critical problem arises to block further use of geometric programming. Often, the optimization problem can not be stated as a posynomial (e.g., equation 6-35). This does not mean that unconstrained geometric programming techniques are useless; it is, at times, possible to circumvent this problem (Beightler and Phillips, 1976). Nevertheless, it would probably behoove the analyst to use another simple technique when this occurs.

Graphical Optimization

The optimization problem outlined in equations 6-1 through 6-4 is easily solved by graphical techniques. Such techniques are not slaves to the functional form of the cost or production functions and advantage can be taken of "form-free" production function or cost functions. When a reasonable functional form cannot adequately fit the data or when analytical optimization begins to become difficult, a graphical procedure can be used to solve the problem. Indeed, it might be advantageous to begin and stay with a graphical procedure. Of course, graphical procedures are limited to problems expressible in two dimensions.

It would be a mistake to label graphical techniques as approximation techniques. The graphical optimization technique used here is mathematically rigorous. Graphical solutions become approximations only because of the necessary visual representation and manipulation. However, is massaging a set of production data to fit an imperfect functional form and generating an "exact" solution via an analytical technique anything but an approximation?

Graphical procedures can be time consuming and, of course, they are limited in the number of inputs and outputs that can be displayed at one time. They also require careful work--the solution becomes more "approximate" as less care is taken. But once the representations are constructed, the insight a graphical technique can provide is probably greater than that of a "blind" analytical procedure.

Graphical optimization can be carried out in "production space" or "cost space." Production-space methods have been used in stormwater storage/release system optimization (Murphy, 1975; Nix et al., 1977; Nix and Melton, 1979; Heaney, 1979) and other applications. The production-space method works nicely for cases in which the cost function is linear but it encounters some difficulty when it is not. Unfortunately, many input cost functions are nonlinear. This problem is avoided by shifting the analysis to cost pace.

Optimization in production space. Optimization in production space is carried out by plotting the cost function (or objective function) in the isoquant plane as shown in Figure 6-9. In production space, a linear cost equation (i.e., $\alpha_1 = \alpha_2 = 1$) is written as

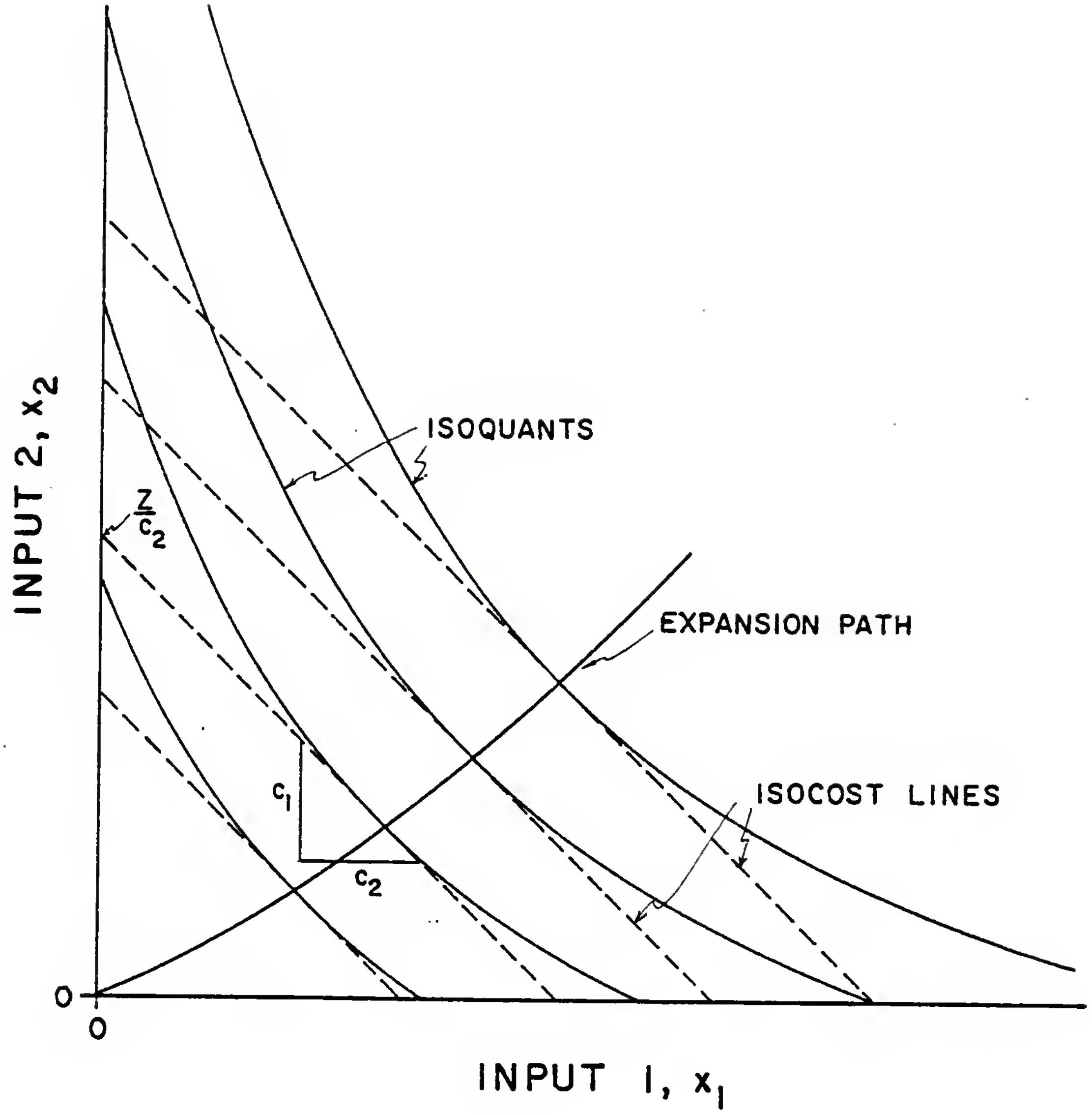


Figure 6-9. Optimization in Production Space.

$$x_2 = \frac{Z}{c_2} - \frac{c_1}{c_2}x_1 \qquad (6\text{-}64)$$

and is known as the isocost line. As Z is increased, the isocost line moves away from the origin with the constant slope $-c_1/c_2$. The intercept at the x_2 axis changes directly with the value of Z. Because the isoquants are convex to the origin, the point at which the isocost line first touches each isoquant (i.e., the point of tangency) is the least-cost solution for that particular value of y. If this process is continued, a line connecting all such points can be constructed. This line is called the expansion path. The expansion path represents the optimal input combinations, x_1* and x_2*, for any particular level of performance, y^o. When dealing with a linear cost function, the expansion path corresponds exactly to an isocline of the production function because at any point of tangency,

$$\frac{c_1}{c_2} = MRS_{21} \qquad (6\text{-}65)$$

For example, if the production function is a Cobb-Douglas equation, the expansion path becomes a straight line, i.e.,

$$x_2 = \frac{c_1}{c_2}\frac{\alpha_2}{\alpha_1}\, x_1 \qquad (6\text{-}66)$$

The expansion path is used to construct the final cost curve relating Z* and y^o (see Figure 6-10). For any selected value of y^o, the optimal strategy (i.e., x_1* and x_2*) is taken off of the expansion path.

When the cost function is nonlinear, the procedure is more difficult. In this case, the isocost curve becomes

$$x_2 = (\frac{Z}{c_2} - \frac{c_1}{c_2}\, x_1{}^{\alpha_1})^{1/\alpha_2} \qquad (6\text{-}67)$$

Consider the isocost curve represented by curve A in Figure 6-11. Curve A is concave to the origin because both α_1 and α_2 are greater than unity. Because the isocost curve is everywhere concave and the

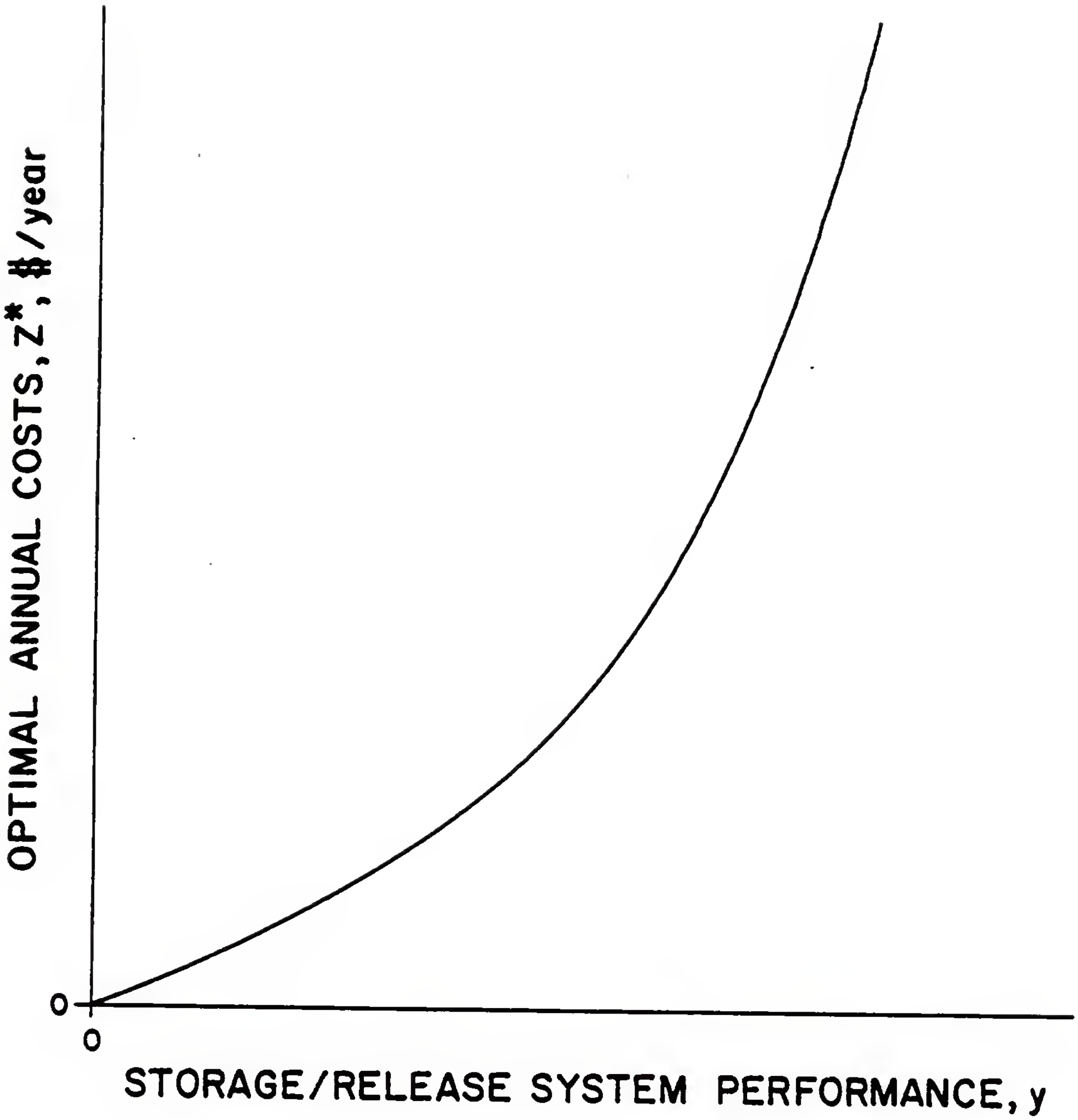


Figure 6-10. Optimal Costs for Any Storage/Release System Performance Level.

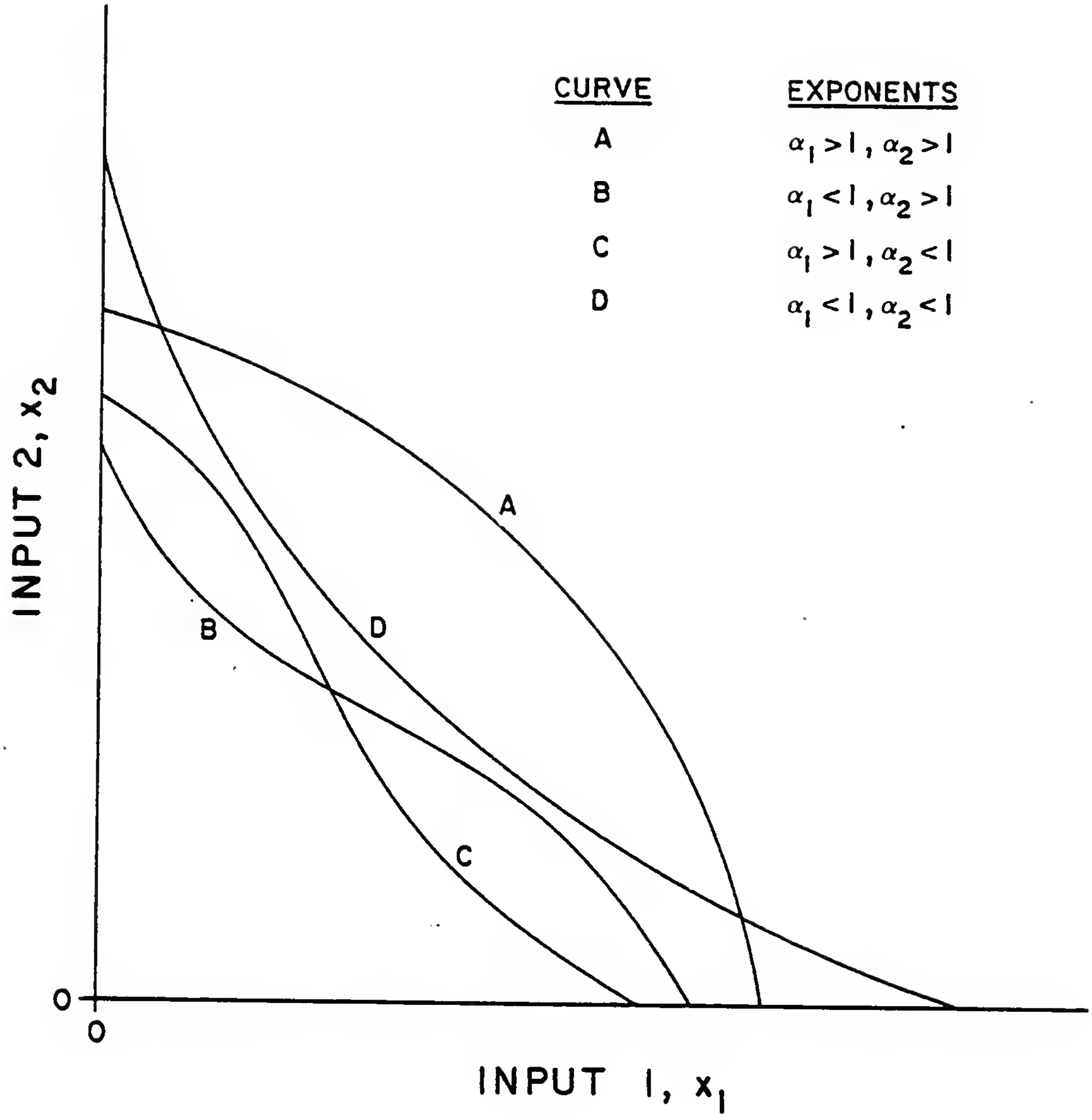


Figure 6-11. Possible Isocost Curves.

isoquants are everywhere convex, it is still possible to locate the points of tangency where the two curves first "touch." An expansion path can still be produced, but it is much more difficult to plot the increasing isocost curves because of their nonlinearity. When one or both of the cost equation exponents is less than one, an isocost curve similar to curve B ($\alpha_1 < 1$, $\alpha_2 > 1$), curve C ($\alpha_1 > 1$, $\alpha_2 < 1$) or curve D ($\alpha_1 < 1$, $\alpha_2 < 1$) emerges. Obtaining the optimal solutions under these conditions is complicated by the partial or total convexity of the isocost curves. The proper points of tangency can be found, but only with great care. When a nonlinear cost equation is used, the expansion path does not necessarily follow an isocline because the slope of the isocost curve can change at each point of tangency.

Optimization in cost space. If the production function is mapped through the cost function into cost space, an important advantage is gained. Figure 6-12 shows the isoquant set or production function in quadrant III and the annaul costs of inputs 1 and 2, Z_1 and Z_2, plotted in quadrants II and IV. Each isoquant can be mapped into cost space (quadrant I) by determining the costs associated with each input for several points along the isoquant and plotting the results in quadrant I. The technique is demonstrated by the two dotted lines emanating from point A and converging at point B. Repeating this for several points along the production isoquant y' in quadrant III produces an isoquant of y' as a function of the input costs Z_1 and Z_2 in quadrant I.

The optimal solution for any particular level of performance is easily determined. This is due to the fact that the total annual cost is a simple sum of the individual input costs, i.e.,

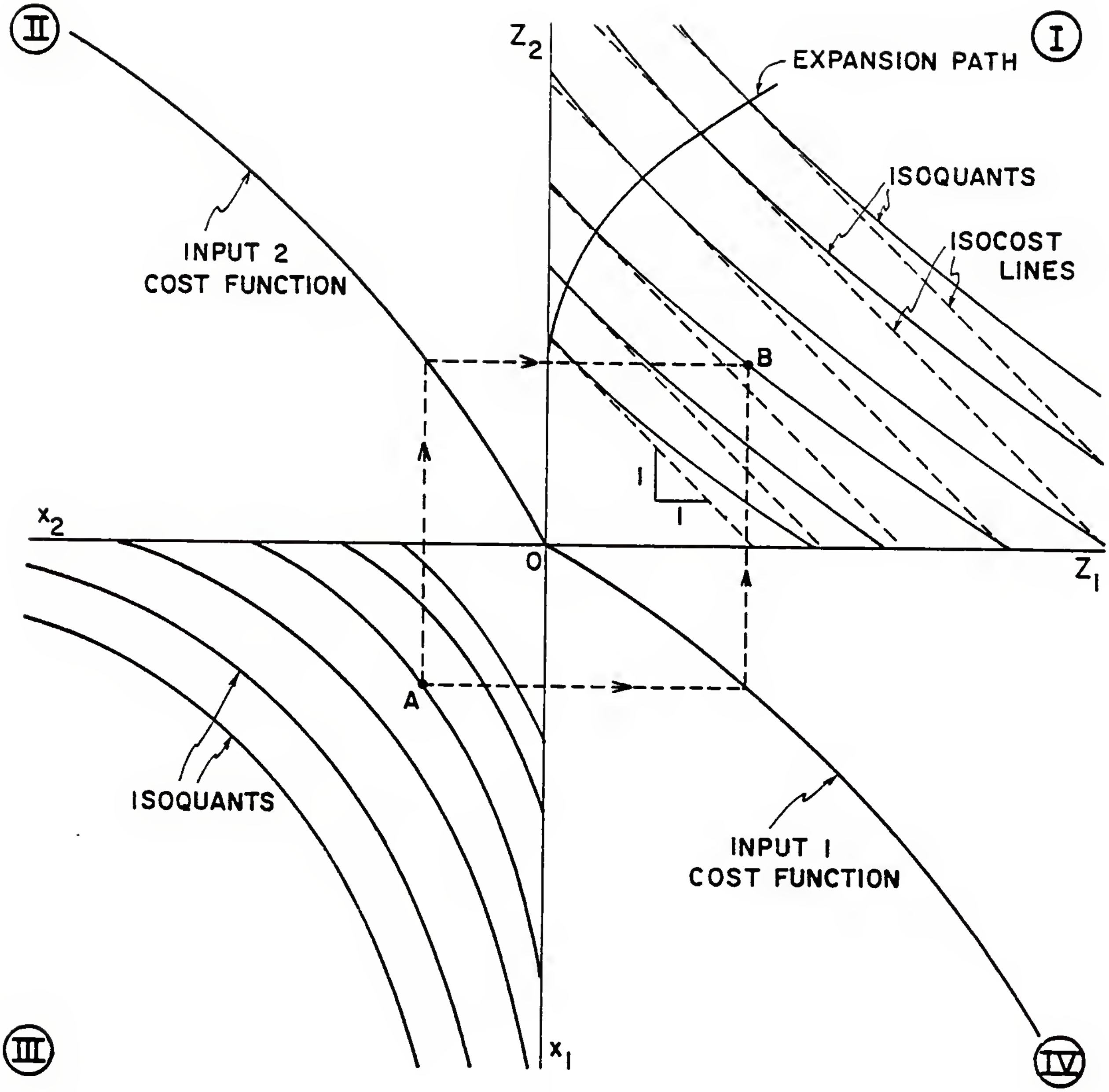


Figure 6-12. Optimization in Cost Space.

$$Z = Z_1 + Z_2 \tag{6-68}$$

Equation 6-68 always generates a isocost line with a slope of -1:

$$Z_2 = Z - Z_1 \tag{6-69}$$

This linear function is readily "moved" out from the origin to locate the expansion path in the same manner shown in Figure 6-9. In turn, the final cost curve relating Z* and y^o is derived (see Figure 6-10). Once a particular performance level is selected, the optimal strategy (i.e., x_1* and x_2*) is easily determined by working "backwards" from quadrant I to quadrant III in Figure 6-12.

Occasionally, mapping the production function through the input cost functions will produce a set of isoquants in cost space that are totally concave or partially concave. This occurs primarily when the exponents of the cost functions are somewhat less than one. Fortunately, the technique is not affected. The rule is simple; the point at which the moving isocost line first touches the each is the optimal solution for that performance level. Another possible problem occurs when the isoquants in quadrant I intersect one or both of the axes. Under these circumstances, it is possible that no point or points of tangency will be found. Again, this is handled by the same simple rule.

The technique described above is remarkably elegant and simple and it places no limitations on the functional form of the production or cost function. The method can be as accurate as necessary to fit the level of the analysis. The primary disadvantage is the amount of time needed to construct the figures. However, if a functional form for costs or the production surface is known, the analysis time can be reduced. The emerging technology of computer graphics may be able to aid this process.

Multiple Site Optimization

A facet of the design optimization problem not discussed up to this point is the question of multiple-site systems. This problem can be handled by classical optimization techniques (although not always easily), including the geometric programming technique (with the proper functional forms), or a graphical technique. Given a total cost relationship like that shown in Figure 6-10 for each site, a graphical technique developed by Nix (1976) and Heaney and Nix (1977) can be used to integrate and optimize the multiple-site system. The reader is referred to these and other reports for details (Nix and Heaney, 1977; Heaney and Nix, 1979; Nix and Melton, 1979).

Summary

Simple analytical and graphical techniques have been examined for their applicability to the storage/release optimization problem. The analytical techniques are useful if the functional forms for cost and production fit the situation and are relatively simple. For those cases in which useful functional forms are not available or the simple analytical techniques become overwhelmed, a simple graphical technique was developed. This technique solves the optimization problem in the cost domain and, as a result, allows a wide range of contingencies to be addressed. The end result of any optimization procedure is a function or curve describing the minimum cost over a wide range of system performance. This function can subsequently be used in a broader analysis to select the most cost-effective alternative and/or to determine the role of the storage/release system in a multiple objective setting.

CHAPTER 7

CASE STUDY

Introduction

The previous chapters outlined and discussed the basic theories governing the behavior of storage/release systems in the context of urban stormwater quality management and the use of the production function to consolidate and present the performance information. Several system optimization procedures were also discussed. This chapter will use a hypothetical scenario to explore these procedures, make some general observations regarding the performance of stormwater storage/release systems, suggest some design rules, and suggest a general methodology.

There is not a large body of data describing the water quality performance of urban sormwater storage/release systems. The data that have been collected are sporadic and usually inadequate for the purpose of establishing long-term performance. The primary cause for this is the expense of collecting sufficiently detailed stormwater quality data. Additionally, the persistence of the design storm concept has misdirected many of the monitoring programs. Summaries of several field studies can be found in Lager and Smith (1974), Lager et al. (1977), and Nix et al. (1981). The Humboldt Avenue detention basin study cited in Chapter 4 was probably one of the better data collection efforts (City of Milwaukee et al., 1975). Perhaps it is not surprising that one of its objectives was to estimate the long-term performance of the basin with a computer simulator. The Nationwide Urban Runoff Program (NURP), created by the EPA in 1978, should alleviate this problem but a review of some of the early results indicates that the problem will still exist to some degree.

Ideally, a long-term data base would become available in the near future for several existing storage/release basins in various parts of the U.S. Unfortunately, this is not likely to occur. However, if such a data base was established, it would be possible to make many general statements and conclusions regarding the behavior of urban stormwater storage/release systems. A reasonable compromise would be to calibrate and verify a mathematical model or models (in the general sense) for several systems through well-designed, but limited, monitoring programs. Such an approach would allow the development of a general performance statement and the results could be used to establish predictive relationships and procedures for the design of future systems. In this regard, this thesis can substantially guide and aid future studies and monitoring by establishing the theoretical background, providing a general procedure or methodology, and presenting a compendium of likely results and general design rules.

Problem Setting

The simple scenario depicted in Figure 7-1 forms the basis of the case study. Stormwater runoff from a 640-acre residential watershed is presently discharged directly to a nearby watercourse. The desire is to reduce the associated pollutant load in order to improve the quality of the receiving water to protect a beneficial use such as a public water supply. Total suspended solids (TSS) is to be the primary indicator of

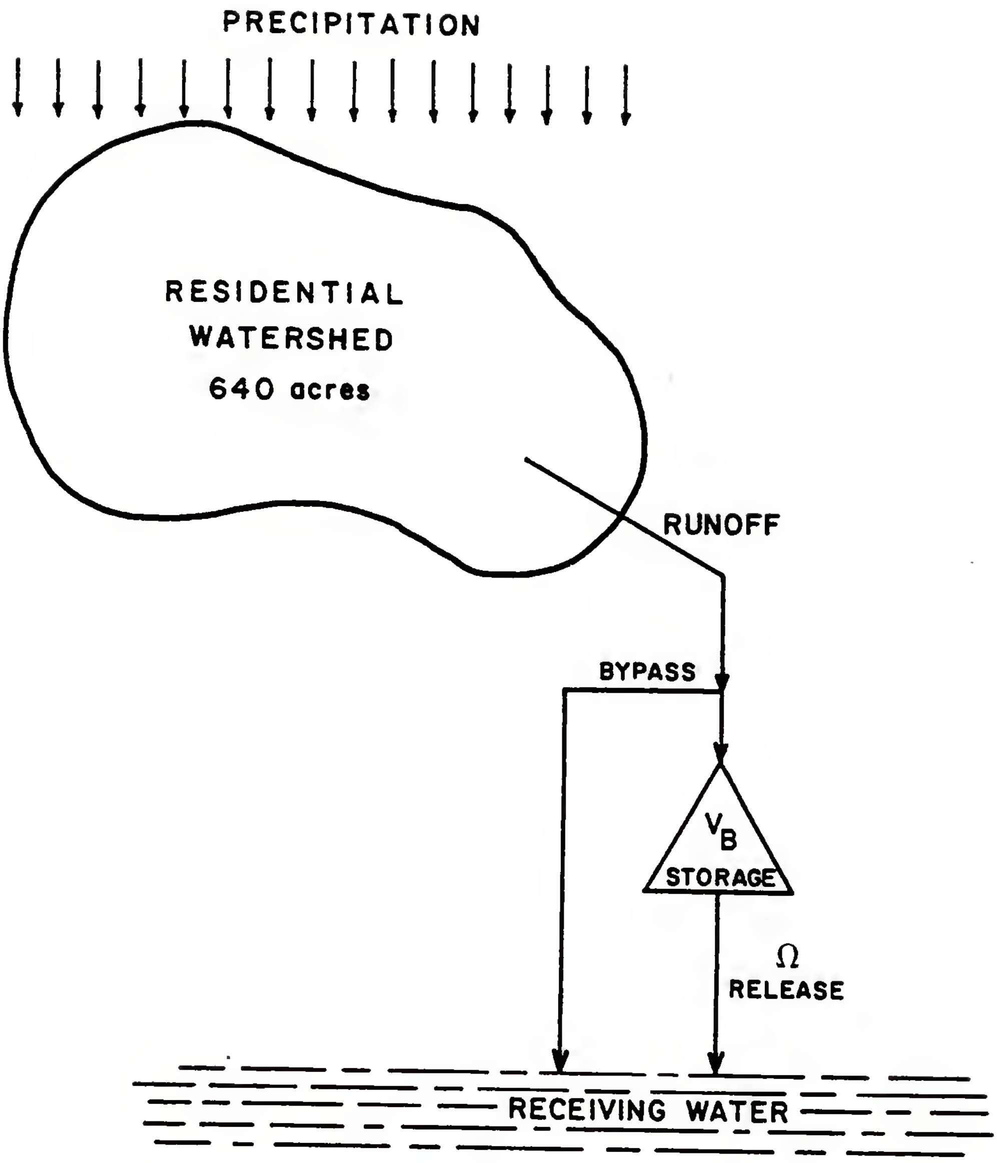


Figure 7-1. Hypothetical Scenario.

quality. Construction of a lined earthen detention basin is considered to be a feasible control alternative because of the availability of several relatively low-cost sites.

The number of specific design permutations is large. In order to maintain a manageable level of analysis, only one of these permutations was studied extensively. The simplest representation is to conceptualize the detention basin as an in-line storage/release system with the bypass mode in place and a constant release rate operating whenever stormwater is in the basin. The basin was assigned a depth of 10 feet and treated as a plug-flow reactor with a simple first-order pollutant removal mechanism. While this simple system formed the basis of the analysis, several variations on this theme were also explored in less detail.

This simple design allowed the necessary flexibility to explore some of the general facets of urban stormwater storage/release systems without obfuscating the analysis. Again, it is emphasized that this is one of many possibilities (e.g., variable release, no bypass, off-line configuration, etc.) and other feasible designs should be analyzed in practical applications. In addition, it is likely that quantity considerations (e.g., peak flow attenuation) will be evident in actual situations.

All of the evaluation techniques require some knowledge of the runoff to the control system. The Corps of Engineers' STORM model was used to generate a year's worth of hourly runoff flows and pollutant loads (see Hydrologic Engineering Center, 1977, and Chapters 2 and 3). The model was driven with the 1971 hourly precipitation record for Minneapolis, Minnesota. Thus, it can be assumed that the urban watershed described above is located in the Minneapolis area. This location was arbitrarily selected from the five cities used in other related studies (Heaney et al., 1976, 1977, 1978, 1979; Nix et al., 1977). These studies and the storage/release production functions they produced were discussed in Chapter 5. Use of this record also enhances the opportunities for comparison. The particular year was selected on the basis of a frequency analysis performed by Heaney et al. (1977) in which the hourly record for 1971 was deemed to be the most representative of the long-term record (1948-1972). This was done to reduce computer costs in this and past studies.

The STORM model requires little input data to generate a runoff flow and pollutant load record. In addition to the rainfall record, a value for the percent of impervious area and a value for maximum detention storage is required to create the runoff record. The percent of impervious area was selected to be 20 percent and the maximum depression storage was assigned a value of 0.10 inches (normalized over the watershed). These are realistic, but not necessarily typical values for residential watersheds (Miller and Viessman, 1972; Hydroscience, Inc., 1979). Depression storage is viewed by STORM as the surface storage that must be satisfied before runoff can occur.

The suspended solids load record was generated by arbitrarily assigning a daily surface accumulation rate of 1.0 lbs/acre. A first-order reaction was used to characterize the removal of suspended solids in the watershed (see later discussion) and, thus, the actual magnitude of the load is unimportant. This is due to the fact that first-order reactions (in the storage basin) are independent of the reactant concentration. STORM utilizes the following equation to wash the accumulated pollutant load from the land surface at each hourly step:

$$f_p = 1 - \exp(-K_p r_o \Delta t) \tag{7-1}$$

where f_p = fraction of the available total suspended solids load washed off the watershed surface during this hour,

K_p = washoff decay coefficient, inch^{-1},

r_o = hourly runoff flow rate, inches/hour, and

Δt = time step, one hour.

Equation 7-1 gives the load record characteristics consistent with a "first-flush" assumption. The value of K_p was assigned a value of 4.6 inch^{-1} in order to be consistent with past studies (Heaney et al., 1976, 1977, 1978, 1979; Nix et al., 1977). The runoff flow rates are normalized over the watershed and reported in inches/hour. The loads are reported in lbs/hour and as concentrations in mg/l (milligrams per liter).

Developing the Production Functions

With the STORM-generated runoff record, various storage/release system evaluation techniques were used to develop production functions relating the storage basin capacity, V_B, and release rate, Ω, to the following outputs:

1) percent volume control (the percent of the total runoff flow entering the detention basin);
2) percent pollutant control (the percent of the total pollutant load entering the detention basin); and
3) percent pollutant removal (the percent of the total pollutant load retained, by settling or other removal mechanisms, in the basin).

Each of these performance parameters is directly related to the volume reliability measure discussed in Chapter 2. In addition, performance was measured in terms of the occurrence or frequency of hourly pollutant loads, but not to the extent of the parameters listed above. This type of parameter is identical to the time reliability measure discussed in Chapter 2. The values for V_B are normalized over the watershed and reported in inches. The values for Ω are also normalized and reported in inches/hour.

The following evaluation techniques were used to develop the production functions:

1) SWMM Level I (see Heaney et al., 1976, 1978 and the discussion of STORM-produced production functions in Chapter 5);
2) Hydroscience statistical method (see DiToro and Small, 1979; Hydroscience, Inc., 1979; and Chapter 3); and
3) SWMM Storage/Treatment (S/T) Block (see Chapter 4 and Huber et al., 1981).

The SWMM Level I and Hydroscience methods were used to provide preliminary assessments of the production surface and establish the ranges of V_B and Ω over which the more sophisticated (and expensive) S/T Block was utilized. Within these ranges, several combinations of V_B and Ω were run and the results plotted. These results were visually interpreted to produce the various isoquant sets or production functions. This is equivalent to developing "form-free" representations of the production functions (see Chapter 5).

SWMM Level I

The STORM-generated volume and pollutant control production functions developed by Heaney et al. (1977, 1978), and discussed in Chapter 5, formed the basis for a simplified preliminary assessment procedure in which the results for five major cities could be used to estimate similar production functions for any other urban area (Heaney et al., 1976, 1978). This is accomplished by adjusting some of the coefficients in the transcendental equations (i.e., equation 5-71) used to represent the volume and pollutant control production functions. These adjustments are based on the ratio of the annual runoff in the urban area of interest to the annual runoff of one of the five reference cities. Each of the reference cities is assumed to represent one of five relatively homogeneous (in terms of rainfall patterns) regions in the U.S. The production functions for the five reference cities were constructed for all developed portions of the urban area and thus, must also be adjusted for any particular area within these cities as well as cities within the congruent region.

The STORM runs reported 8.16 inches of runoff (normalized over the watershed) for the hypothetical Minneapolis watershed during 1971. The runoff reported for all developed areas in Minneapolis for 1971 is 10.50 inches (Heaney et al., 1977). With these values and the necessary adjustments, the volume control production function shown in Figure 7-2 was constructed. The pollutant control production function shown in Figure 7-3 was produced in a similar manner. The interested reader can find the details of the necessary adjustments in the reports by Heaney et al. (1976, 1977, 1978). The manner in which STORM models a storage/release system is described in Chapters 2 and 5. STORM models the

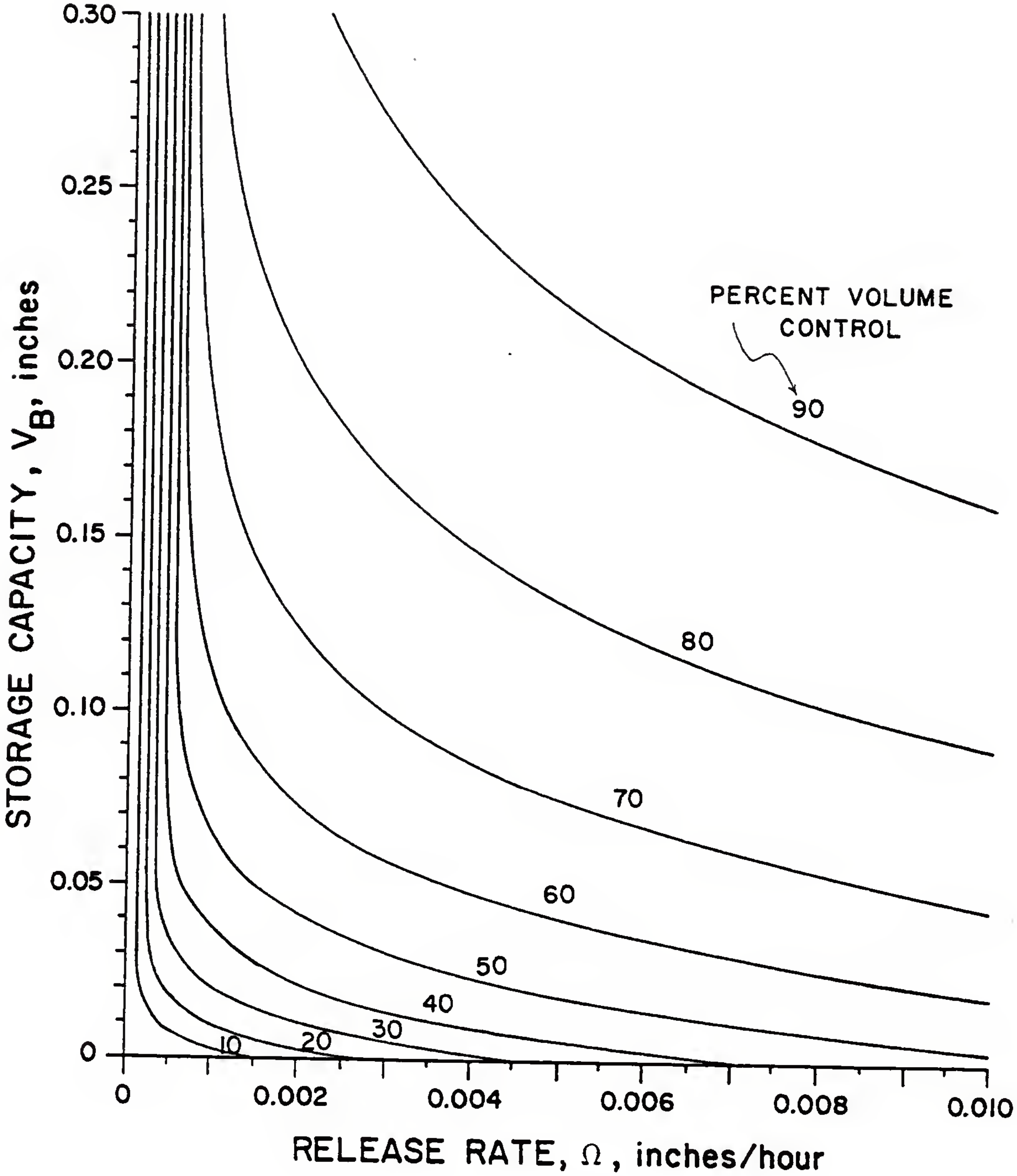


Figure 7-2. Percent Volume Control Production Function, SWMM Level I.

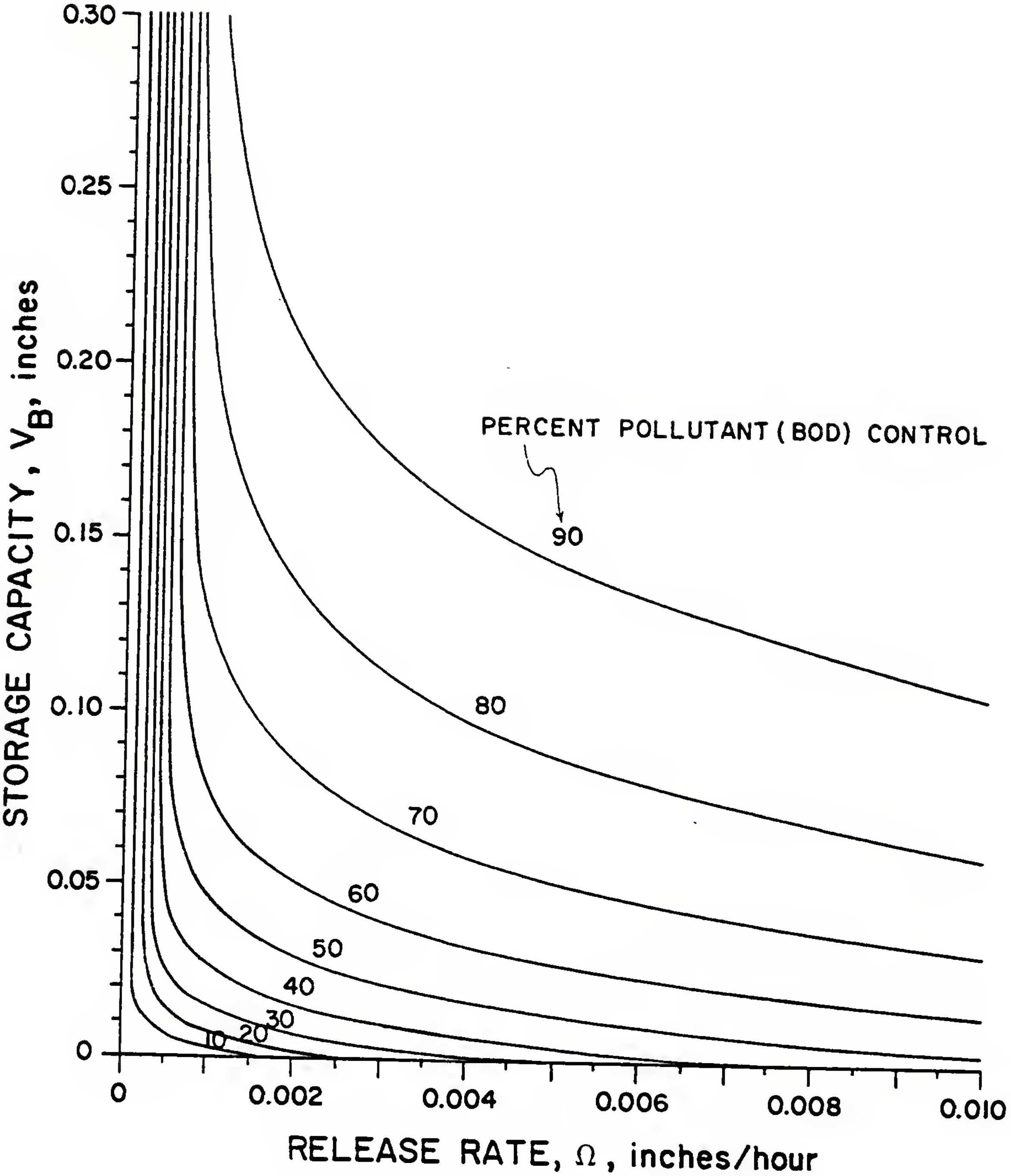


Figure 7-3. Percent Pollutant (BOD) Control Production Function, SWMM Level I.

system as an off-line configuration; but, because the release rate from the hypothetical basin is constant and operates whenever the basin contains stormwater, the two systems are essentially identical. However, STORM calculates the percent volume or pollutant control as the portion that passes through "treatment," either directly or by releases from storage. This slight difference in definitions causes the isoquants to cross the Ω axis. The definitions for volume and pollutant control given earlier imply that the basin volume is limitational. Nevertheless, the results shown in Figures 7-2 and 7-3 quickly established approximate ranges of 0.0 to 0.30 inches for V_B and 0.0 to 0.01 inches/hour for Ω. The criteria for establishing ranges were not precise but, in general, they were selected to expose most of the performance range and the "interesting" production regions (i.e., the "knees" of the isoquants). However, it will become evident that economic considerations can also help to define these ranges.

The pollution control production functions developed by Heaney et al. (1977) were based on the same first-flush assumptions used in producing the hypothetical runoff record. Because of this and the fact that the bypass mode was used, the system appears to be more efficient as a pollutant control device than as a volume control device. However, the volume control isoquants would be identical to the pollutant control isoquants if pollutant concentrations were constant or totally independent of runoff flows. This production function is based on biochemical oxygen demand (BOD) loads generated by STORM, but the analogy to suspended solids is not lost.

Hydroscience Statistical Method

This method requires the analysis of a long-term record of independent runoff events (or with some simplifying assumptions, a precipitation record). The STORM-generated runoff record was separated into independent events in accordance with the assumption that the interevent times (between event midpoints) are exponentially distributed and, thus, the coefficient of variation for interevent times is 1.0. The minimum dry period (separating independent events) most closely meeting this criterion was seven hours. Of course, the fact that the coefficient of variation is 1.0 does not guarantee that the interevent times are exponentially distributed, especially for a single year. However, a cumulative frequency plot of the interevent times, shown in Figure 7-4, demonstrates the assumption is reasonable because it closely follows the cumulative gamma distribution for $\nu = 1.0$ (i.e., the cumulative exponential distribution).

The statistics characterizing the resulting 93 independent runoff events occurring in 1971 are given below:

Parameter	Mean	Coefficient of Variation
Flow rate, q_R	Q_R = 0.018 inches/hour	$\nu_{qR} = 2.59$
Duration, d_R	D_R = 7.3 hours	$\nu_{dR} = 1.10$
Volume, v_R	V_R = 0.088 inches	$\nu_{vR} = 1.40$
Time between event midpoints (inter event time), δ_R	Δ_R = 94 hours	$\nu_{\delta R} = 0.96$

The values of Q_R and V_R are normalized over the catchment. The cumulative frequency plots for event flows, durations, and volumes are also

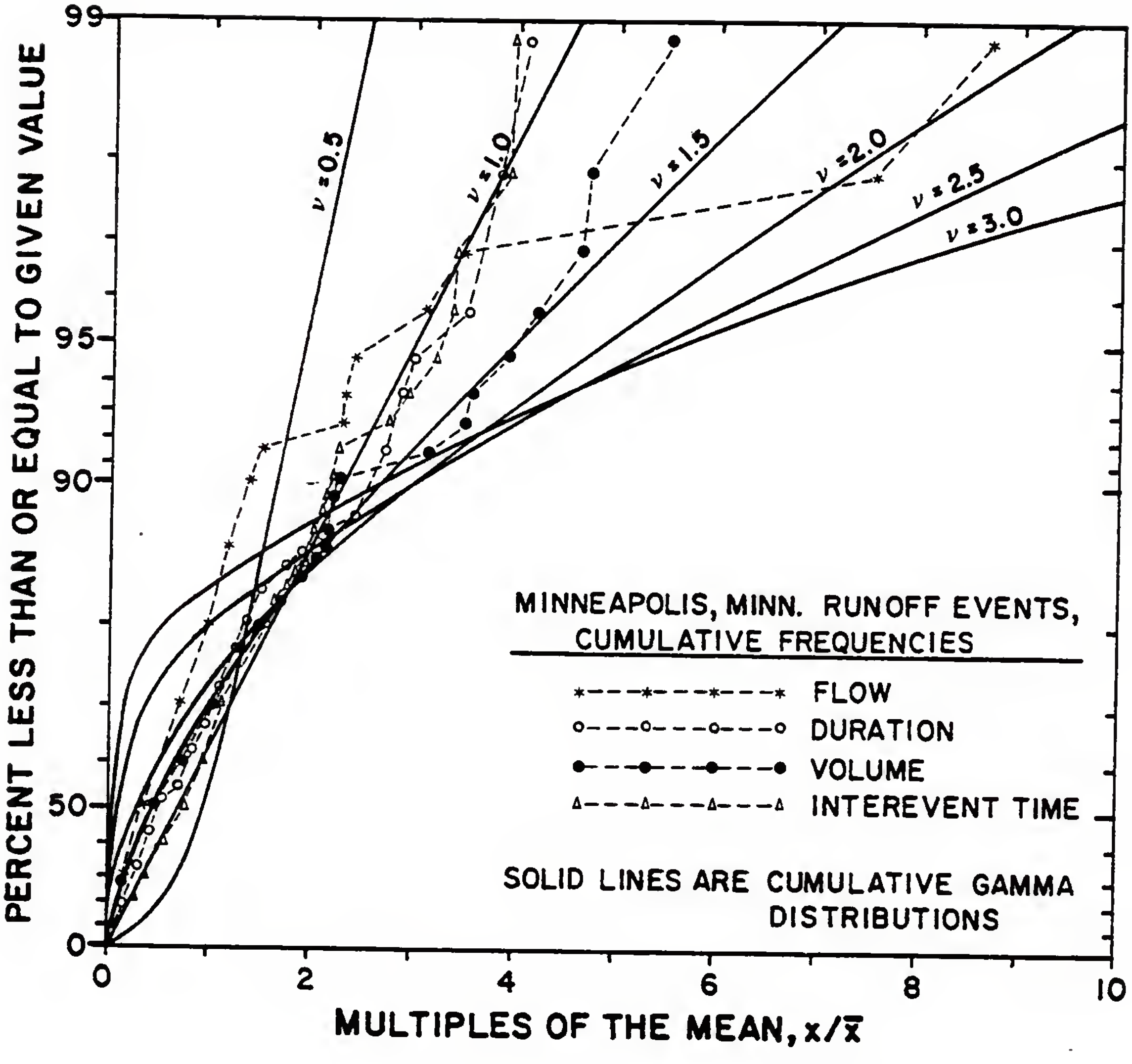


Figure 7-4. Cumulative Frequency Plots for Event Flows, Durations, Volumes, and Interevent Times and Several Cumulative Gamma Distributions.

shown in Figure 7-4. Comparison of the plot for event durations with the cumulative gamma distributions indicates reasonable agreement with the method's assumption that they are gamma distributed (see Chapter 3). The event volumes also appear to closely follow a gamma distribution, although this is not explicitly assumed by the Hydroscience method. The assumption that flow rates are also gamma distributed is not well suited in this case. This is unfortunate, because much of this method's analysis of storage/release systems is based on the assumption that flows and durations are gamma or, in some cases, exponentially distributed. The event durations approach this assumption, but the event flows are not close. Of course, the use of a single year tends to bias the statistics. In fact, an analysis of the 25-year Minneapolis precipitation record with the SYNOP program (see Chapter 3) shows that the record for 1971 is not typical in terms of the distribution of precipitation event intensities. Nevertheless, the statistical method was used to generate a preliminary estimate of the pertinent production surfaces. It is important to note, however, that the longest available record should be used in an actual analysis.

The event statistics given above, along with Figure 5-20, were used to produce the volume/pollutant control production function shown in Figure 7-5. The Hydroscience method assumes that pollutant concentrations and runoff event flows and durations are independent and thus, this set of isoquants describes the volume and pollutant control capability of the system. The most apparent feature distinguishing these isoquants from those produced by the SWMM Level I procedure is the relatively small range over which significant changes in the marginal rate of substitution occur. This occurs because the effective or

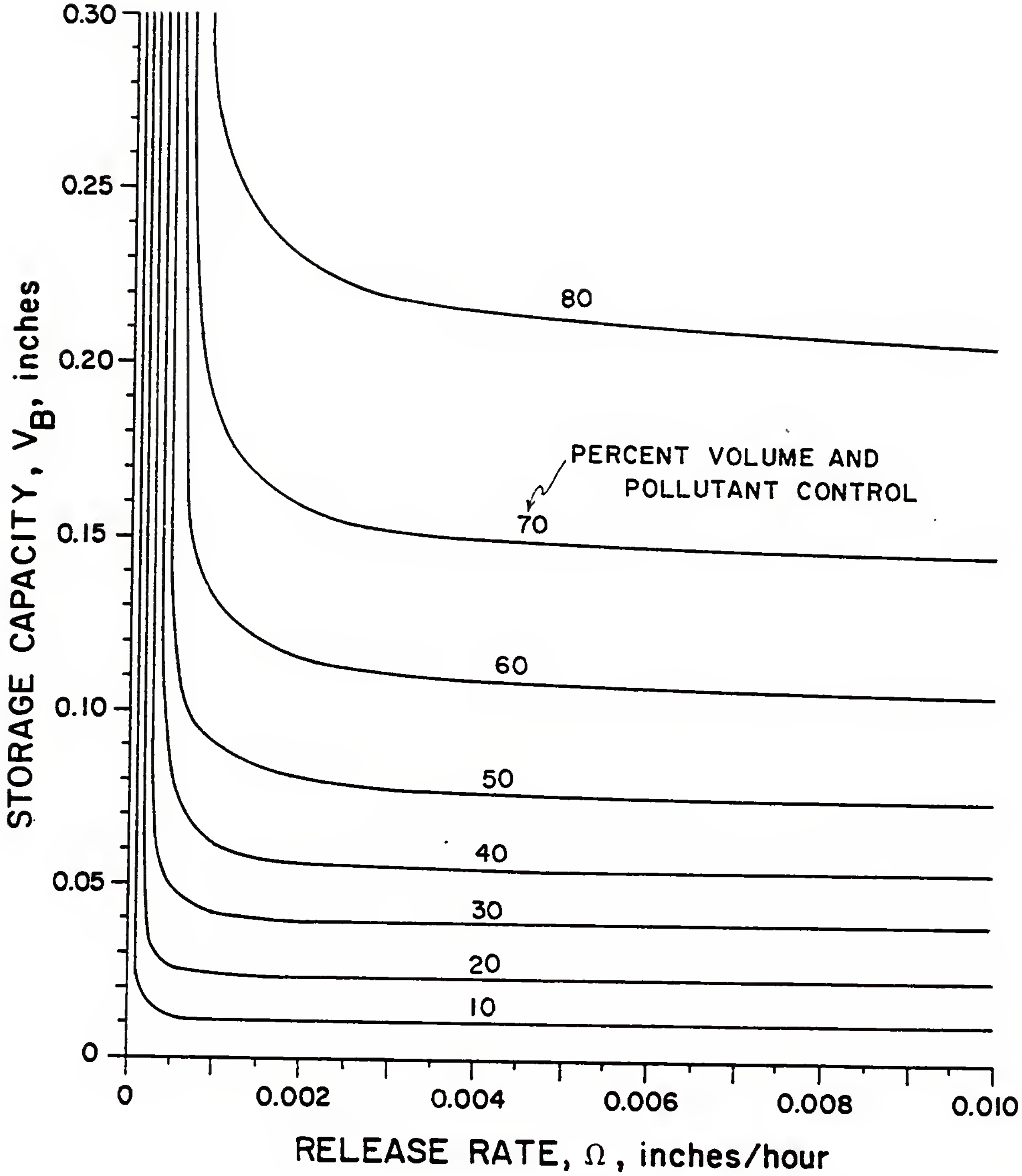


Figure 7-5. Percent Volume and Pollutant Control Production Function, Hydroscience Statistical Method.

available storage capacity at the start of any runoff event is assumed to be equal to the mean effective storage capacity. As a result, the release rate, Ω, and the maximum storage capacity, V_B, more quickly approach a marginal productivity of zero than they would in reality because the distribution of effective storage capacities is missing. Regardless, the Hydroscience method generally confirms the ranges of Ω or V_B determined by the SWMM Level I procedure.

SWMM Storage/Treatment (S/T) Block

The SWMM S/T Block was set up to receive the STORM-generated runoff record and to simulate the hypothetical system outlined earlier. The resulting production functions for volume and pollutant or total suspended solids (TSS) control are shown in Figures 7-6 and 7-7, respectively. Both figures show the combinations of V_B and Ω used to develop these curves and the associated control percentages.

The volume and TSS control results are much more like those produced by the SWMM Level I procedures than those generated by the Hydroscience method. This is not surprising since, in terms of volume or pollutant control, the storage/release algorithms in STORM and SWMM operate in a similar manner for this hypothetical system. However, because STORM is specifically designed to simulate a system very similar to the one used here, a different type of system might cause the SWMM Level I procedures to produce results very different from those generated by the S/T Block.

An interesting feature of these two production functions emerges when a plot is made of the corresponding percent TSS control-to-percent volume control ratios. Such a plot is shown in Figure 7-8. As expected

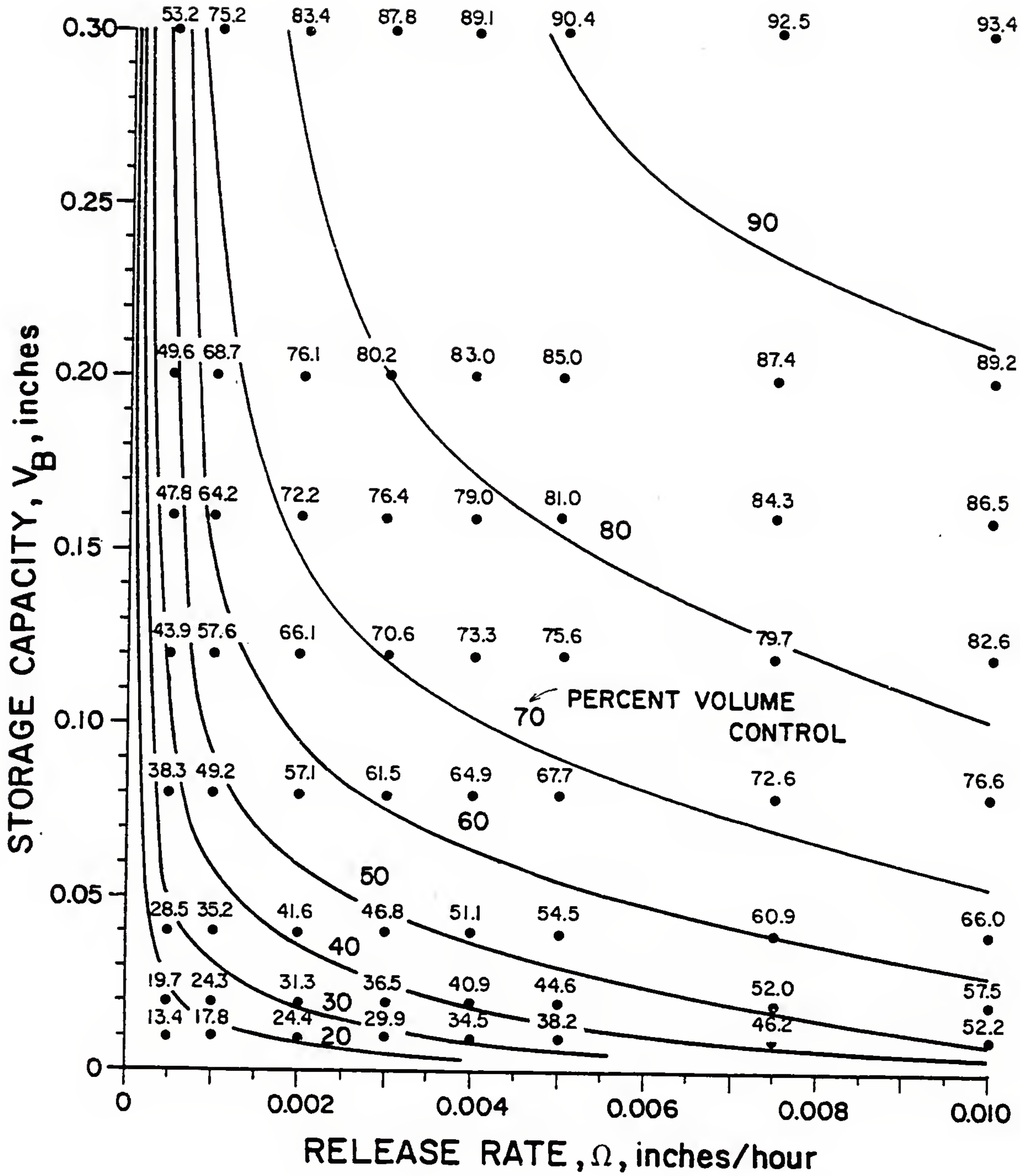


Figure 7-6. Percent Volume Control Production Function, Results of S/T Block Simulation.

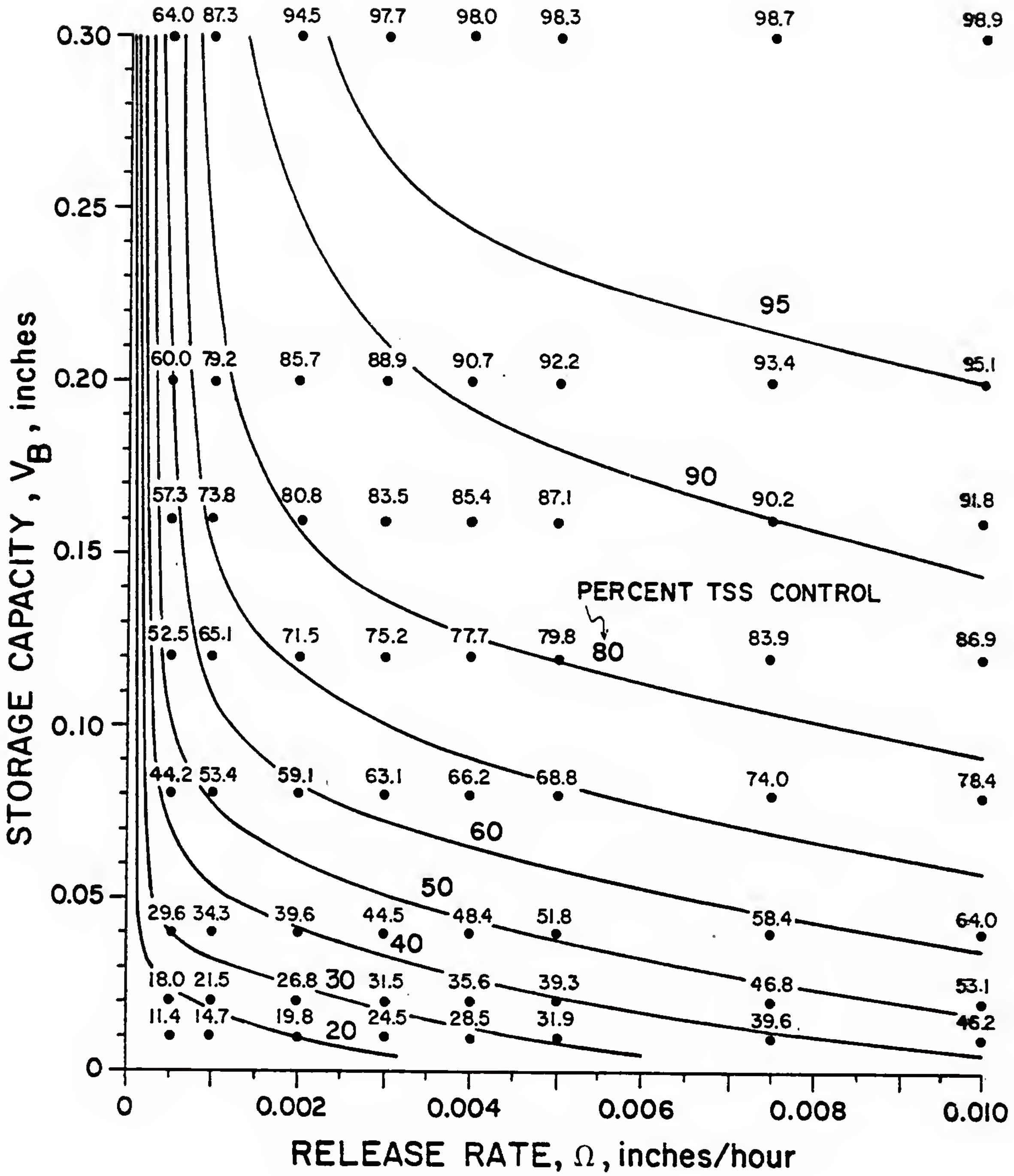


Figure 7-7. Percent TSS Control Production Function, Results of S/T Block Simulation.

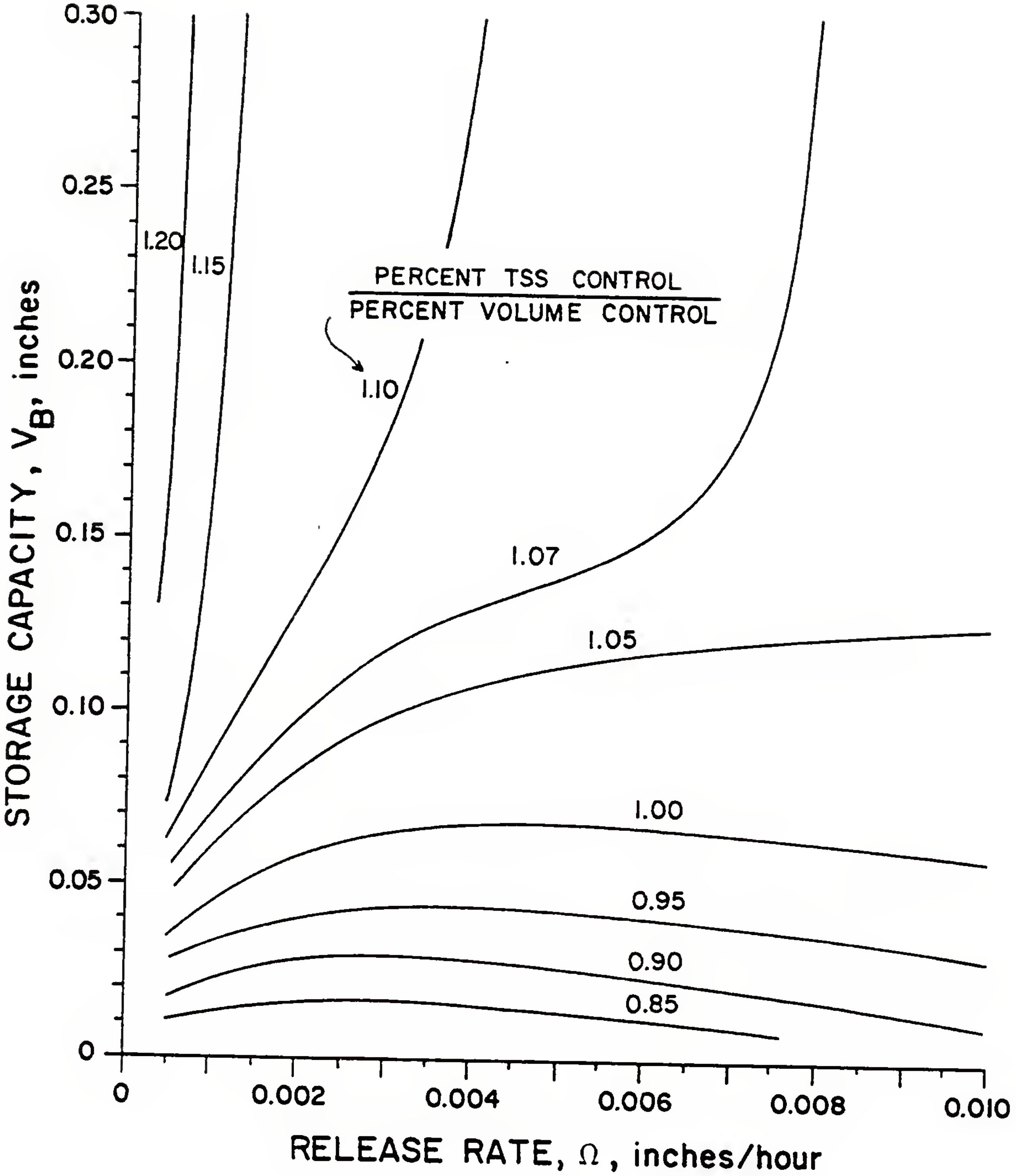


Figure 7-8. Percent TSS Control-to-Percent Volume Control Ratios.

with the first-flush assumption used in generating the runoff record, the system is more efficient at capturing the TSS load than the runoff volume over large ranges of V_B and Ω. Interestingly, a region exists where the reverse is true. This occurs when the value of V_B/Ω is relatively low because the relatively high release rates are relieving the basin fast enough to allow it to capture more of the less concentrated loads in the later stages of many events. In addition, as the release rate is increased for a particular storage capacity in this region, the efficiency decreases until a point is reached where the release rate's effect on the provision of more initial storage capacity begins to offset the effect of capturing the later runoff event stages. Regardless of the region, movement in any direction away from the origin will eventually reach a point where this ratio is 1.0. This occurs when the storage capacity and/or release rate is large enough to capture all runoff flows (and, thus, all TSS loads).

As stated previously, the basin was modeled as a plug-flow reactor with a simple first-order removal mechanism representing the settling of suspended solids (see Chapter 4), i.e.,

$$R_{TSS} = R_{max}[1 - \exp(-K\theta_j)] \qquad (7\text{-}2)$$

where R_{TSS} = total suspended solids removal fraction, $0 \leqq R_{TSS} \leqq R_{max}$,

R_{max} = maximum removal fraction,

K = rate constant, $\sec^{-1}$, and

θ_j = detention time of plug j, sec.

In this case, R_{max} represents the settleable portion of the TSS load. The calibrated results for total suspended solids in the Humboldt Avenue detention basin described in Chapter 4 produced the values $K = 0.00003$

sec^{-1} (0.108 $hour^{-1}$) and $R_{max} = 0.50$. However, this basin serves a combined sewer area and the magnitude and settleability of pollutant loads are quite different for combined sewage and surface runoff alone (Lager et al., 1977; Huber et al., 1981; E. D. Driscoll et al., 1981). Thus, the values of K and R_{max} can also be very different. Whipple and Hunter (1981) studied the settleability of several pollutants in surface runoff for five urban catchments in New Jersey. Their results for total suspended solids are shown in Figure 7-9. Interestingly, the value of K determined for the Humboldt Avenue basin appears to be appropriate. However, the value of R_{max} is probably closer to 0.75. While these results are not conclusive, they do suggest that a value of 0.00003 sec^{-1} for K is representative for suspended solids in stormwater-related flows. This value corresponds to a detention time of 21.3 hours to remove 90 percent of the settleable material.

The values of $K = 0.00003$ sec^{-1} (or 0.108 $hour^{-1}$) and $R_{max} = 0.75$ were used to develop the TSS removal production function shown in Figure 7-10. These results do not clearly reveal the area of substitution and, thus, more runs were made at very low values of Ω and the range of 0 to 0.0025 inches/hour expanded in Figure 7-11. The results show that the marginal product of Ω becomes negative (and the total product decreases) as the release rate is increased for any particular storage capacity. The value of Ω at which this occurs is higher for increasing values of V_B. Increasing the release rate beyond this value allows more of the total load to be captured or "controlled" (see Figure 7-7) but this factor is overwhelmed by the loss of detention capacity.

When the marginal product of Ω is zero, the average load-weighted detention time for all TSS loads (including those bypassing the basin)

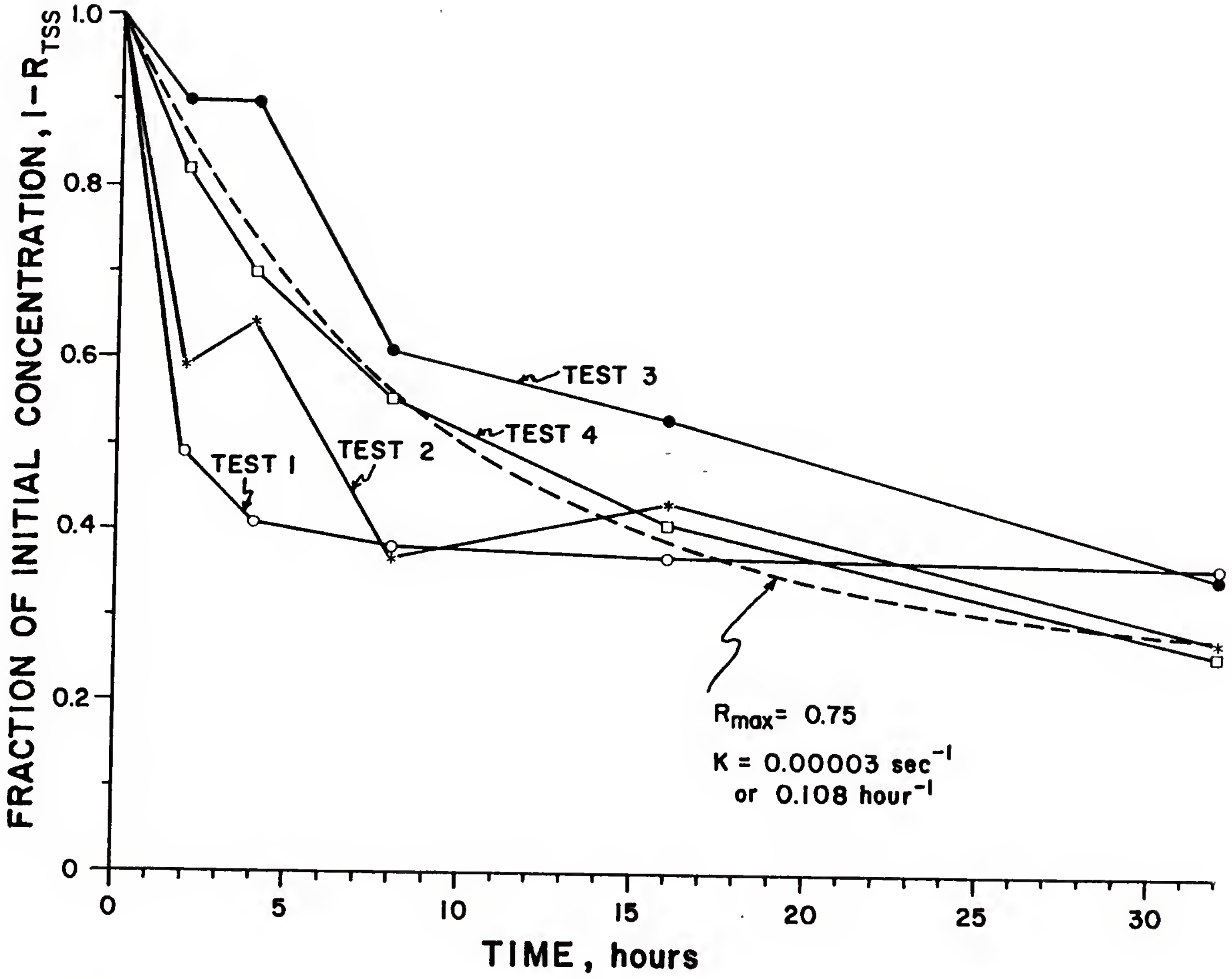


Figure 7-9. Results of Settleablility Tests for Total Suspended Solids (Whipple and Hunter, 1981).

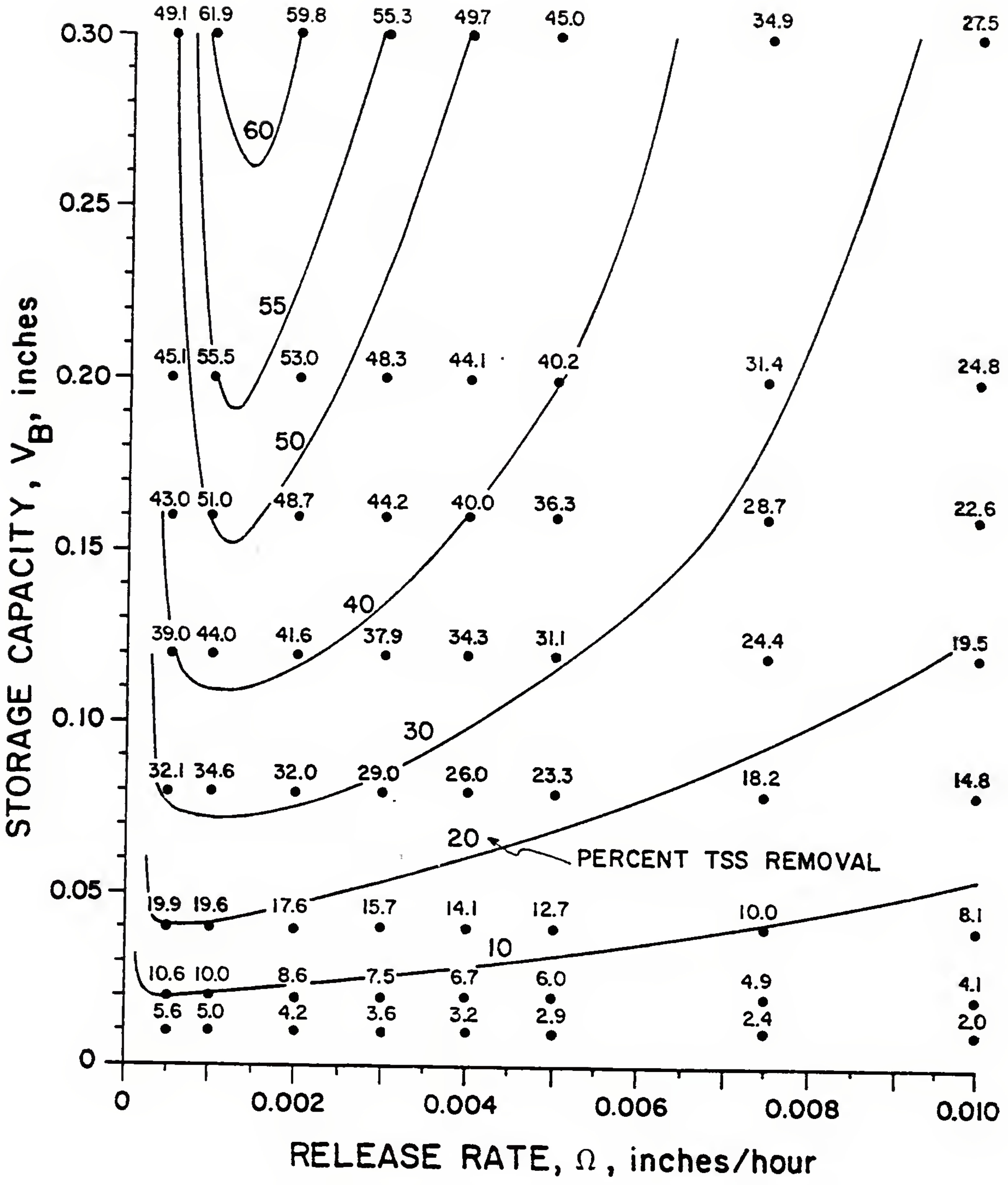


Figure 7-10. Percent TSS Removal Production Function, Results of S/T Block Simulation.

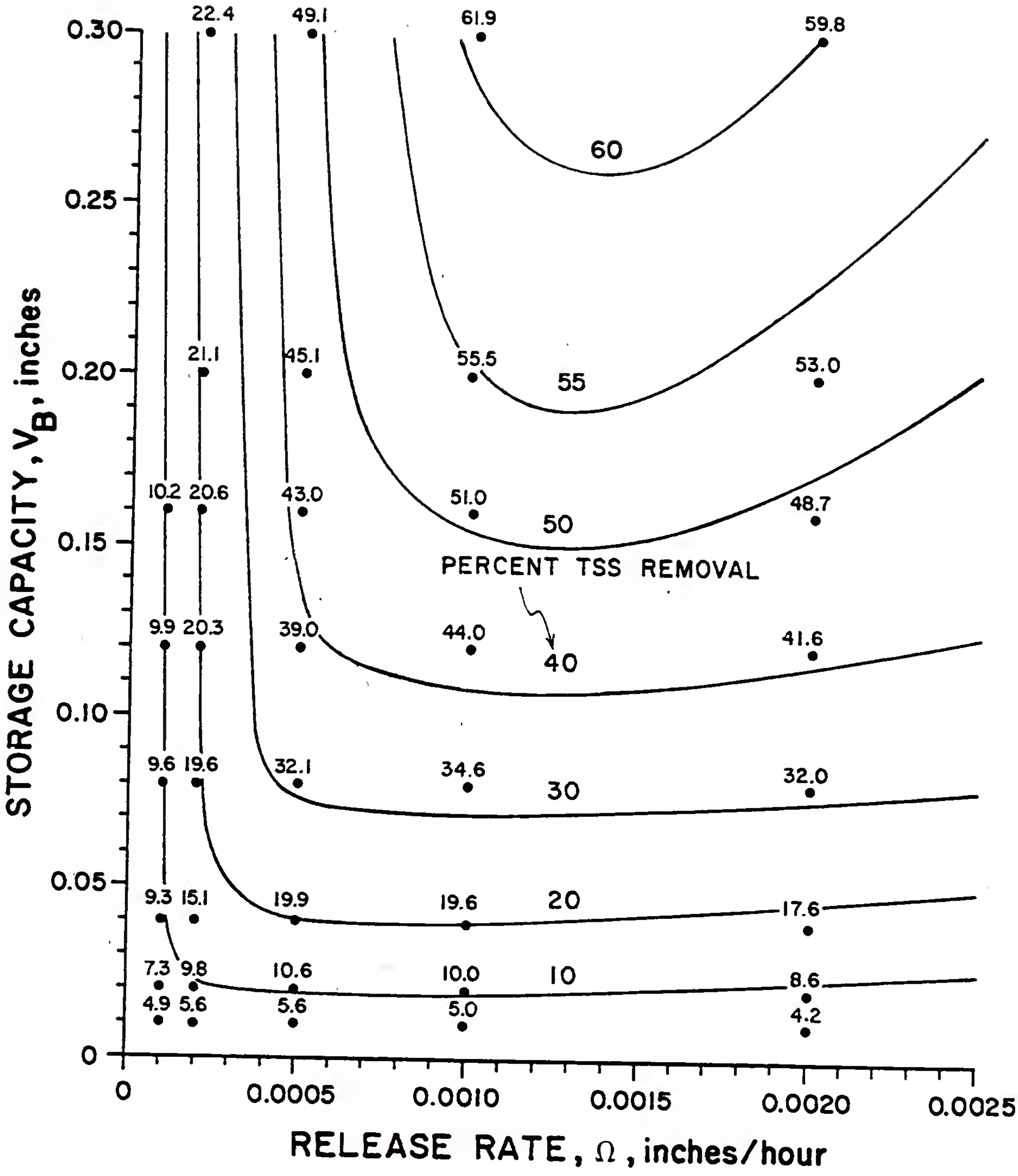


Figure 7-11. Percent TSS Removal Production Function, Results of S/T Block Simulation (Enlarged Range).

is at its maximum. This is true because equation 7-2 is monotonically increasing and the average detention time is a weighted sum of this equation applied to each plug. However, each TSS removal isoquant does not correspond to an average detention time isoquant because of the nonlinearity of equation 7-2. The average flow-weighted detention time may not be at its maximum at this same point because of the first-flush assumption. Conceptually, the average load-weighted detention time is similar to the definition presented in Chapter 2 (i.e., equation 2-41) but instead of applying it only to the loads passing through the basin, it is applied to all loads (with the bypassed loads having a detention time of zero). Since plug flow is assumed, the average detention time for loads passing through the basin always decreases when the release rate increases. This is evident in Figure 7-12 where the percent TSS removal-to-percent TSS control ratios are plotted. A similar trend is seen for the percent TSS removal-to-percent volume control ratios (see Figure 7-13). However, because of the first-flush assumption, this ratio is higher than the corresponding TSS control/volume control ratio in the region where the TSS control/volume control ratio is greater than 1.0. The reverse is true when the TSS control/volume control ratio is less than 1.0 (see Figure 7-8).

Inspection of Figure 7-13 and the method used by Howard et al. (1979) to produce a similar TSS removal production function emphasizes an important point. The method of Howard et al. determines the pollutant removal isoquants by multiplying the percent volume control by a specific overall removal fraction that is identical for all combinations of V_B and Ω with the same value of V_B/Ω. In other words, the curves in Figure 7-13 are replaced with a series of rays emanating from the origin. What

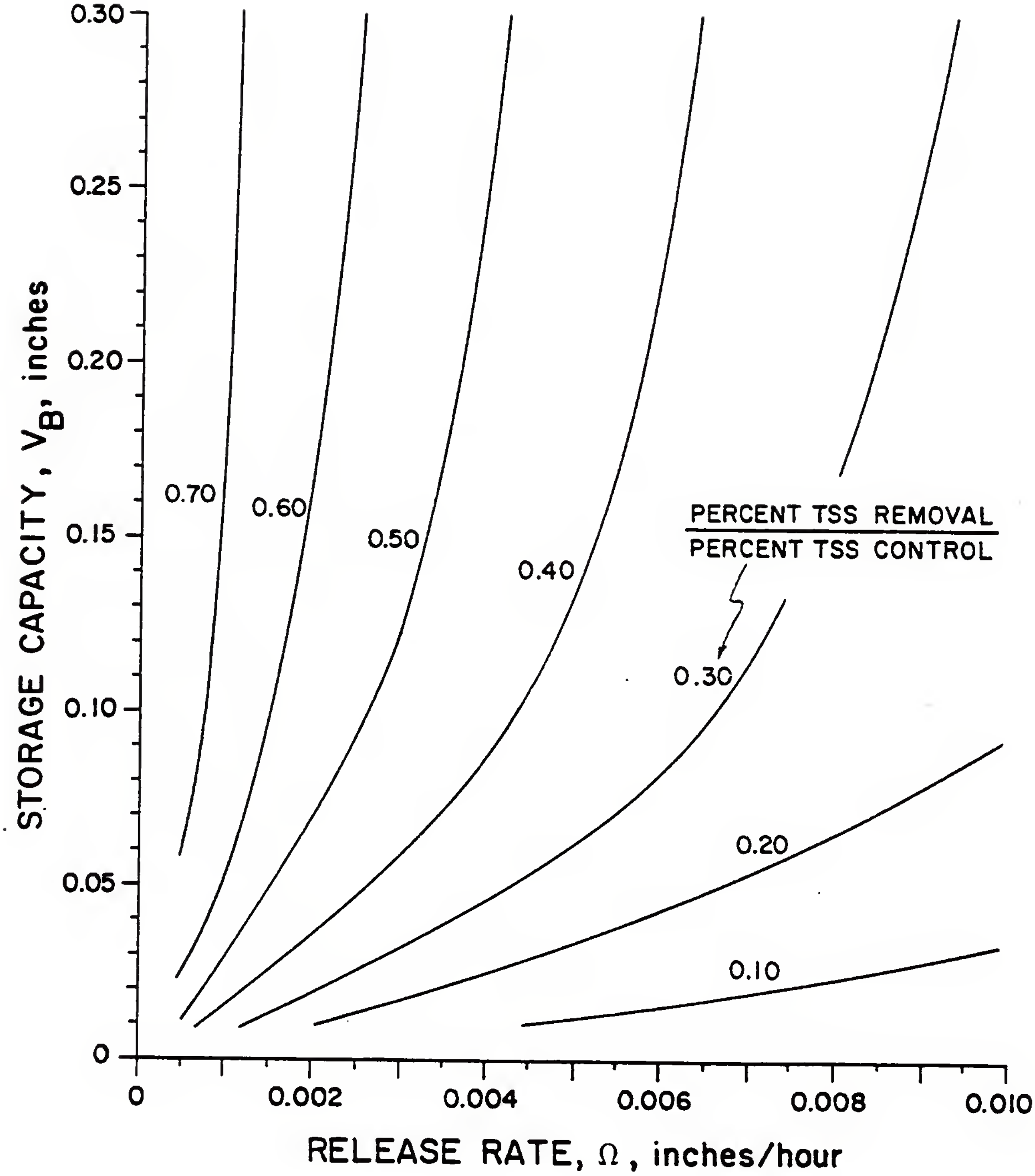


Figure 7-12. Percent TSS Removal-to-Percent TSS Control Ratios.

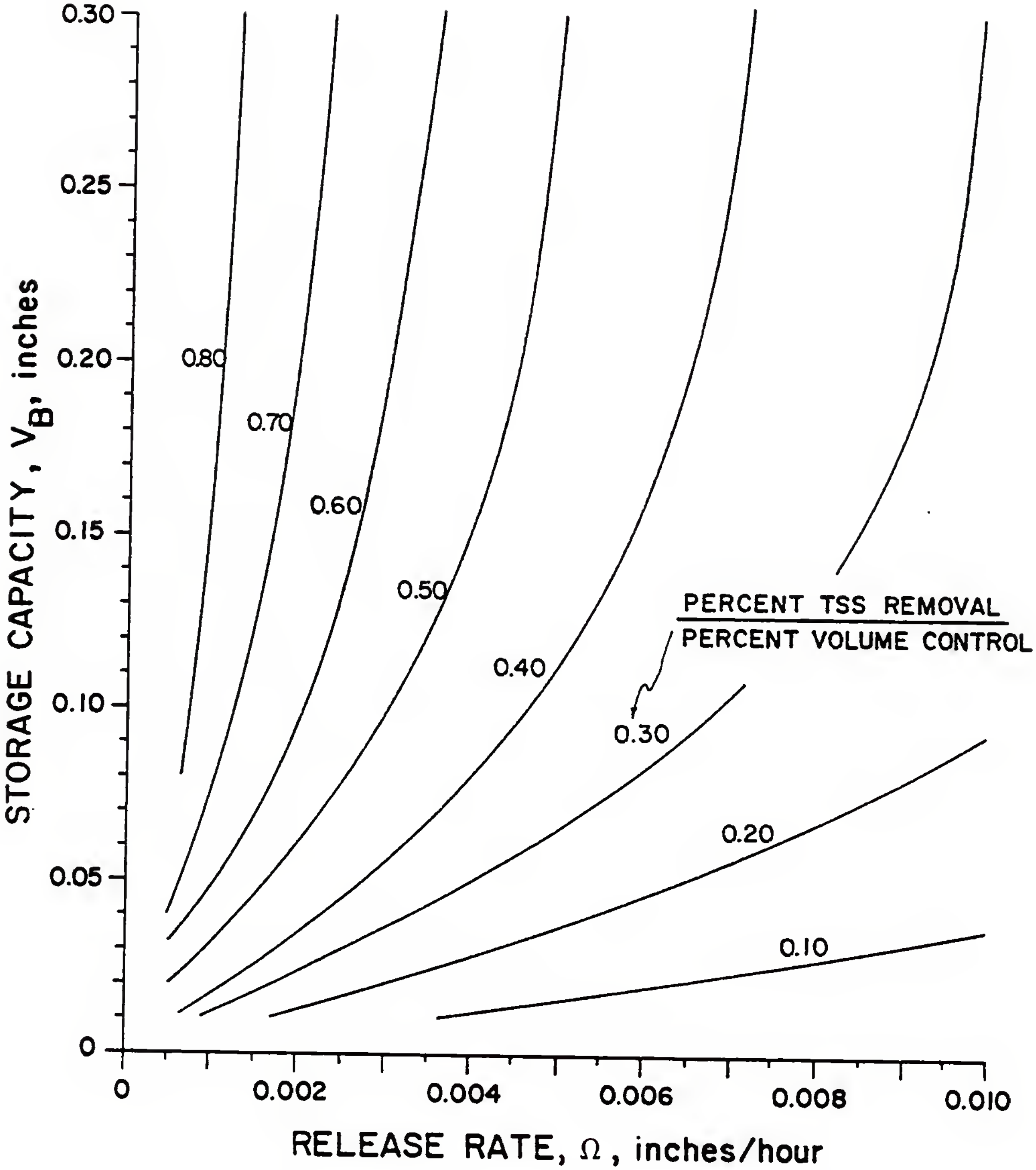


Figure 7-13. Percent TSS Removal-to-Percent Volume Control Ratios.

the method of Howard et al. fails to account for is the fact that a change in V_B does not necessarily have the same effect on pollutant removal as an inversely proportional change in Ω. For example, doubling V_B (and holding Ω constant) allows more flow parcels or plugs to be queued, but it does not increase the detention time of the plugs that would have been queued through a basin with the original capacity. Halving Ω allows fewer plugs to be queued through the basin and increases the detention time of the fewer number of queued plugs. These two effects are quite different and this is evident in Figure 7-13.

The output from the S/T Block was also linked with the Statistical Block of SWMM to generate a cumulative distribution of the hourly TSS loads (in lbs/hour) released and bypassed by the system. The amount of time (as a percent of the year) that loads of 50, 100, and 500 lbs/hour are each exceeded is plotted for a limited number of storage capacity/ release rate combinations in Figure 7-14. The Statistics Block is relatively expensive to run and, thus, the number of runs was restricted. Because of the limited array of points, no attempt was made to construct isoquants. Two of the three target loads (i.e., 50 and 100 lbs/hour) experience regions in which the presence of the storage/ release system actually creates a larger number of hours in which the specified load is exceeded. This is not surprising when one considers the effect of dispersing large single-hour loads over several hours through the release mechanism. This type of information could be useful in analyzing receiving water impacts. In its simplest form, the load from the system could be linked to a critical point in the receiving water through a linear transfer coefficient that relates the load to a concentration at the point of interest. Alternatively, the S/T Block results could be linked

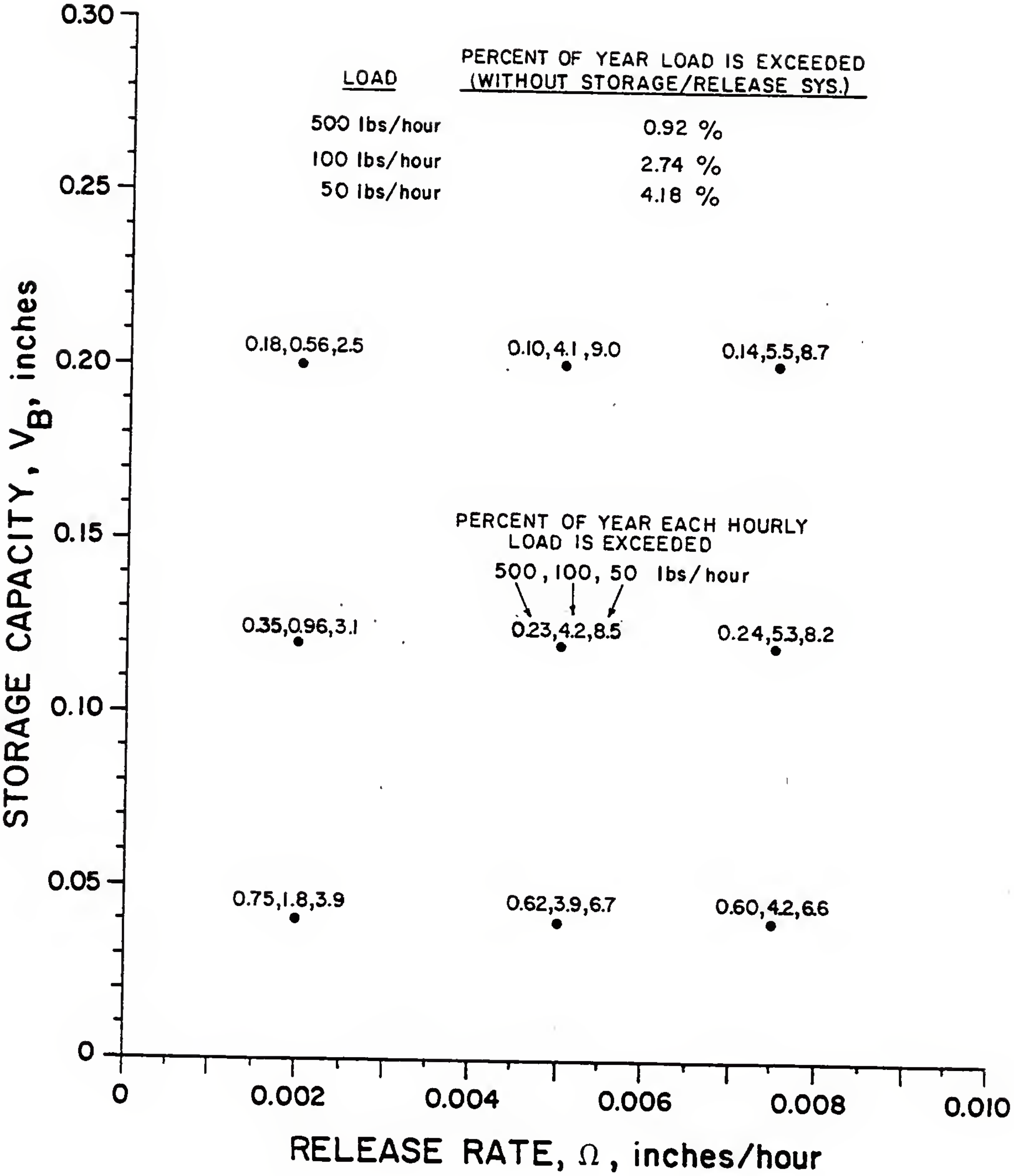


Figure 7-14. Percent of the Time that the Hourly TSS Load from System Exceeds 50, 100, or 500 lbs/hour.

to a receiving water model or the results of the Statistics Block used in a statistical analysis of receiving water impacts.

Variations on the Theme (with the S/T Block)

The pollutant routing mechanism was changed to the completely mixed mode to generate the TSS removal production function shown in Figure 7-15. Obviously, completely mixed conditions are not ideal for TSS removal, but this is a useful comparison of the two modes. In order to directly compare these results and the results for the plug-flow assumption, the rate constant, K, was set at 0.00003 $\sec^{-1}$ and the results multiplied by 0.75 to account for R_{max}. Figure 7-15 shows that a completely mixed basin is not as efficient as a plug-flow basin (see Figure 7-10). This occurs because the parcels of water entering the basin are immediately dispersed, and some exit with little or no detention. This effect is particularly severe because releases occur at all depths. Goforth (1981) demonstrated that the dilution effect provided by a minimum pool is beneficial. Of course, some treatment processes require mixing to provide conditions which lead to an overall improvement in performance (e.g., activated sludge).

The original scenario was formulated with the bypass mode in place. This was removed and replaced with an overflow weir. The TSS removal production functions for the plug-flow and completely mixed basins are shown in Figures 7-16 and 7-17. The weir was modeled with the discharge formula for rectangular weirs (Streeter and Wylie, 1979), i.e.,

$$Q_w = 3.33\, L_w (d_e - d_{eo})^{1.5} \qquad (7\text{-}3)$$

where Q_w = discharge over the weir, ft^3/sec,

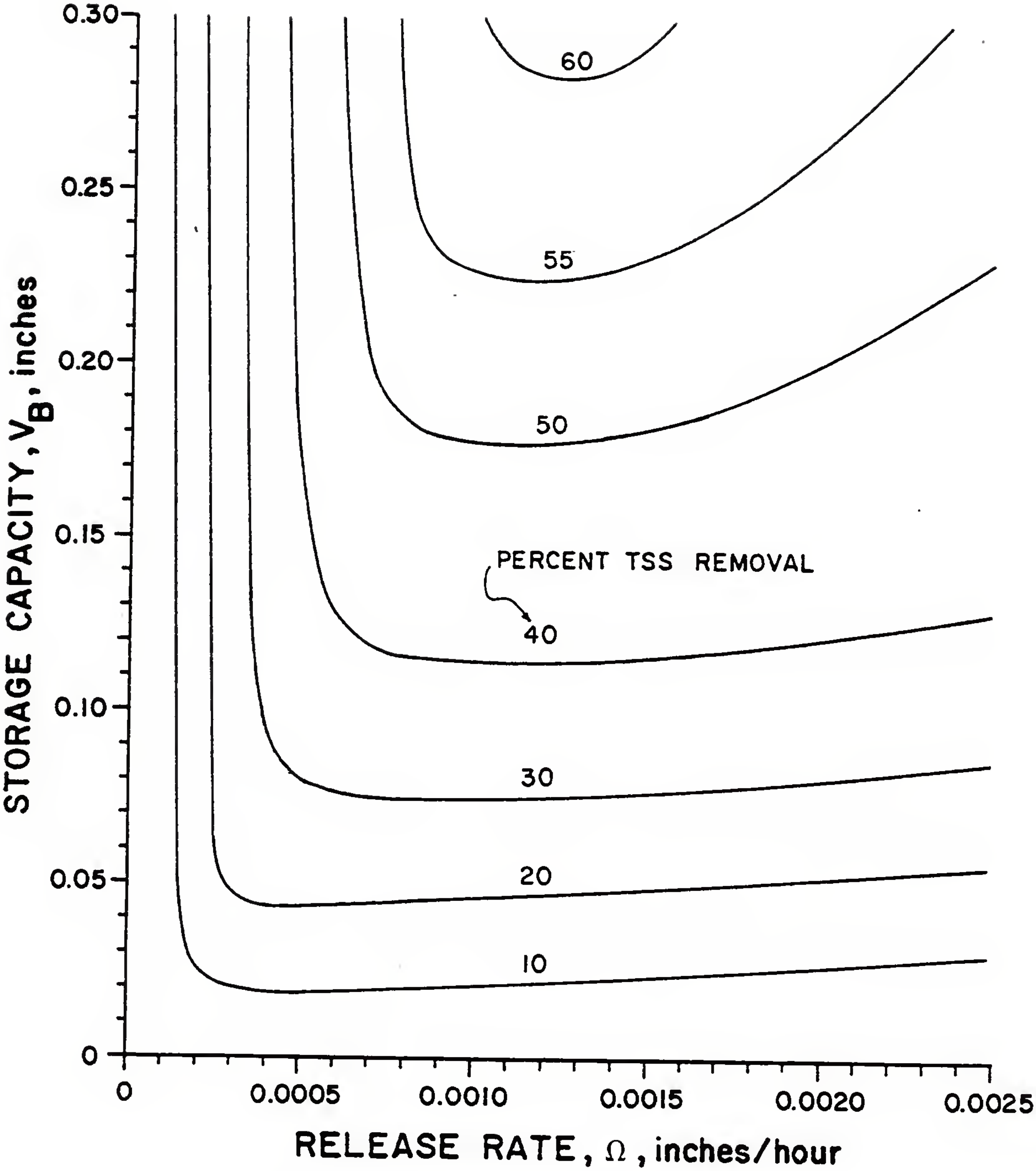


Figure 7-15. Percent TSS Removal Production Function, Completely Mixed Storage Basin with Bypass.

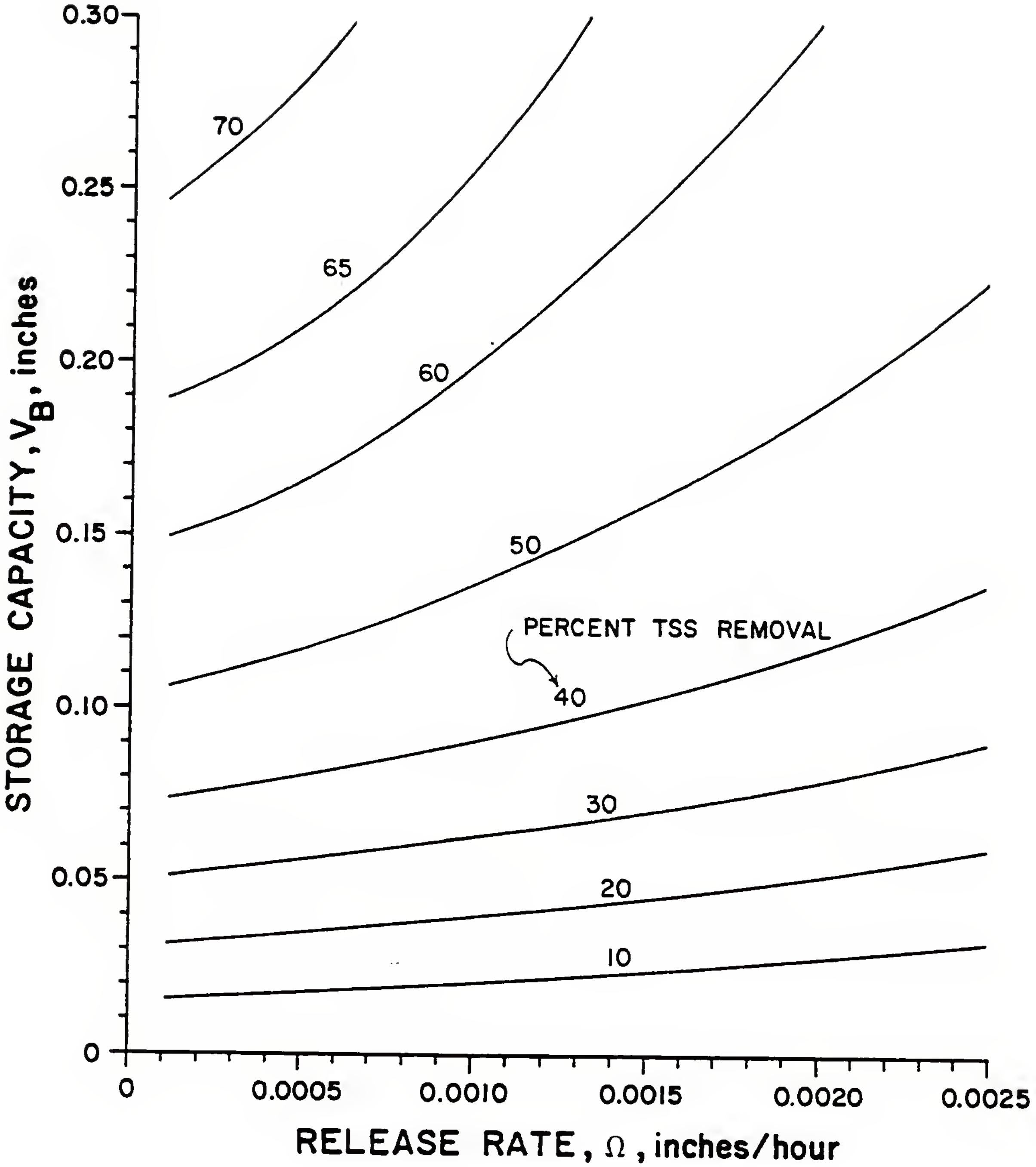


Figure 7-16. Percent TSS Removal Production Function, Plug-Flow Storage Basin without Bypass.

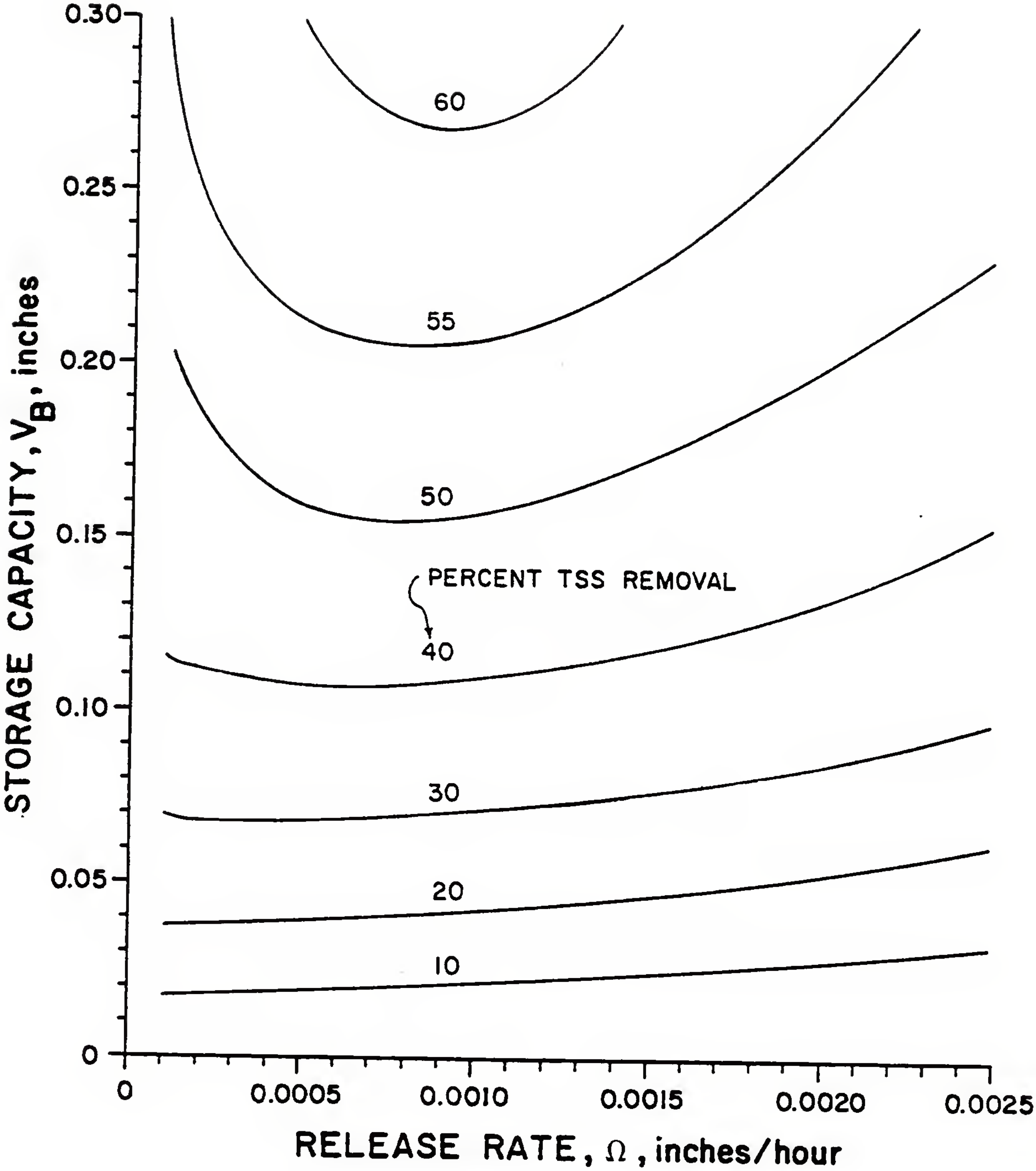


Figure 7-17. Percent TSS Removal Production Function, Completely Mixed Storage Basin without Bypass.

L_w = length of the weir, ft,

d_e = depth of water in the detention basin, ft, and

d_{eo} = depth of water below which no discharge occurs, ft.

The weir length was assumed to extend over the width of the basin. The basin width was calculated for each value of V_B by assuming a length-to-width ratio of 3.0. The basin was given 2 feet of freeboard in all cases. The value of d_{eo} is equivalent to the original depth of 10 feet. Strictly, the discharge over the weir should be accounted for in the value of Ω in some manner, but the weir was always large enough to cause the outflow to roughly equal the inflow during the periods of overflow. In addition, the weir was large enough in each case to pass the flows without exceeding the freeboard. Thus, the constant release rate is the only release mechanism affecting performance. The results for the plug-flow assumption (Figure 7-16) show that the marginal product of Ω is always negative. The reason for this is directly related to the nature of plug flow. Without the constant release rate, the plugs still in the basin when the overflow ceases are held until the next storm. Thus, as a group, they have longer detention times than they would with the constant release mechanism. The flows from the next storm are simply queued through the basin behind the plugs remaining from the last storm. The large weir causes the remaining plugs to have virtually no effect on the ability of the basin to capture and detain flows from the new storm. In other words, the old plugs are "pushed" out as the storage capacity is needed.

The production function for the completely mixed assumption shows somewhat different results (Figure 7-17). In this case, there is a region where the marginal product of Ω is positive. Routing all flows through the basin creates a situation in which it is possible to increase Ω and delay the expulsion of water parcels by providing an initial capacity storage. However, the effect is only noticeable for values of V_B that can provide significant levels of effective storage in the first place.

Sensitivity of the S/T Block to Settleability Characteristics

The original scenario was formulated with the value of K at 0.00003 sec^{-1} and $R_{max} = 0.75$. These values were determined from the calibrated Humboldt Avenue results discussed in Chapter 4 and results presented by Whipple and Hunter (1981) for several urban catchments in New Jersey. They probably represent a fairly typical case and form the basis for investigating the sensitivity of the S/T Block (and the basin under consideration) to changes in K. Since R_{max} is a constant multiplier, the results change only by the ratio of the new and old values. Thus, it is not important in this analysis and it is left at 0.75 for all situations.

The analysis was carried out for an order of magnitude in either direction from the original value of K, i.e., K = 0.0003 sec^{-1} and 0.000003 sec^{-1}. The resulting first-order removal equations (equation 7-2) are plotted in Figure 7-18. The value of K = 0.0003 sec^{-1} corresponds closely to the well-known curve presented by Fair et al. (1968) for the removal of suspended solids found in sanitary sewage by sedimentation when $R_{max} = 0.65$. The resulting production functions for K = 0.0003, 0.00003, and 0.000003 sec^{-1} are shown in Figures 7-19, 7-20, and 7-21, respectively. The results for K = 0.00003 sec^{-1} are repeated from Figure 7-10.

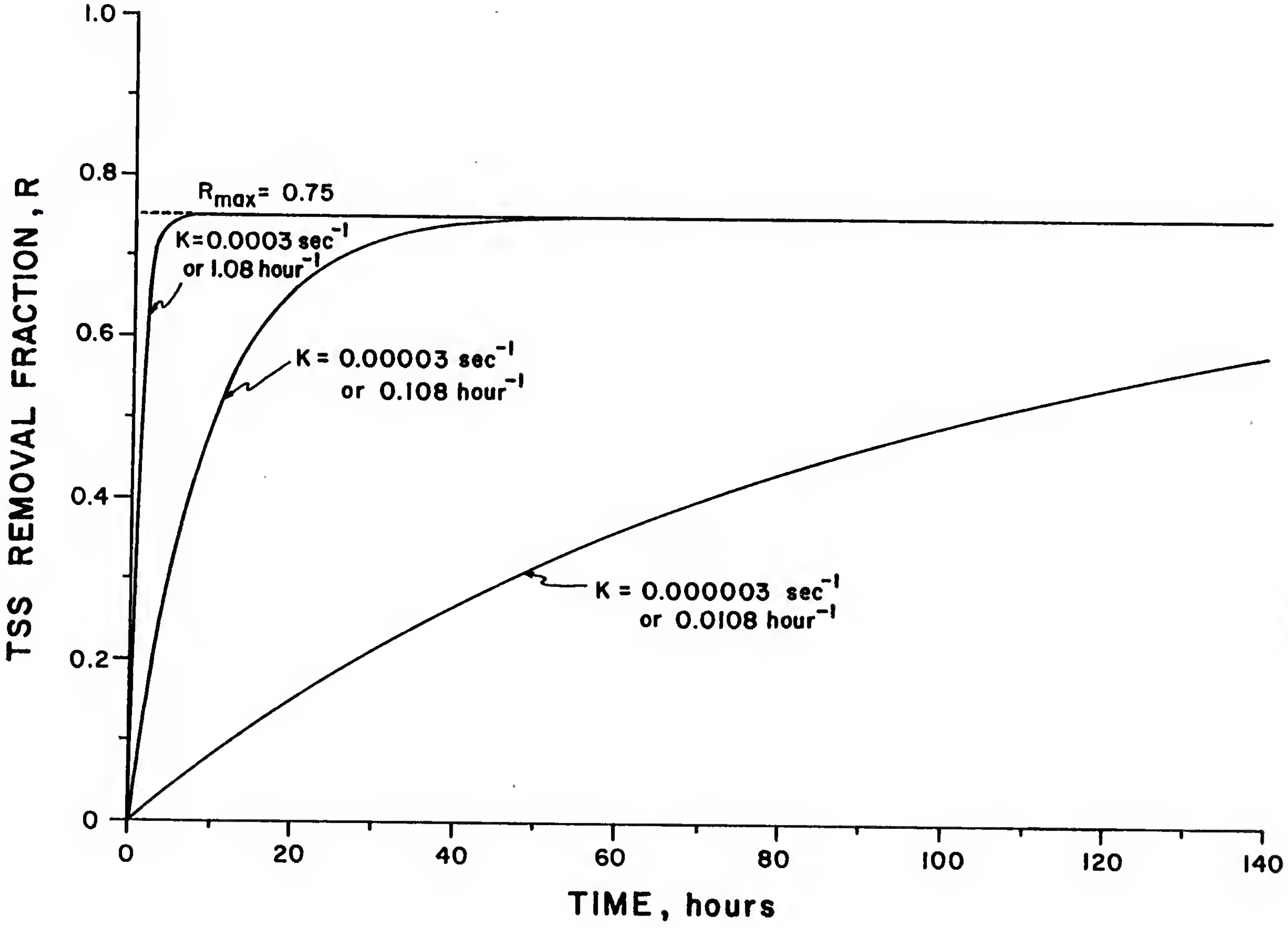


Figure 7-18. First-Order TSS Removal Equations.

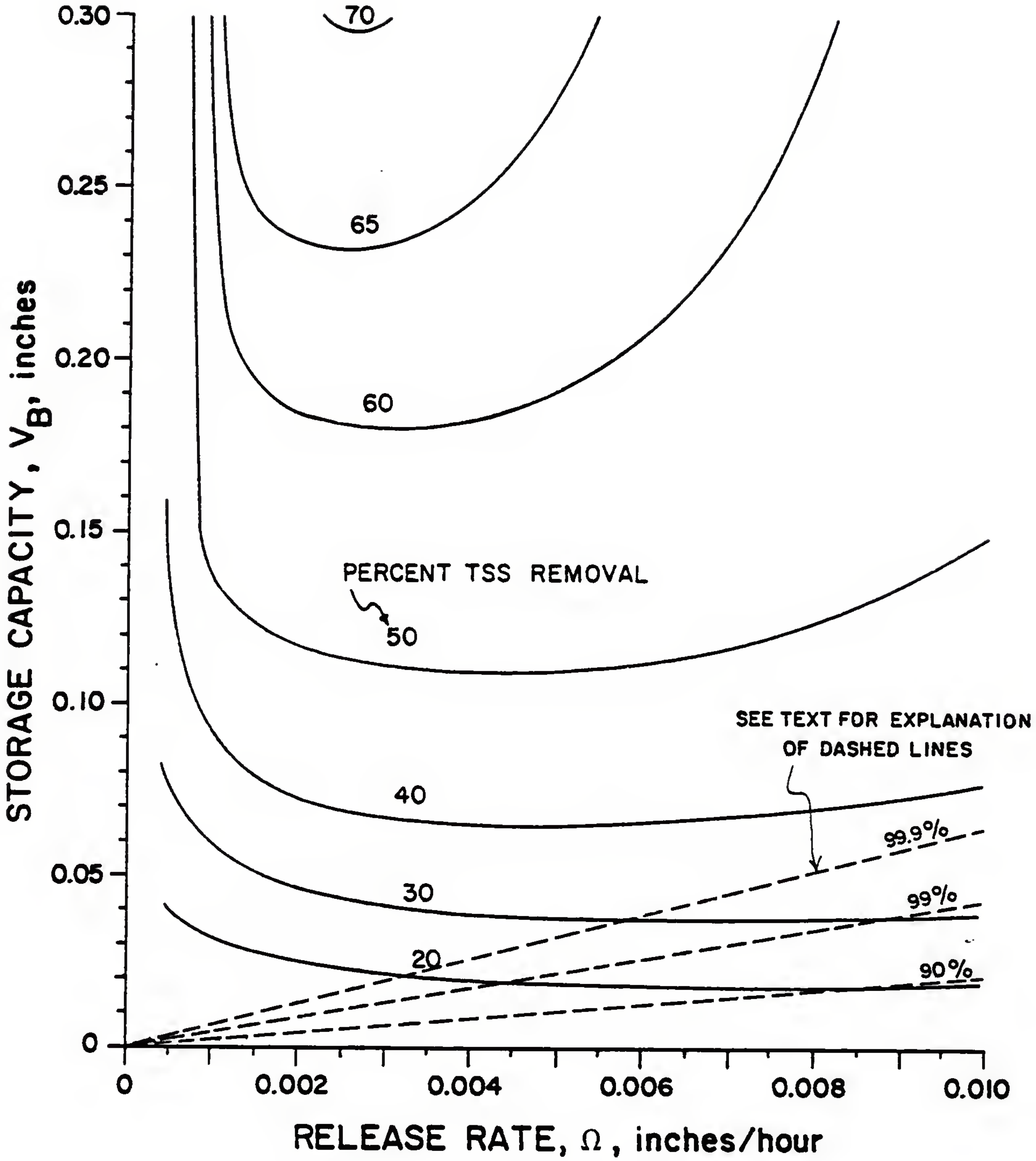


Figure 7-19. Percent TSS Removal Production Function, $K = 0.0003 \text{ sec}^{-1}$.

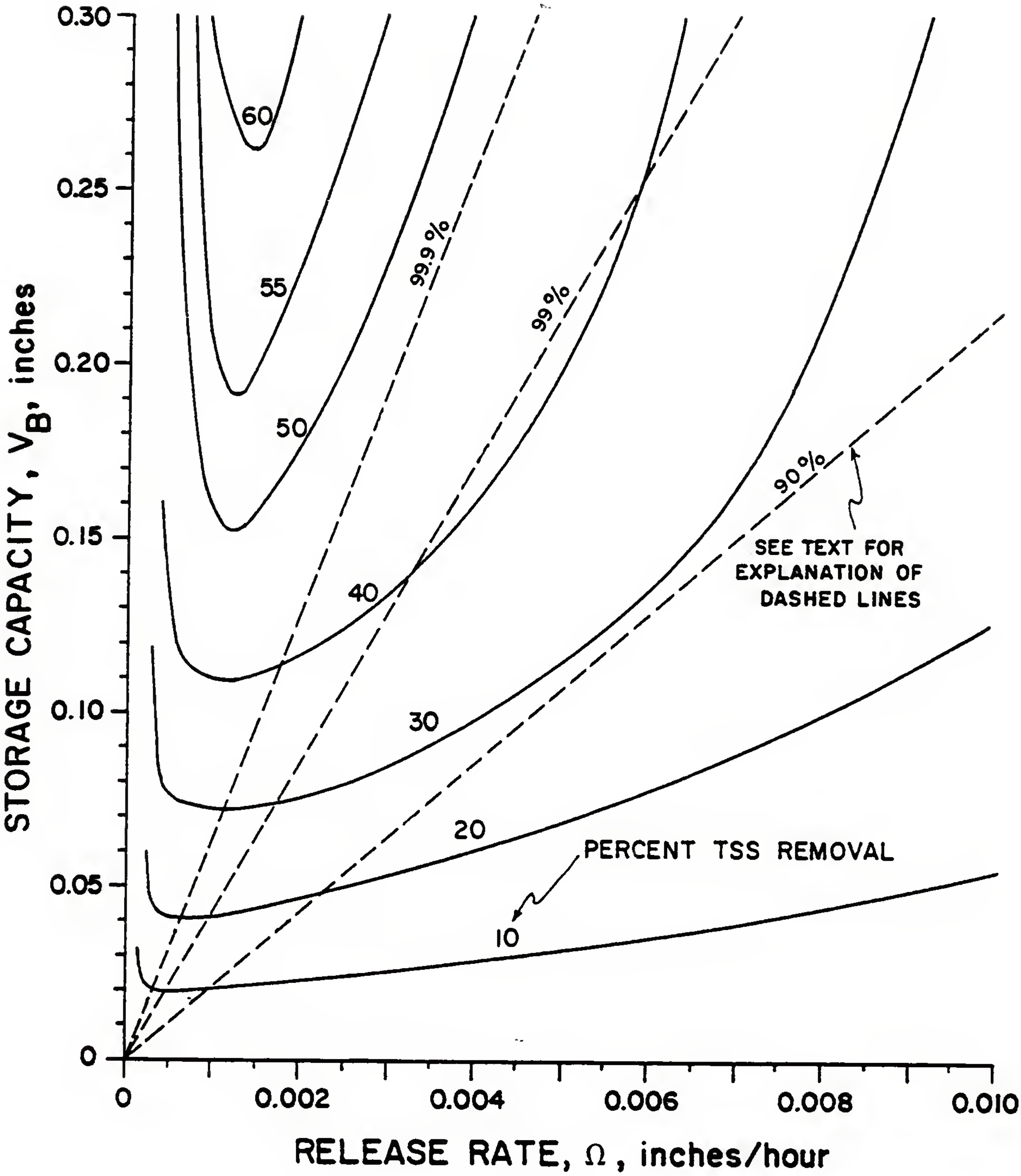


Figure 7-20. Percent TSS Removal Production Function, $K = 0.00003 \text{ sec}^{-1}$.

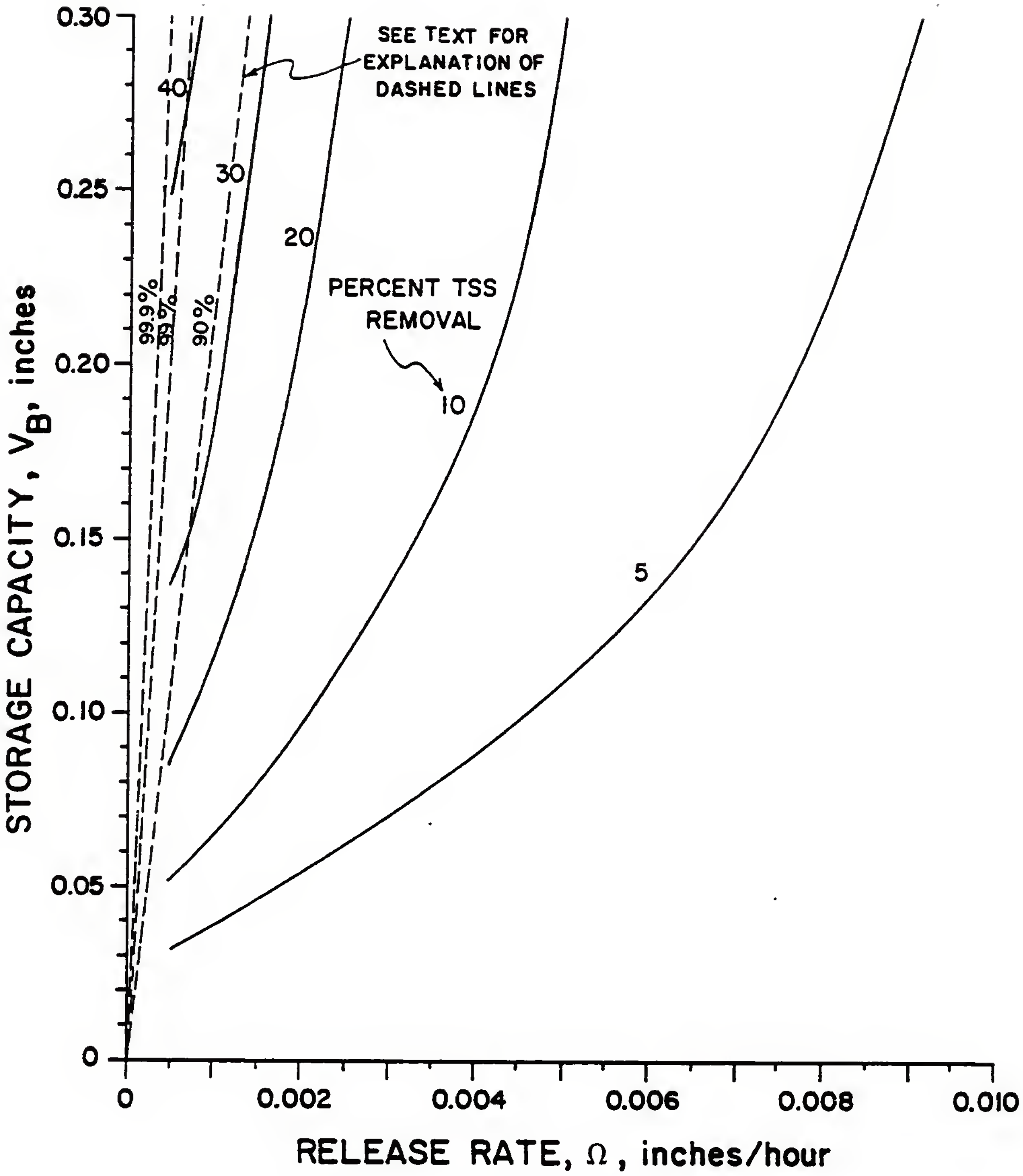


Figure 7-21. Percent TSS Removal Production Function, $K = 0.000003 \text{ sec}^{-1}$.

Each production function shows a radically different area of substition, with the decreasing value of K reducing the region. This region is the primary area of interest because, for nearly all input cost functions, the optimal solution for any level of removal will be located here. A useful design rule would serve to eliminate some of the areas outside this region before any modeling begins. One readily determined value is the time it takes to empty a full basin (i.e., the drawdown time). Also easily determined from a settleability or treatability test is the time it takes to reach any particular percent of the maximum removal fraction, R_{max} (e.g., 90 percent of 0.75, see Figure 7-18). These two values were combined to determine if they could be useful in this task. Several rays are shown in Figures 7-19, 7-20, and 7-21. Each represents the drawdown time (i.e., V_B/Ω) associated with a particular percentage of the maximum removal fraction, R_{max}. From these results, it appears as though the region to the left of the 90-percent ray always contains the area of substitution. In fact, it can probably be said that for all but very low removal levels, the analyst needs to be concerned only with the region to the left of the 99.9-percent ray. This is a significant result because it substantially reduces the simulation region--this would have been appreciated in the earlier phases of this application. Of course, this analysis was performed with the plug-flow assumption and the bypass mode, but Figures 7-15, 7-16, and 7-17 suggest that this general rule will encompass the area of substitution under a variety of conditions. As a "rough" design rule, it can probably be said that the optimal release rate for a basin with a given volume will be less than the rate it takes to empty the filled basin in the time dictated by the 90 percent (or perhaps as high as the 99.9 percent) criterion.

Mathematical Representations of the Production Functions

The graphical or "form-free" representations for the various production functions produced by the S/T Block (for the system as originally conceived) were presented in the previous section. The matrices for volume and TSS control and TSS removal were also fit to the Cobb-Douglas, Mitscherlich, and quadratic equations with the appropriate least squares method (see Chapter 5). For TSS removal, the expanded-range results shown in Figure 7-11 were used instead of the full field shown in Figure 7-10. The estimated coefficients for each functional form are given in Table 7-1, along with the mean of the absolute deviations and the corresponding coefficients of variation. The use of the mean absolute deviation to judge the fit of each equation to the date was discussed in Chapter 5. The estimated functions are displayed along with the corresponding graphical representations in Figures 7-22 through 7-30.

Of the functional forms considered, the absolute deviations displayed in Table 7-1 suggest that the quadratic equation provides the best representation of the volume control and TSS control production functions and that the Mitscherlich equation gives the best representation of the TSS removal values. However, visual inspection of Figures 7-22 through 7-27 would probably favor the Cobb-Douglas equation for the volume and TSS control functions. The Cobb-Douglas equation agrees more favorably with the general shape of these production functions and appears to be overly penalized by the mean absolute deviation criterion for its poor reproduction of the higher-valued isoquants. Visual inspection of the TSS removal results (Figures 7-28, 7-29, and 7-30) seems to confirm the selection of the Mitscherlich function by the mean absolute

Table 7-1. Coefficients for the Cobb-Douglas, Mitscherlich, and Quadratic Equations Representing the Production Functions Produced by the S/T Block

	Cobb-Douglas				Mitscherlich				Quadratic						
Type	A	β_1	β_2	MAD[a] (CVAD)[b]	A	β_1	β_2	MAD[a] (CVAD)[b]	β_0	β_1	β_2	β_3	β_4	β_5	MAD[a] (CVAD)[b]
Volume control	574.22	0.2572	0.3152	4.724 (1.031)	82.50	1418.41	33.082	6.160 (0.919)	6.889	9177.33	412.86	-513949	-765.05	-3562.72	3.188 (0.849)
Pollutant control	632.53	0.2299	0.4079	5.787 (1.074)	92.51	1837.41	20.634	6.837 (0.825)	3.031	7985.80	541.91	-424715	-1007.11	-4765.64	3.256 (0.887)
Pollutant removal	719.79	0.2525	0.6254	4.887 (0.865)	62.86	3283.87	10.348	2.463 (0.618)	-5.462	33403.41	246.41	-15410920	-579.47	75266.84	4.236 (0.630)

a. Mean absolute deviation.

b. Coefficient of variation of the absolute deviations.

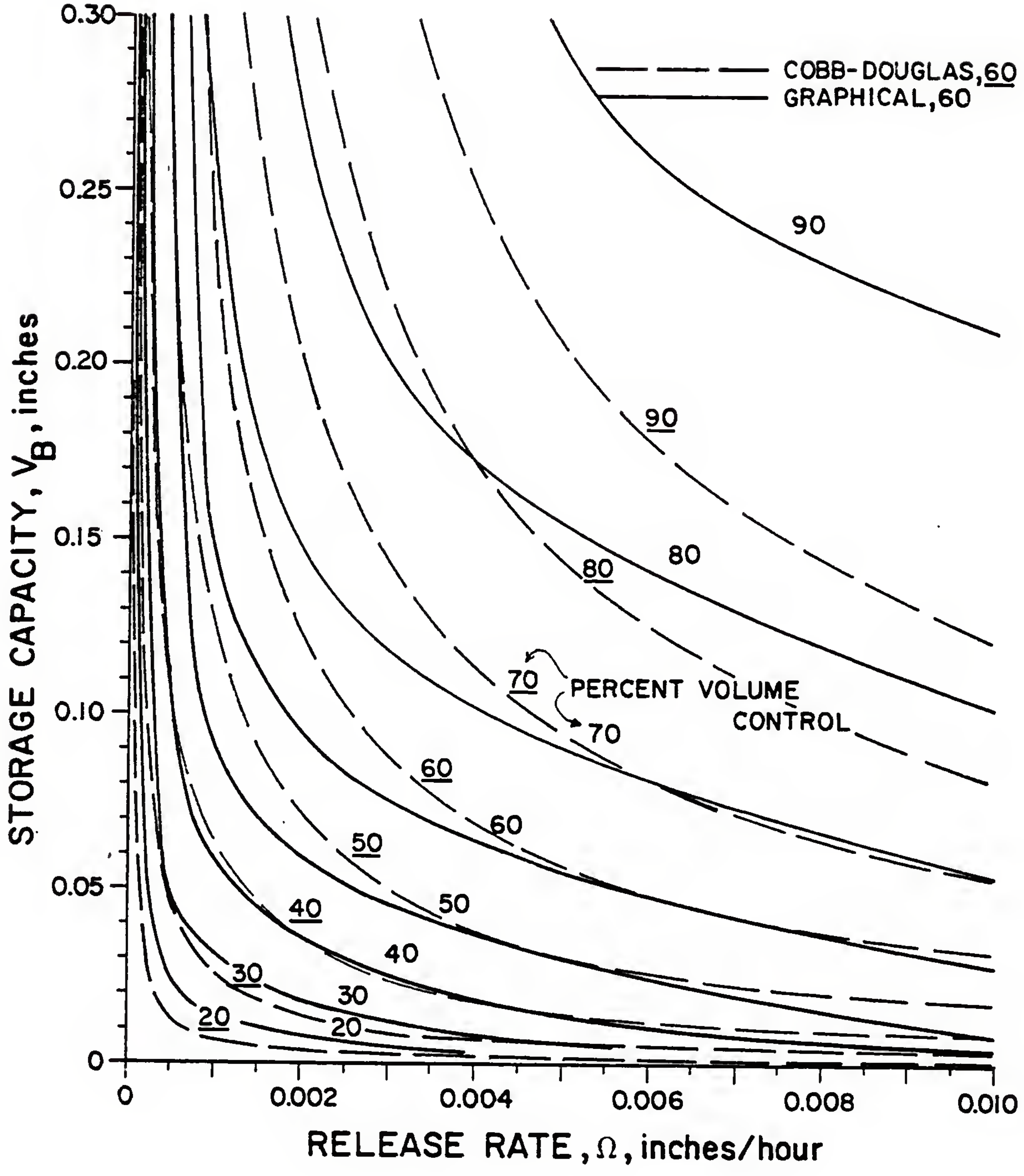


Figure 7-22. Percent Volume Control Production Function, Represented by the Cobb-Douglas Equation.

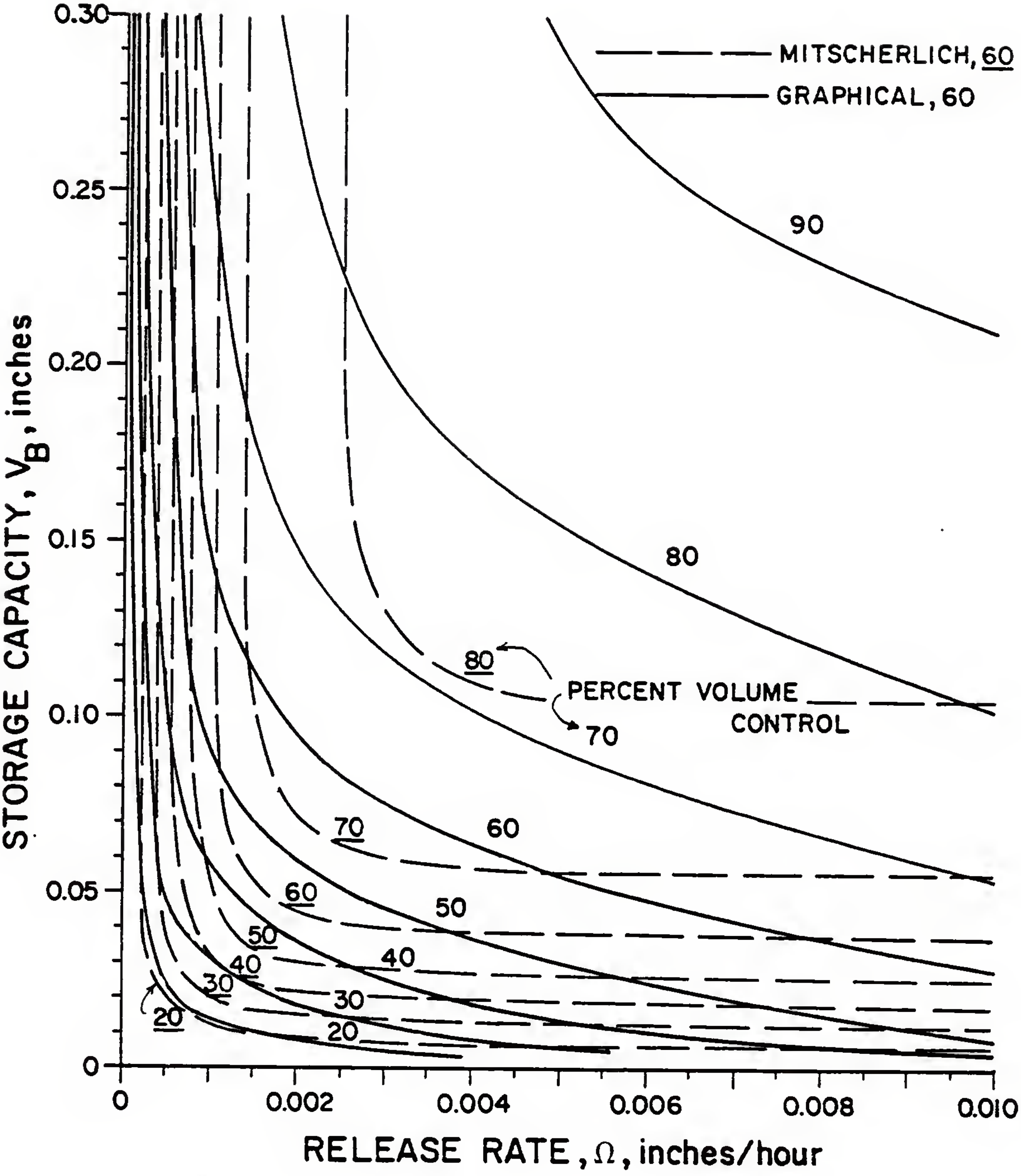


Figure 7-23. Percent Volume Control Production Function, Represented by the Mitscherlich Equation.

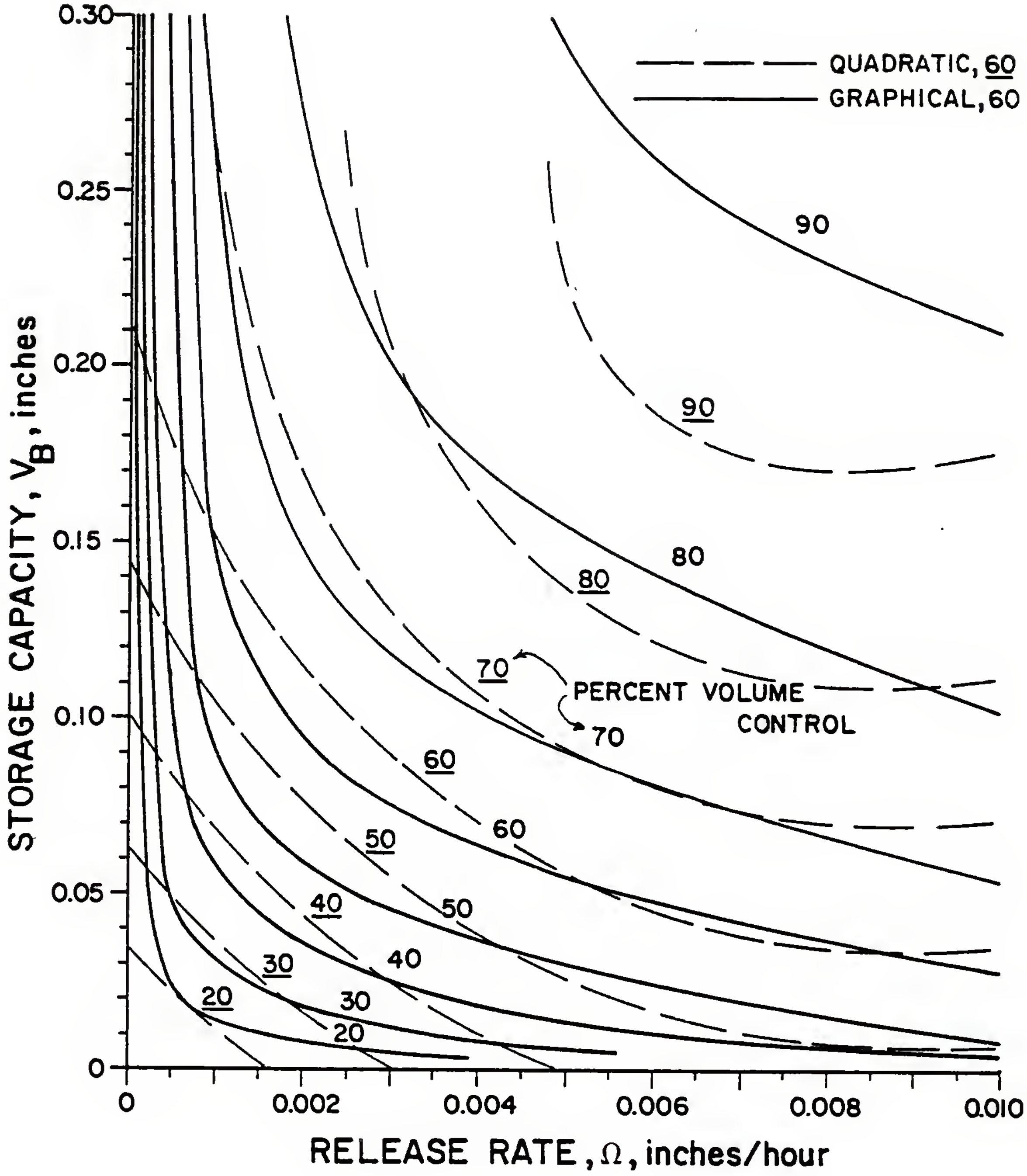


Figure 7-24. Percent Volume Control Production Function, Represented by the Quadratic Equation.

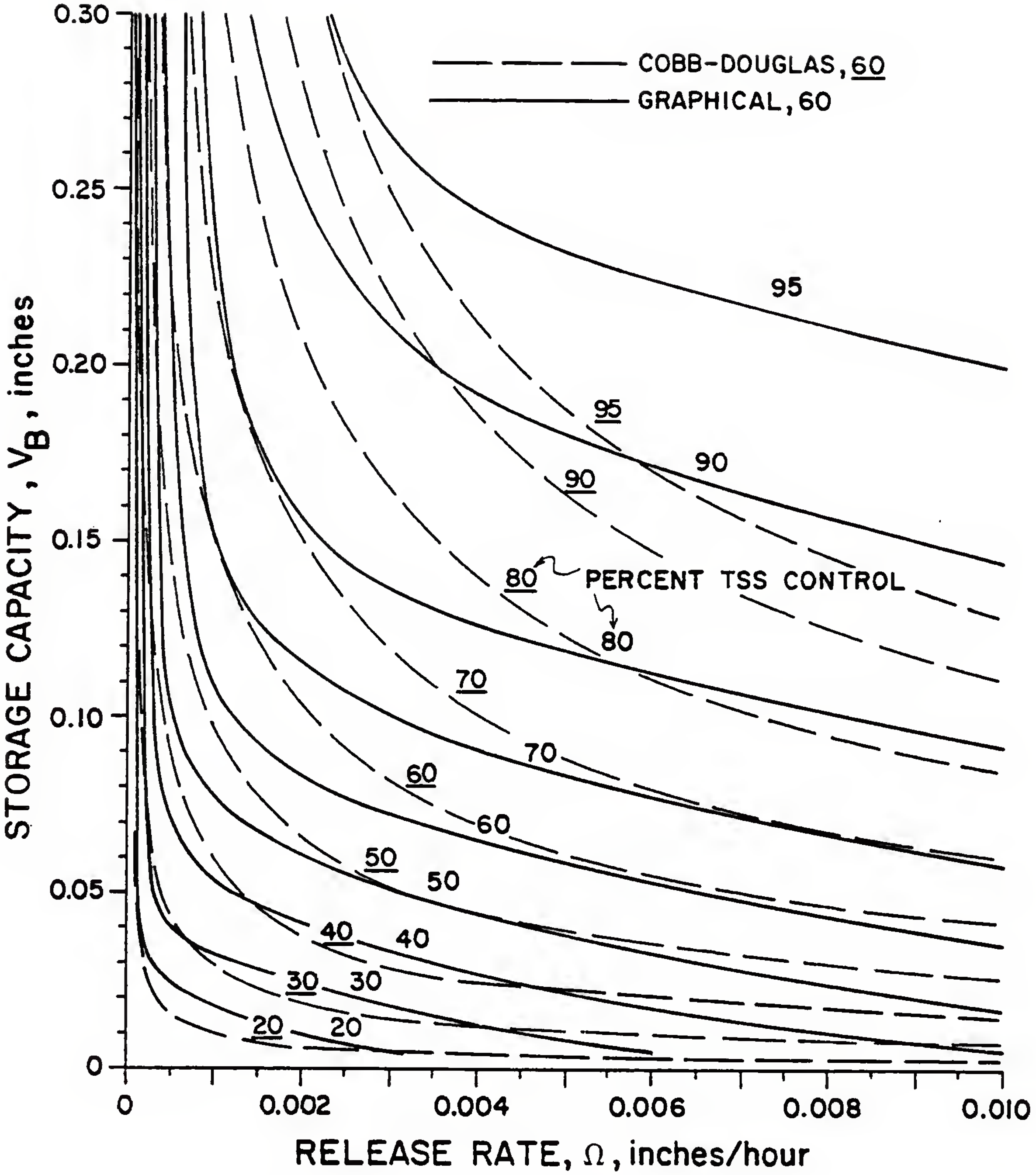


Figure 7-25. Percent TSS Control Production Function, Represented by the Cobb-Douglas Equation.

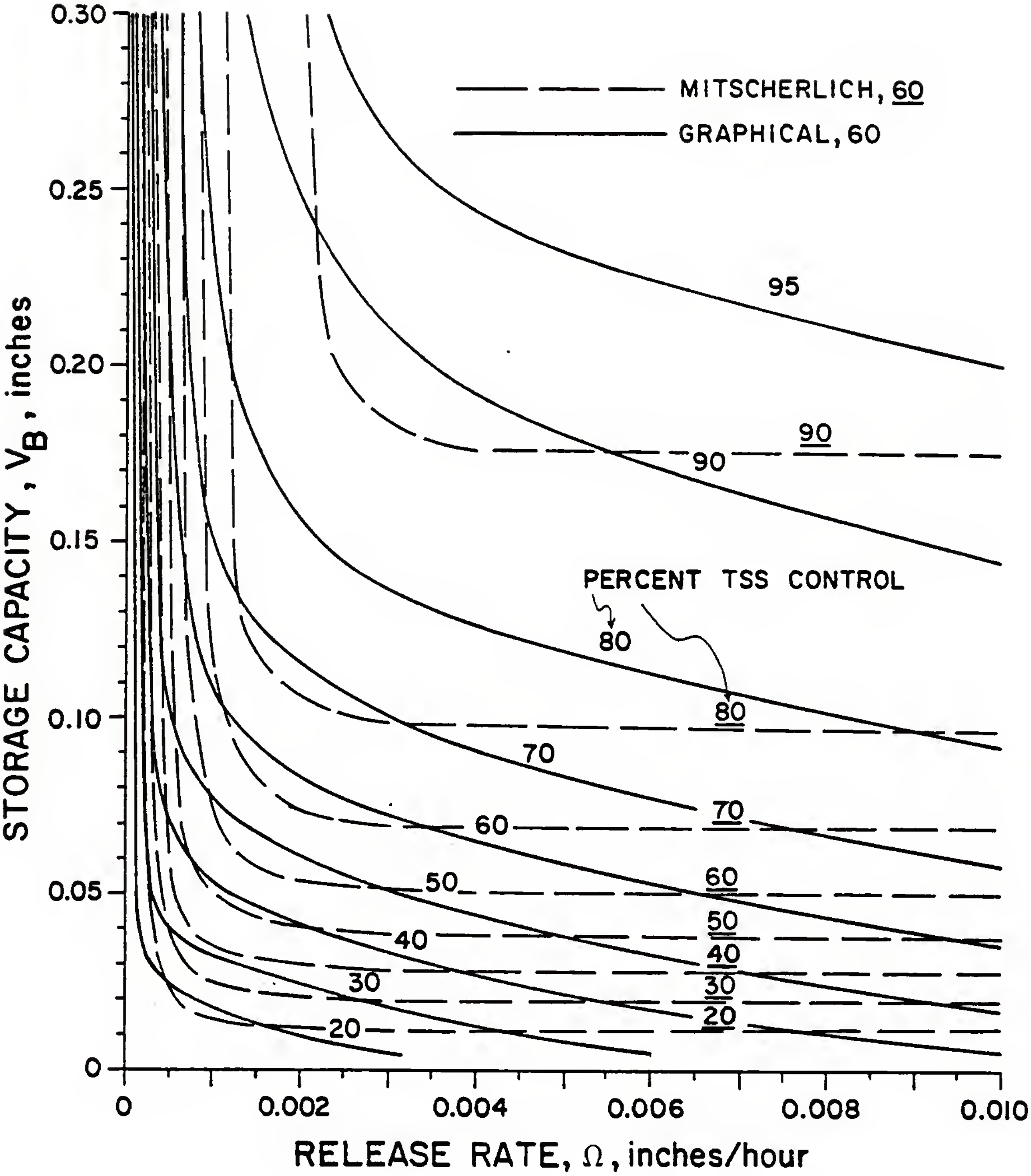


Figure 7-26. Percent TSS Control Production Function, Represented by the Mitscherlich Equation.

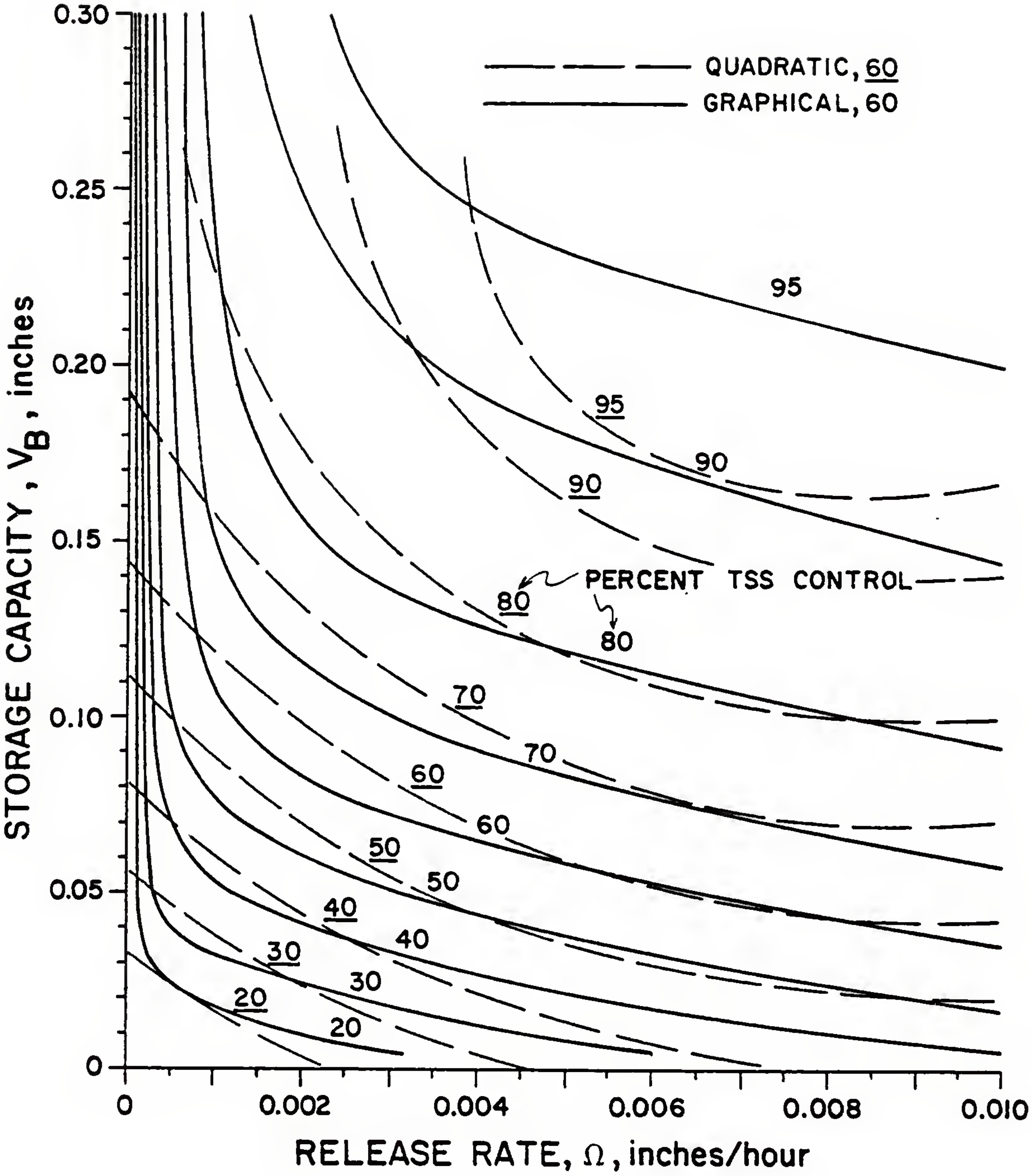


Figure 7-27. Percent TSS Control Production Function, Represented by the Quadratic Equation.

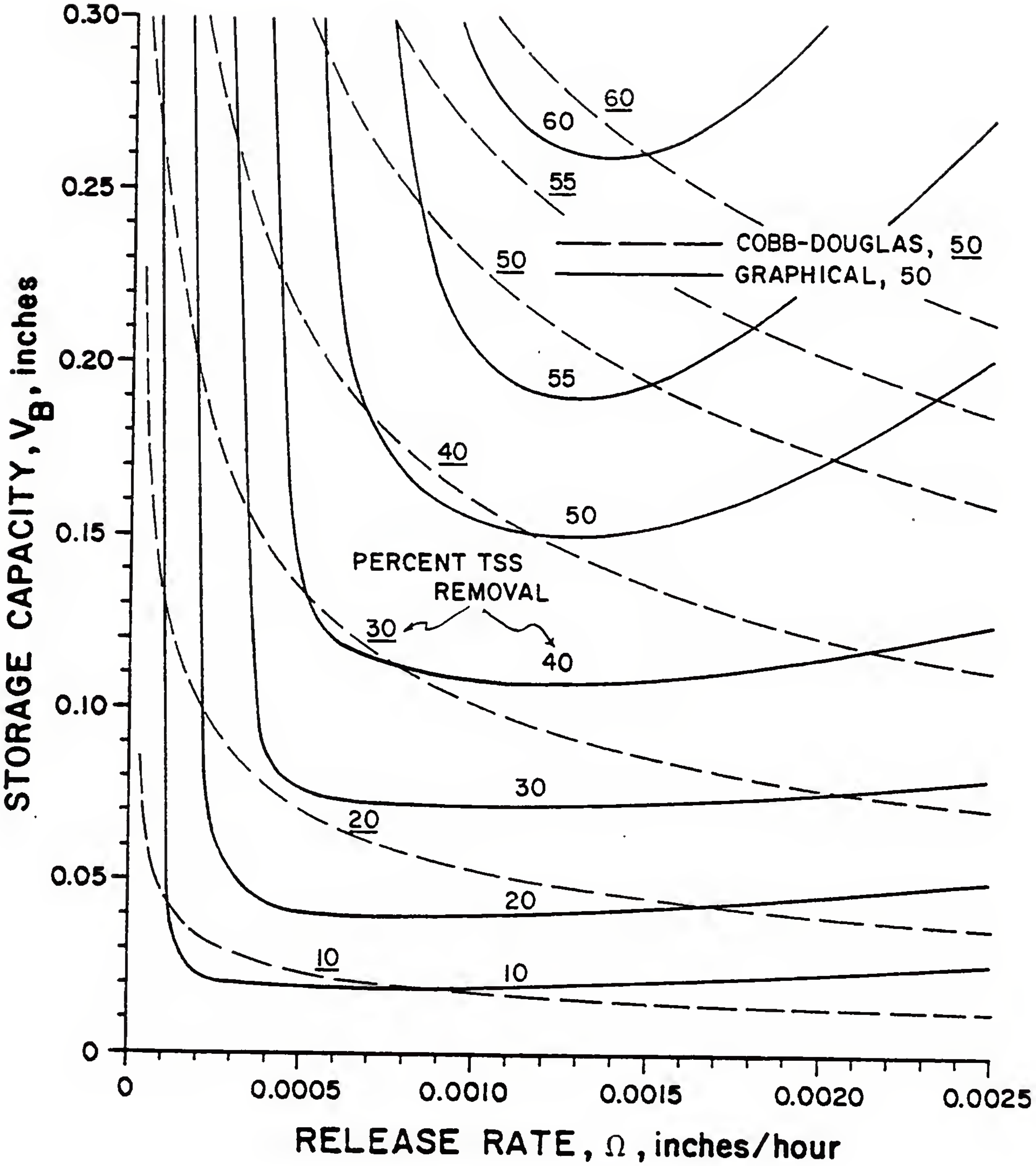


Figure 7-28. Percent TSS Removal Production Function, Represented by the Cobb-Douglas Equation.

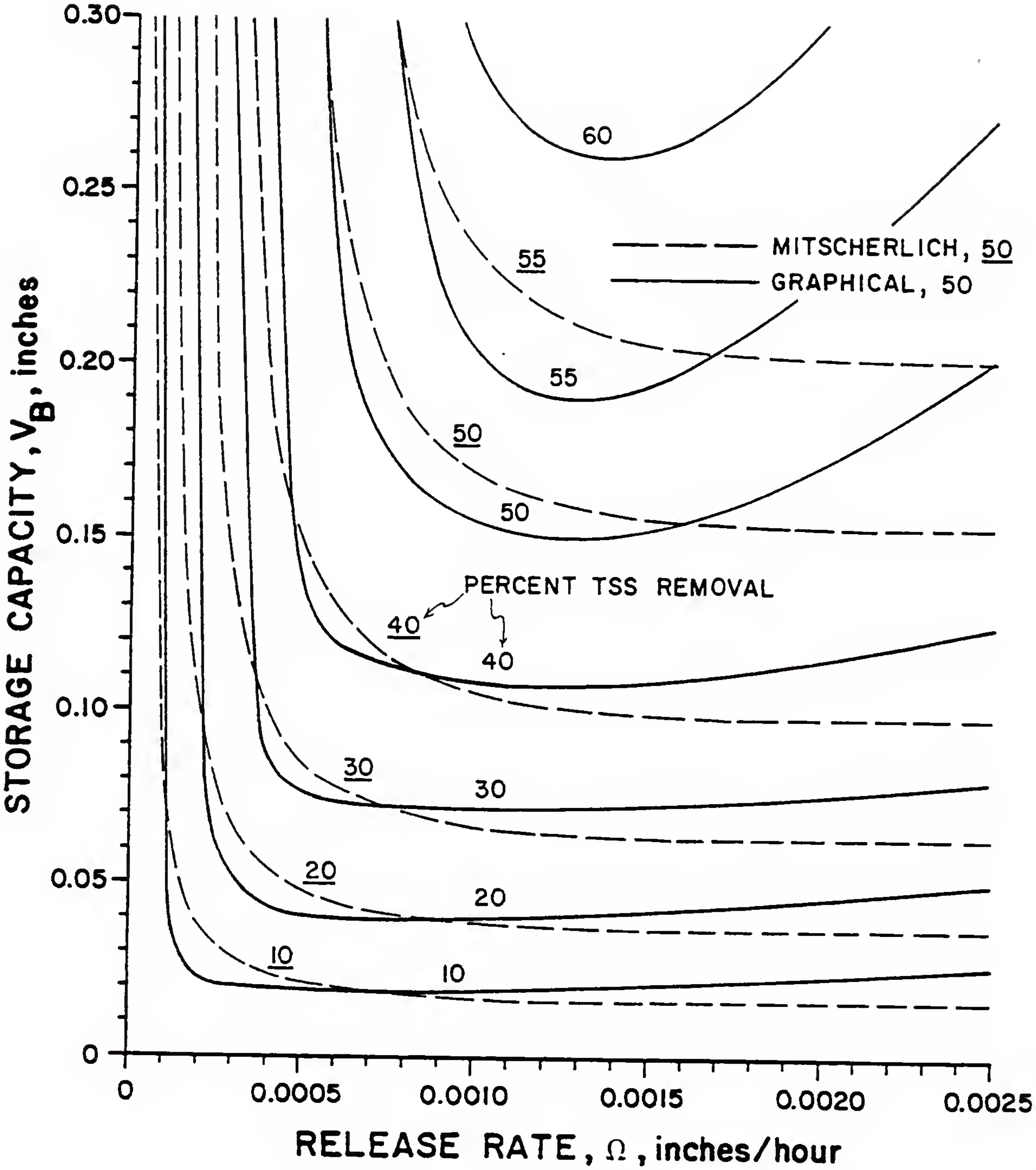


Figure 7-29. Percent TSS Removal Production Function, Represented by the Mitscherlich Equation.

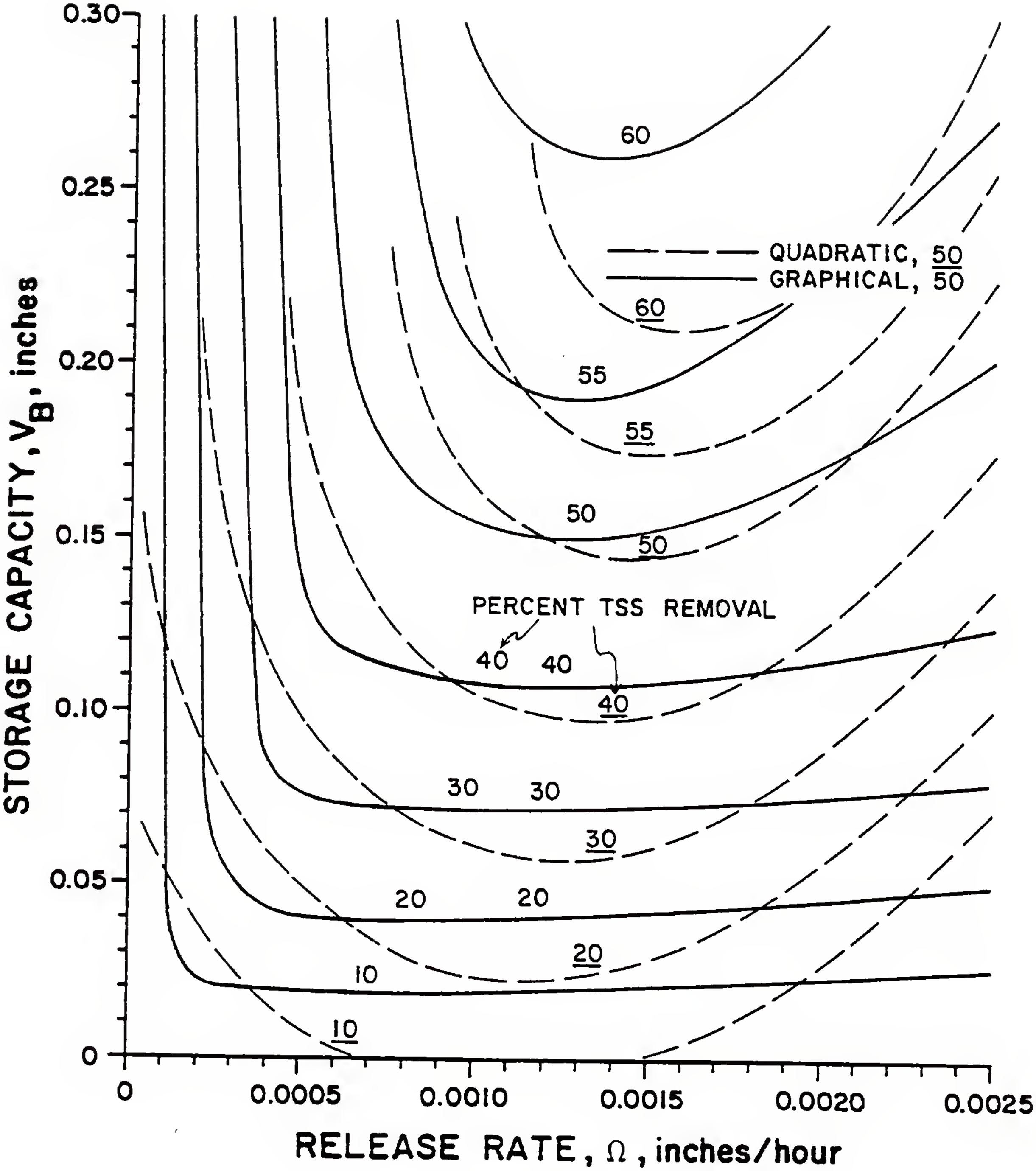


Figure 7-30. Percent TSS Removal Production Function, Represented by the Quadratic Equation.

deviation criterion; although none of the functional forms appears to be a very good representation. However, the region of interest is the area of substitution (see Chapter 5) and within this area the Mitscherlich equation provides a credible representation. If the points outside the area of substitution were excluded, some improvement might be expected for some or all of the equations. Considering all of the results, it appears that none of the functional forms is particularly well suited to the control or removal production functions.

System Optimization

Optimization with the Graphical and Functional Representations

The system optimization techniques discussed in Chapter 6 were utilized to gauge the appropriateness of the Cobb-Douglas, Mitscherlich, quadratic and graphical representations as tools in the economic analysis of storage/release systems. This was accomplished by comparing the expansion paths obtained by the graphical and functional representations with a linear cost equation and various cost coefficient ratios (i.e., c_1/c_2). The analysis was performed for the volume and TSS control representations and the TSS removal representations. The expansion paths for the graphical representations were derived by the production-space procedures described in Chapter 6. Expansion paths for the three functional representations were calculated by the simple classical (or analytical) optimization techniques also described in Chapter 6.

Three cost coefficient ratios, 1, 10, and 100, were used to derive the expansion paths. Recall that expansion paths developed with linear cost equations are equivalent to isoclines (or lines of equal marginal rates of substitution). The expansion paths for the graphical representations are shown in Figures 7-31, 7-32, and 7-33. Following each of these figures is a table (Table 7-2, 7-3, or 7-4) listing the optimal values of V_B and Ω, corresponding to several levels of control or removal as calculated by the simple analytical procedure appropriate to each of the three functional forms (see Chapter 6). The graphical results are also given in each table for comparative purposes. The differences between the analytical and graphical solution are expressed as the percent of error produced by the analytical solution when compared to the graphical solution. Since the graphical representations of the production functions are "form-free," it is natural that they should form the standard by which the functional representations are evaluated.

The comparison of "error percentages" can be misleading, but examination of these results produces a few general conclusions:

1) The Mitscherlich function produces the most accurate results for the TSS removal production function.
2) The Cobb-Douglas equation produces the most accurate results for the volume and TSS control production functions.
3) None of the functional forms produces very accurate results or performs very consistently.

Based on the previous discussion of the representational aptitude of the three functional forms, none of these conclusions is very surprising. However, it may be possible to use one of these forms as a workable, generalized representation of the performance of basins within a specific region or the U.S. as a whole (e.g., in a manner similar to the SWMM Level I procedures). For localized or more detailed analyses, a graphical approach is probably preferable. In fact, fitting functions to the simulation results appears to be counterproductive at this level.

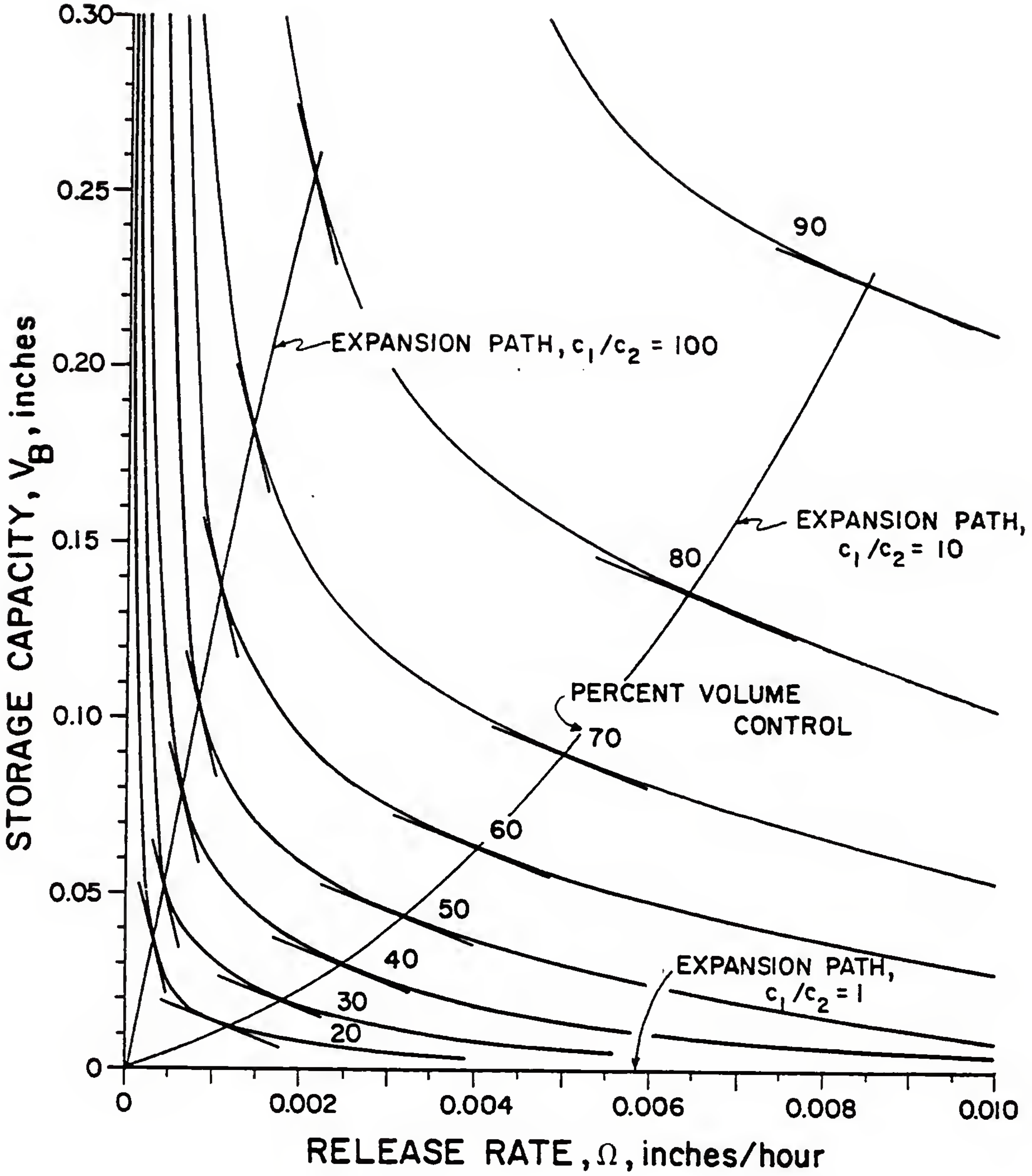


Figure 7-31. Expansion Paths for the Percent Volume Control Production Function, Graphical Representation.

Table 7-2. Optimal Storage/Release Combinations for Volume Control

Percent Control	$\frac{c_1}{c_2}$	Optimal Storage/Release Combination[a] (and Percent Error from Graphical Solution)							
		Graphical		Cobb-Douglas		Quadratic		Mitscherlich	
		V_B	Ω	V_B	Ω	V_B	Ω	V_B	Ω
20	1	b	b	0.0031(-)	0.00254(-)	0.0^c (-)	0.00157(-)	0.0091(-)	0.00190(-)
	10	0.012	0.0011	0.0087(-28)	0.00071(-25)	0.0^c (-100)	0.00157(43)	0.0130 (8)	0.00083(-25)
	100	0.037	0.0003	0.0246(-34)	0.00020(-33)	0.0339(-8)	0.0^c	0.0283(-24)	0.00036(20)
30	1	b	b	0.0063(-)	0.00515(-)	0.0^c (-)	0.00303(-)	0.0144(-)	0.00226(-)
	10	0.020	0.0018	0.0178(-11)	0.00145(-19)	0.0^c (-100)	0.00303(68)	0.0188(-6)	0.00108(-40)
	100	0.054	0.0004	0.0500(-7)	0.00041(3)	0.0634(17)	0.0^c (-100)	0.0375(-31)	0.00051(28)
50	1	b	b	0.0154(-)	0.01257(-)	0.0059(-)	0.00854(-)	0.0289(-)	0.00294(-)
	10	0.043	0.0031	0.0433(1)	0.00354(14)	0.0246(-43)	0.00521(68)	0.0340(-21)	0.00161(-48)
	100	0.107	0.0008	0.1220(14)	0.00100(25)	0.1415(32)	0.0^c (-100)	0.0578(-46)	0.00088(10)
70	1	b	b	0.0277(-)	0.02263(-)	0.0673(-)	0.00842(-)	0.0578(-)	0.00383(-)
	10	0.092	0.0050	0.0780(-15)	0.00637(27)	0.0818(-11)	0.00587(17)	0.0633(-31)	0.00242(-52)
	100	0.185	0.0014	0.2196(19)	0.00179(28)	0.2229(20)	0.00117(-16)	0.0911(-51)	0.00157(12)
90	1	b	b	0.0430(-)	0.03510(-)	0.1647(-)	0.00823(-)	d	d
	10	0.224	0.0084	0.1210(-46)	0.00988(18)	0.1727(-23)	0.00691(-18)	d	d
	100	b	b	0.3407(-)	0.00278(-)	0.2474(-)	0.00475(-)	d	d

a. V_B is in inches and Ω is in inches/hour.
b. Unable to determine the solution because it is out of range.
c. The computed solution is negative; the actual solution is located where the isoquant intersects the nearest axis.
d. The equation reached its maximum at a value less than 90 percent.

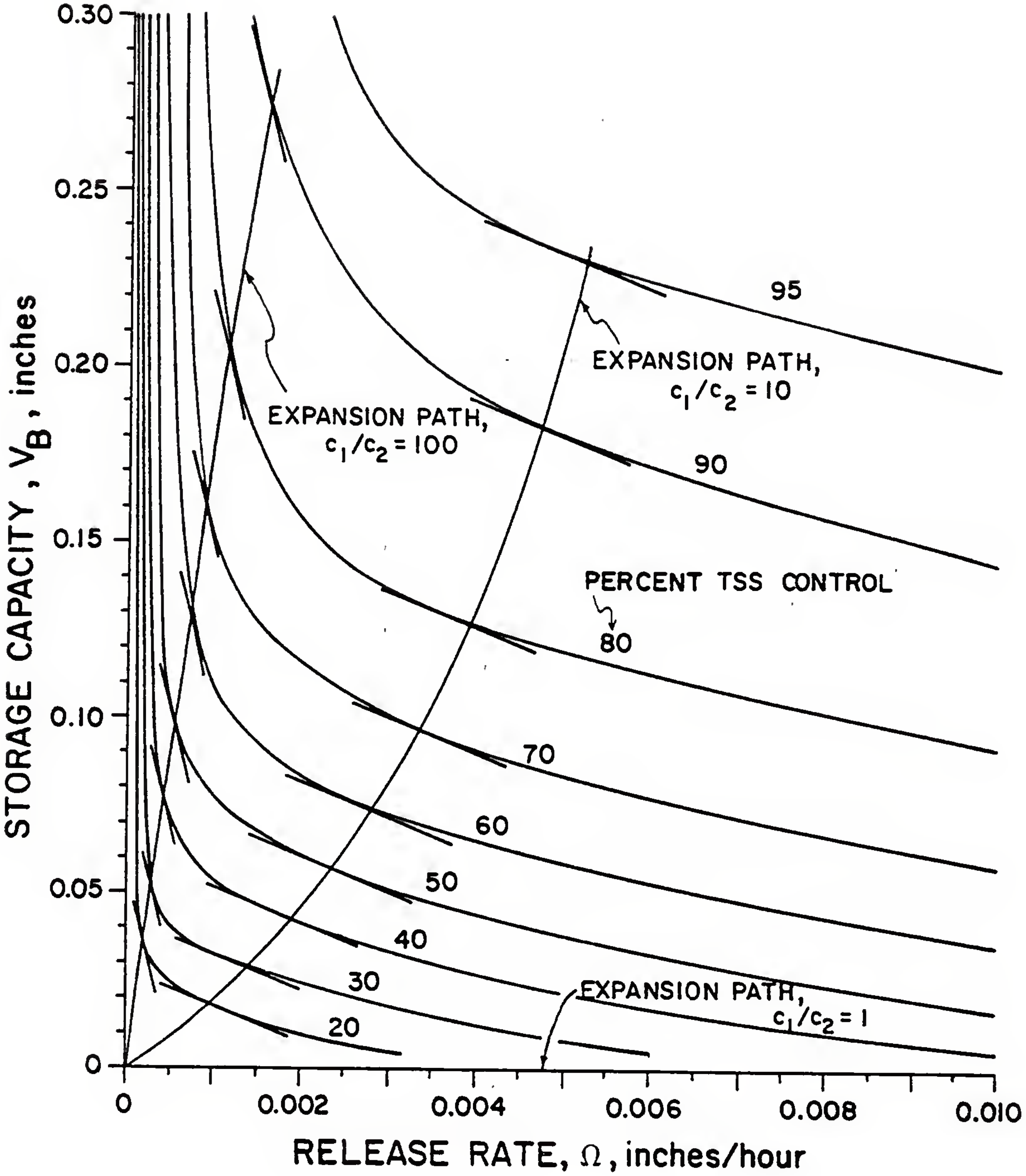


Figure 7-32. Expansion Paths for the Percent TSS Control Production Function, Graphical Representation.

Table 7-3. Optimal Storage/Release Combinations for TSS Control

Percent Control	$\frac{c_1}{c_2}$	Optimal Storage/Release Combination[a] (and Percent Error from Graphical Solution)							
		Graphical		Cobb-Douglas		Quadratic		Mitscherlich	
		V_B	Ω	V_B	Ω	V_B	Ω	V_B	Ω
20	1	b	b	0.0055(-)	0.00308(-)	0.0^c (-)	0.00245(-1)	0.0124(-)	0.00172(-)
	10	0.017	0.0009	0.0125(-26)	0.00071(-21)	0.0^c(-100)	0.00245(172)	0.0159(-6)	0.00081(-10)
	100	0.033	0.0002	0.0288(-13)	0.00016(-20)	0.0334(1)	0.0^c (-100)	0.0317(-4)	0.00033(65)
30	1	b	b	0.0103(-)	0.00582(-)	0.0^c (-)	0.00441(-)	0.0196(-)	0.00201(-)
	10	0.028	0.0014	0.0237(-15)	0.00133(-5)	0.0108(-61)	0.00324(131)	0.0235(-16)	0.00101(-28)
	100	0.050	0.0003	0.0543(9)	0.00031(3)	0.0555(11)	0.0^c (-100)	0.0425(-15)	0.00044(47)
50	1	b	b	0.0230(-)	0.01296(-)	0.0200(-)	0.00875(-)	0.0383(-)	0.00248(-)
	10	0.057	0.0023	0.0527(-8)	0.00297(-29)	0.0469(-18)	0.00389(69)	0.0425(-25)	0.00141(-39)
	100	0.098	0.0005	0.1210(23)	0.00068(36)	0.1086(11)	0.0^c (-100)	0.0666(-32)	0.00070(40)
70	1	b	b	0.0390(-)	0.02197(-)	0.0689(-)	0.00859(-)	0.0691(-)	0.00302(-)
	10	0.097	0.0034	0.0894(-8)	0.00504(48)	0.0907(-6)	0.00469(38)	0.0736(-24)	0.00189(-44)
	100	0.160	0.0009	0.2050(28)	0.00116(29)	0.1923(20)	0.0^c (-100)	0.1014(-37)	0.00108(20)
90	1	b	b	0.0578(-)	0.03258(-)	0.1371(-)	0.00837(-)	0.1755(-)	0.00438(-)
	10	0.182	0.0047	0.1326(-27)	0.00747(59)	0.5121(-16)	0.00581(24)	0.1800(-1)	0.00320(-32)
	100	0.277	0.0016	0.3040(10)	0.00171(7)	0.2459(-11)	0.00255(59)	0.2110(-24)	0.00231(44)

a. V_B is in inches and Ω is in inches/hour.
b. Unable to determine the solution because it is out of range.
c. The computed solution is negative; the actual solution is located where the isoquant intersects the nearest axis.

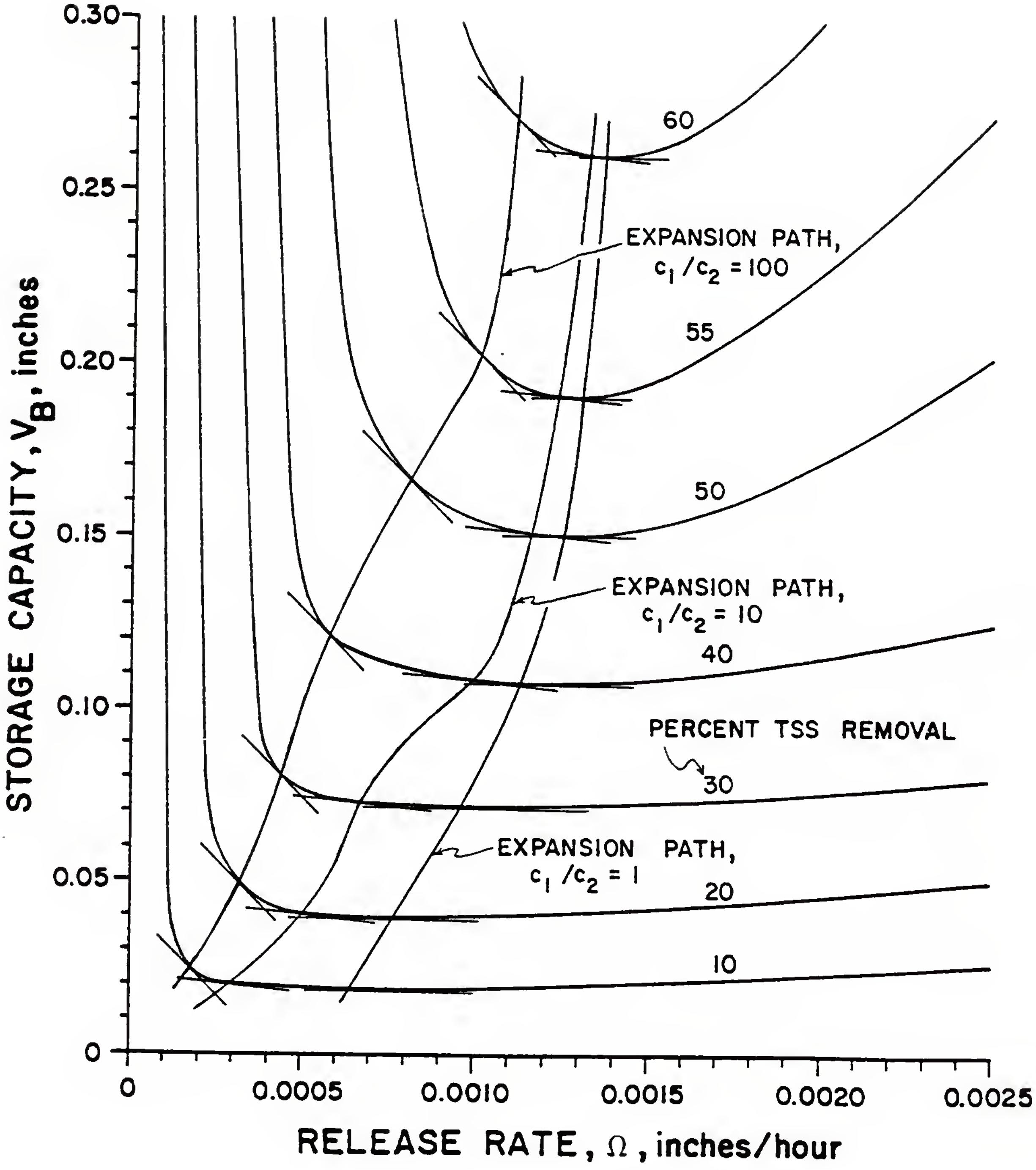


Figure 7-33. Expansion Paths for the Percent TSS Removal Production Function, Graphical Representation.

Table 7-4. Optimal Storage/Release Combinations for TSS Removal

Percent Removal	$\frac{c_1}{c_2}$	Optimal Storage/Release Combination[a] (and Percent Error from Graphical Solution)							
		Graphical		Cobb-Douglas		Quadratic		Mitscherlich	
		V_B	Ω	V_B	Ω	V_B	Ω	V_B	Ω
20	1	0.040	0.00077	0.0219(-45)	0.00885(1062)	0.0233(-42)	0.00113(47)	0.0374(-7)	0.00145(88)
	10	0.041	0.00051	0.0425(4)	0.00172(237)	0.0235(-43)	0.00107(110)	0.0399(-3)	0.00086(69)
	100	0.049	0.00032	0.0824(68)	0.00033(3)	0.0443(-10)	0.00056(75)	0.0567(16)	0.00038(19)
30	1	0.072	0.00097	0.0348(-52)	0.01404(1347)	0.0579(-20)	0.00122(26)	0.0630(-13)	0.00171(76)
	10	0.073	0.00068	0.0674(-8)	0.00272(300)	0.0581(-20)	0.00116(71)	0.0656(-10)	0.00105(54)
	100	0.081	0.00043	0.1308(61)	0.00053(23)	0.0739(-9)	0.00074(72)	0.0849(5)	0.00052(21)
40	1	0.108	0.00113	0.0483(-55)	0.01948(1624)	0.0976(-10)	0.00131(16)	0.0981(-9)	0.00189(67)
	10	0.109	0.00100	0.0936(-14)	0.00378(278)	0.0977(-10)	0.00128(28)	0.1008(-8)	0.00123(23)
	100	0.121	0.00058	0.1815(50)	0.00073(26)	0.1081(-11)	0.00095(64)	0.1217(1)	0.00067(16)
50	1	0.150	0.00126	0.0622(-59)	0.02512(1894)	0.1453(-3)	0.00143(13)	0.1537(2)	0.00213(69)
	10	0.151	0.00116	0.1207(-20)	0.00487(320)	0.1454(-4)	0.00141(22)	0.1564(4)	0.00147(27)
	100	0.167	0.00081	0.2340(40)	0.00094(16)	0.1500(-10)	0.00121(49)	0.1785(7)	0.00088(9)
60	1	0.260	0.00137	0.0766(-71)	0.03092(2157)	0.2099(-19)	0.00159(16)	0.2988(15)	0.00266(94)
	10	0.260	0.00133	0.1485(-43)	0.00600(351)	0.2099(-19)	0.00159(20)	0.3015(16)	0.00199(50)
	100	0.271	0.00111	0.2880(6)	0.00116(5)	0.2099(-23)	0.00158(42)	0.3247(20)	0.00134(23)

a. V_B is in inches and Ω is in inches/hour.

Graphical "Cost-Space" Optimization

The graphical representation of the TSS removal production function, given by Figure 7-11, was used to demonstrate the graphical "cost-space" optimization procedure described in Chapter 6. The cost curves presented in Chapter 6 (Figure 6-4) for earthen basins were used to develop the cost curve for V_B. The construction cost curve was amortized over 20 years at an interest rate of 8 percent. The operation and maintenance cost curve was added to the amortized curve. The resulting annual cost curve for V_B is shown in quadrant II of Figure 7-34. The cost curve for the release rate, Ω, is shown in quadrant IV of Figure 7-34. This curve was arbitrarily chosen to insure that the expansion path would be well within the domain of the displayed production function. This strongly suggests that the cost information should play a role in selecting the ranges over which the simulator should be operated. This could be done by using the SWMM Level I procedure and/or the Hydroscience method along with the relevant cost information to develop preliminary expansion paths and, thus, provide more assurance that the simulation work will not be wasted or need to be extensively (and expensively) enhanced.

Since the cost functions are nonlinear, the cost-space technique is preferred to the production-space method. At the heart of this procedure is the transfer of the production function from production space to cost space. This was accomplished by mapping the production-space isoquants shown in quadrant III of Figure 7-34 through the cost curves into the cost space provided by quadrant I. In cost space, the isocost relationship is a simple linear function, i.e.,

$$Z_2 = Z - Z_1 \tag{7-4}$$

where Z = total annual cost, dollars/year,

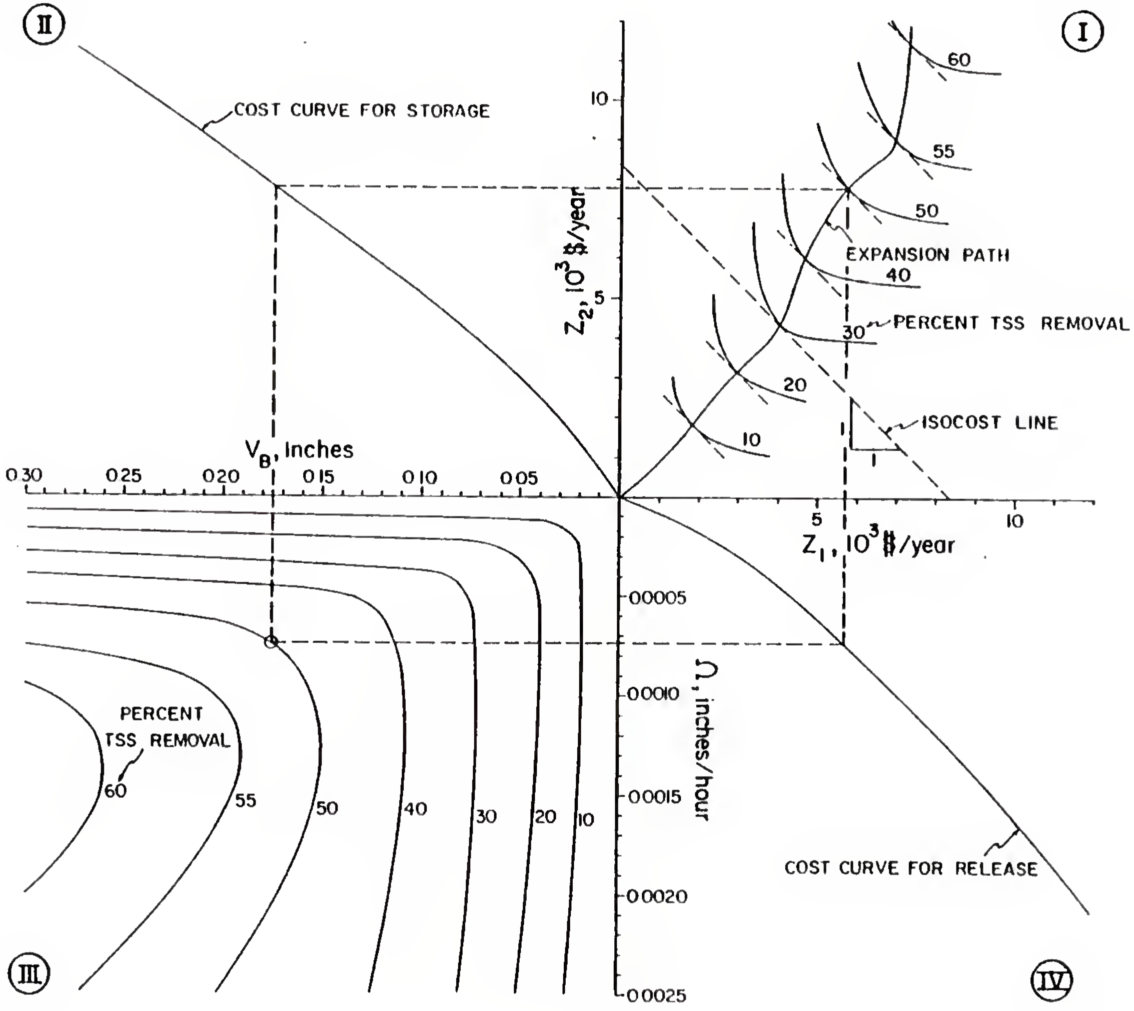


Figure 7-34. Application of Cost-Space Optimization Procedure.

Z_1 = annual cost of input Ω, dollars/year, and

Z_2 = annual cost of input V_B, dollars/year.

This isocost line was used to produce the expansion path shown in quadrant I by locating the point on each isoquant tangent to a line with a slope of -1 and connecting the resulting points. From the expansion path, the optimal cost of achieving a range of pollutant removal levels was obtained and plotted as shown in Figure 7-35. This curve becomes the input for a decision concerning the desired level of pollutant removal. In effect, it and Figure 7-34 summarizes the entire analysis. Once a particular level is selected, Figure 7-34 can be used to quickly extract the desired design. For example, if the 50 percent pollutant level is selected, the corresponding point on the expansion path is located and the design values of Ω and V_B found by reversing the mapping procedure used to develop the cost-space isoquants (see Figure 7-34). In this case, the resulting values of Ω and V_B are 0.00073 inches/year and 0.175 inches, respectively.

Summary

Many of the procedures developed and/or presented in the previous chapters were demonstrated through the use of a simple, hypothetical storage/release system. This simple application also allowed several general observations to be made concerning the performance of storage/release systems in urban stormwater quality management. Particularly close attention was paid to the volume and pollutant control and pollutant removal production functions generated by the SWMM Storage/Treatment Block. The Cobb-Douglas, Mitscherlich, and quadratic equations were also used to represent these production functions. The simple

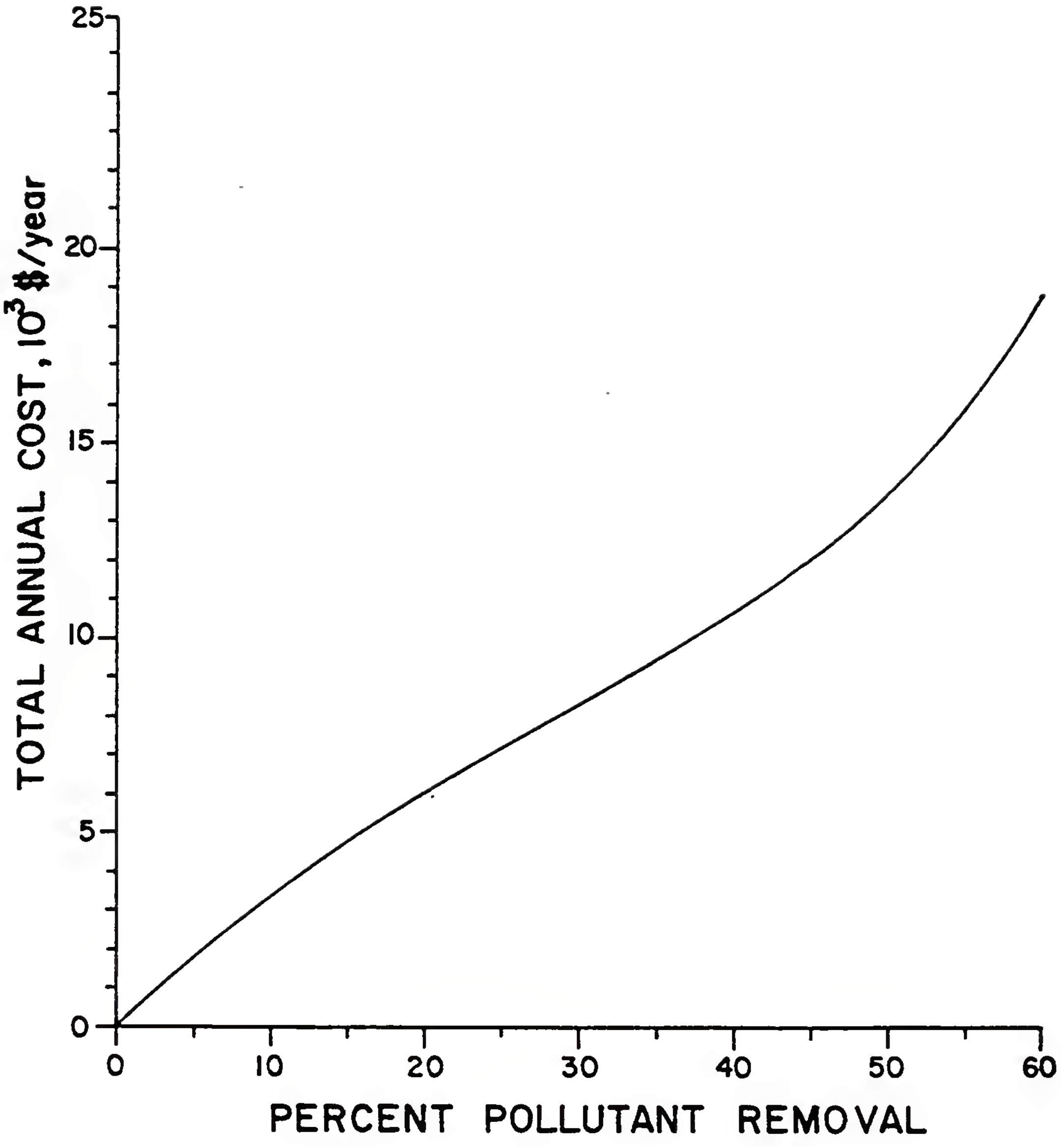


Figure 7-35. Final Cost Curve for Hypothetical Scenario.

storage/release system was modified in different ways to explore variations on the primary theme of the example. Also investigated was the effect of various pollutant settling characteristics. The graphical production-space optimization technique was used to demonstrate the merits or demerits of the functional forms. Finally, the graphical cost-space optimization technique was demonstrated with a set of partially hypothetical cost data and the graphically derived production function for percent pollutant removal.

The sequence of operations was intended to suggest a general methodology for the design of urban stormwater storage/release systems. The actual methodology used in practice a function of

1) budgetary constraints;
2) the magnitude of the problem;
3) local, state, and federal regulations;
4) the availability of field and laboratory data;
5) the accessibility of computer facilities; and
6) the confidence of the analyst in the available procedures and techniques.

Because of these factors, the approach taken here was to exhibit several techniques in a logical sequence and, thus, suggest a very general methodology. The analyst can tailor the general methodology to the needs of the specific problem.

CHAPTER 8

SUMMARY AND CONCLUSIONS

A comprehensive approach to the analysis and design of storage/release systems for use in urban stormwater quality management has been presented. A review of theoretical foundations served as a prelude to a discussion of techniques capable of determining the long-term performance as a function of one or more design parameters. Unfortunately, most of these techniques are inadequate in terms of sophistication or flexibility. To meet the need for a better analytical tool, a computer simulator known as the Storm Water Management Model (SWMM) Storage/Treatment Block was developed as part of the larger SWMM model (Huber et al., 1981) and this study. Considerable attention was also given to the statistical method developed by Hydroscience, Inc. (1979) and DiToro and Small (1979). In general, performance was classified in terms of volume, time, and occurrence-based reliability.

Emphasis was placed on long-term analyses rather than the more traditional design storm or single event approach. While the design storm method has been very useful in the design of flood control structures and drainage networks, it is inappropriate for stormwater quality problems. The production function, a concept taken from economic theory, was introduced as a medium through which long-term performance could be displayed as a function of one or more design parameters or inputs (e.g., storage basin capacity and release rate). The properties of various types of production functions were discussed, as well as several potentially useful mathematical representations. Graphical and other "form-free" representations were also discussed.

Simple analytical and graphical optimization techniques, which use the information provided by the production function, were employed to determine the most cost-effective designs over a wide range of performance. This information forms the base from which the appropriate performance level and design can be selected. Essentially, the approach taken in this study maintains the original spirit of the design storm method by searching a wide range of possibilities for the "best" solution. In more recent years, the design storm has come to be associated with a pre-selected set of conditions, in the form of a single storm with a specified set of characteristics, about which a design is concocted. In other words, the design storm has become, in many applications, a design standard rather than the result of a more comprehensive technique.

A hypothetical case study illustrated the general approach and provided a vehicle through which the behavior of storage/release systems could be explored. Several of the evaluation techniques, including the Storage/Treatment Block, were set up to produce a series of production functions relating long-term volume and pollutant control and pollutant removal to the storage basin capacity and a constant release rate. Several functional forms were fit to the production functions generated by the Storage/Treatment Block and tested for their appropriateness. These mathematical representations, along with simple anlytical optimization techniques, were also tested for their ability to reproduce the results generated by a "production-space" graphical procedure for different linear cost coefficients. With a given set of cost data, a "cost-space" graphical procedure was used to develop a final cost curve as a function of pollutant removal. The sensitivity of the Storage/Treatment Block to different modeling assumptions was also explored.

Several conclusions concerning the analysis and design of storage/release systems within the context of urban stormwater quality management are in order.

1) Storage/release systems operating under the conditions presented by urban stormwater are complex entities in which temporal variabilities can not be ignored.
2) It is much more valuable to analyze these systems in a comprehensive, yet relatively simple, fashion than to explore the theoretical nuances of one particular facet of the problem. Up to this time, little progress has been made toward understanding the general behavior of stormwater storage/release systems, much less the intricacies.
3) Simple estimators of the long-term pollutant removal capabilities of storage/release systems should be carefully scrutinized. For example, some of these methods are based on the ratio of the storage capacity and the release rate or the average inflow to the system. The production functions generated and displayed throughout this study demonstrate that the relationship between performance and design is much more complex.
4) The concept of detention time has been frequently misused in the field of stormwater management. This appears to have stemmed from the emphasis placed on steady-state analysis in the training of sanitary and environmental engineers. The calculation of detention time under unsteady-state conditions is

relatively complicated and should not be performed with steady-state assumptions.

5) Statistical and computer simulation models are complements to, not replacements for, monitoring and pilot plant programs and treatability studies. It is equally important to realize that the reverse is also true. Collecting stormwater of any kind is an expensive, time-consuming proposition; thus, the development of a data base large enough to singularly make conclusive observations is unlikely. However, models that are calibrated to a limited data base can be used to substantially fill the void.

6) Analytical optimization techniques that require mathematical representations of the production and cost functions may be inappropriate for local design problems. The process of fitting functional forms can destroy much of the information contained in "form-free" or graphical representations and even some of the simpler analytical techniques are unwieldy in certain situations. Graphical techniques, while potentially tedious, are easier to understand and are more enlightening. However, analytical techniques might be very useful in regional or national assessments or in nomographic procedures developed for preliminary evaluations of local situations.

7) There are several levels of analysis applicable to storage/release systems. Simple evaluation techniques, preliminary system cost data, and simple optimization procedures can do much to minimize the expense and time committed to more sophisticated techniques (e.g., the Storage/Treatment Block).

In addition to the above conclusions, several recommendations for further research are listed below.

1) The evaluation techniques, especially the modeling algorithms in the SWMM Storage/Treatment Block, need to be verified for a substantial number of systems and improved when necessary.
2) Graphical optimization procedures appear to be useful in urban stormwater management. The use of computer graphics could greatly enhance their applicability.
3) More cost data, collected and analyzed in a consistent manner, are needed.
4) Urban stormwater storage/release systems commonly serve more than one purpose. Thus, the methods presented here should be integrated into a methodology that addresses the multi-purpose system.
5) The procedures presented in this study were directed toward the management of urban stormwater quality problems. However, the holistic nature of the approach and the ubiquity of storage/release systems in the hydrologic cycle suggest that these techniques are potentially useful to a wide range of water resources problems.

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BIOGRAPHICAL SKETCH

Stephan Jack Nix was born on July 20, 1952, in Forest, Mississippi. After brief stays in Mississippi and New York, he resided in Homestead, Florida, until graduating from South Dade High School in June, 1970. He entered the University of Florida in September, 1970, and was awarded a Bachelor of Science in Engineering Science in December, 1974. Mr. Nix entered the Graduate School of the University of Florida in January, 1975, and received the degree of Master of Engineering in environmental engineering sciences in December, 1976. Since that time, he has remained at the University of Florida to pursue the degree of Doctor of Philosophy in environmental engineering sciences. In addition to holding various graduate assistantships and research assistantships, Mr. Nix has been engaged by several consulting firms since 1977.

Mr. Nix is a member of Tau Beta Pi and Phi Kappa Phi honorary societies, the American Society of Civil Engineers, the Water Pollution Control Federation, and the American Water Resources Association. He is the author or co-author of more than fifteen research reports, conference presentations, and journal articles dealing with stormwater management. His most recent publications include "Storm Water Management Model User's Manual, Version III," with Huber et al., U.S. Environmental Protection Agency, 1981, and "Water Quality Benefits of Detention," with J.P. Heaney and W.C. Huber, Chapter 12 of "Urban Stormwater Management," American Public Works Association, 1981.

Mr. Nix has been married to Aulaire Bloski of Pensacola, Florida, since January, 1975. He also has two children, Jeffrey and Stephanie.

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

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I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

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I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

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